EXTERNALITIES AND PUBLIC GOODS

Externality

An <u>externality</u> occurs whenever the activities of one economic agent affect the activities of another economic agent in ways that are not reflected in market transactions

- chemical manufacturers releasing toxic fumes
- noise from airplanes
- motorists littering roadways

Interfirm Externalities

Consider two firms, one producing good *x* and the other producing good *y*

The production of *x* will have an external effect on the production of *y* if the output of *y* depends not only on the level of inputs chosen by the firm but on the level at which *x* is produced

$$y = f(k, l; x)$$

Negative externalities

Detrimental effect of x on the production of y

→ $\frac{dy}{dx} < 0$

Beneficial (positive) externalities

The relationship between the two firms can be beneficial. For example, two firms, one producing honey and the other producing apples

→ $\frac{dy}{dx} > 0$

Externalities in Utility

Externalities can also occur if the activities of an economic agent directly affect an individual's utility

externalities can directly decrease or increase utility

It is also possible for someone's utility to be dependent on the utility of another

$$utility = U_S(x_1, \ldots, x_n; U_J)$$

Public good externalties

Public goods are nonexclusive

- once they are produced, they provide benefits to an entire group
- it is impossible to restrict these benefits to the specific groups of individuals who pay for them

Externalities and Allocative Inefficiency

Externalities lead to inefficient allocations of resources because market prices do not accurately reflect the additional costs imposed on or the benefits provided to third parties

We can show this by using a general equilibrium model with only one individual and two firms, each one producing a different good

Suppose that the individual's utility function is given by

$$U(x_c, y_c)$$

where x_c and y_c are the levels of x and y consumed

The individual has initial stocks of x^* and y^*

He can consume them or use them in production

Assume that good x is produced using only good y according to

$$x_0 = f(y_i)$$

Assume that the output of good y depends on both the amount of x used in the production process and the amount of x produced

$$y_0 = g(x_i, x_0)$$

For example, y could be produced downriver from x and thus firm y must cope with any pollution that production of x creates

This implies that $g_1 > 0$ and $g_2 < 0$

The quantities of each good in this economy are constrained by the initial stocks available (x^* and y^*) and by the additional production that takes place (x_0 and y_0)

$$x_c + x_i = x_0 + x^*$$

$$y_c + y_i = y_0 + y^*$$

Finding the efficient allocation

The problem is to maximize the individual utility, subject to the four constraints:

$$\max_{x_c, y_c, x_i, y_i} U(x_c, y_c)$$

s.t.

$$x_0 = f(y_i)$$
$$y_0 = g(x_i, x_0)$$
$$x_c + x_i = x_0 + x^*$$
$$y_c + y_i = x_0 + y^*$$

The Lagrangian for this problem is

$$L = \\ = U(x_c, y_c) + \lambda_1[f(y_i) - x_0] + \lambda_2[g(x_i, x_0) - y_0] \\ + \lambda_3(x_c + x_i - x_0 - x^*) + \lambda_4(y_c + y_i - x_0 - y^*)$$

$$L =$$
$$= U(x_c, y_c) + \lambda_1[f(y_i) - x_0] + \lambda_2[g(x_i, x_0) - y_0\ ]$$
$$+ \lambda_3(x_c + x_i - x_0 - \ x^*) + \lambda_4(y_c + y_i - y_0 - y^*)$$

The six first-order conditions are:

1) $\frac{dL}{dx_c} = U_1 + \lambda_3 = 0$

2) $\frac{dL}{dy_c} = U_2 + \lambda_4 = 0$

3) $\frac{dL}{dx_i} = \lambda_2 g_1 + \lambda_3 = 0$

4) $\frac{dL}{dy_i} = \lambda_1 f_1 + \lambda_4 = 0$

5) $\frac{dL}{dx_0} = -\lambda_1 + \lambda_2 g_2 - \lambda_3 = 0$

6) $\frac{dL}{dy_0} = -\lambda_2 - \lambda_4 = 0$

Taking the ratio of the first two, we find

$$MRS = \frac{U_1}{U_2} = \frac{\lambda_3}{\lambda_4}$$

The third and sixth equation also imply that

$$MRS = \frac{\lambda_3}{\lambda_4} = \frac{\lambda_2 g_1}{\lambda_2} = g_1$$

Optimality in *y* production requires that the individual's *MRS* in consumption equals the marginal productivity of *x* in the production of *y*

To achieve efficiency in *x* production, we must also consider the externality this production poses to *y*

Combining the last three equations gives

$$MRS = \frac{\lambda_3}{\lambda_4} = \frac{-\lambda_1 + \lambda_2 g_2}{\lambda_4} = -\frac{\lambda_1}{\lambda_4} + \frac{\lambda_2 g_2}{\lambda_4}$$

$$MRS = \frac{1}{f_y} - g_2$$

This equation requires the individual's *MRS* to equal *dy*/*dx* obtained through *x* production

- $1/f_y$ represents the reciprocal of the marginal productivity of *y* in *x* production (how many units of y are necessary to produce one unit of x)
- g_2 represents the negative impact that added *x* production has on *y* output
  - allows us to consider the externality from *x* production

Inefficiency of the Competitive Allocation

Reliance on competitive pricing will result in an inefficient allocation of resources

A utility-maximizing individual will opt for

$$MRS = P_x/P_y$$

and the profit-maximizing producer of *y* would choose *x* input according to

$$P_x = P_y g_1 \rightarrow MRS = P_x/P_y = g_1$$

But the producer of *x* would choose *y* input so that

$$P_y = P_x f_y$$

$$MRS = P_x/P_y = 1/f_y$$

This means that the producer of *x* would disregard the externality that its production poses for *y* and will overproduce *x*

Example: Production Externalities

- Suppose that two newsprint producers are located along a river
- The upstream firm has a production function of the form

$$x = 2{,}000 l_x^{0.5}$$

The downstream firm has a similar production function but its output may be affected by chemicals that firm x pours in the river

$$y = 2{,}000 l_y^{0.5}(x - x_0)^{\alpha} \quad (\text{for } x > x_0)$$

$$y = 2{,}000 l_y^{0.5} \quad (\text{for } x \leq x_0)$$

where x_0 represents the river's natural capacity for pollutants

Assuming that newsprint sells for $1 per foot and workers earn $50 per day, firm x will maximize profits by setting this wage equal to the labor's marginal product

The problem of firm x is

$$\max_{l_x} 2000 l_x^{0.5} - 50 l_x$$

FOC is $1000 l_x^{-0.5} - 50 = 0$

$$l_x^* = 400$$

If $\alpha = 0$ (no externalities) firm y faces a similar problem, then $l_y^* = 400$

When firm *x* does have a negative externality ($\alpha < 0$), its profit-maximizing decision will be unaffected (l_x* = 400 and x* = 40,000)

But the marginal product of labor will be lower in firm *y* because of the externality

If $\alpha = -0.1$ and $x_0 = 38000$, firm *y* will maximize profits solving the problem

$$\max_{l_y} 2000 l_y^{0.5}(40000 - 38000)^{-0.1} - 50 l_y$$

The FOC is $1000 l_y^{-0.5}(40000 - 38000)^{-0.1} - 50 = 0$

Then solving it, l_y^* = 87 and *y* output will be 8,723

Then the externality of firm x, reduces the optimal level of employment and production of firm y

Inefficiency

Suppose that these two firms merge and the manager must now decide how to allocate the combined workforce

If one worker is transferred from *x* to *y*, output of *x* becomes

$$x = 2{,}000(399)^{0.5} = 39{,}950$$

and output of *y* becomes

$$y = 2{,}000(88)^{0.5}(1{,}950)^{-0.1} = 8{,}796$$

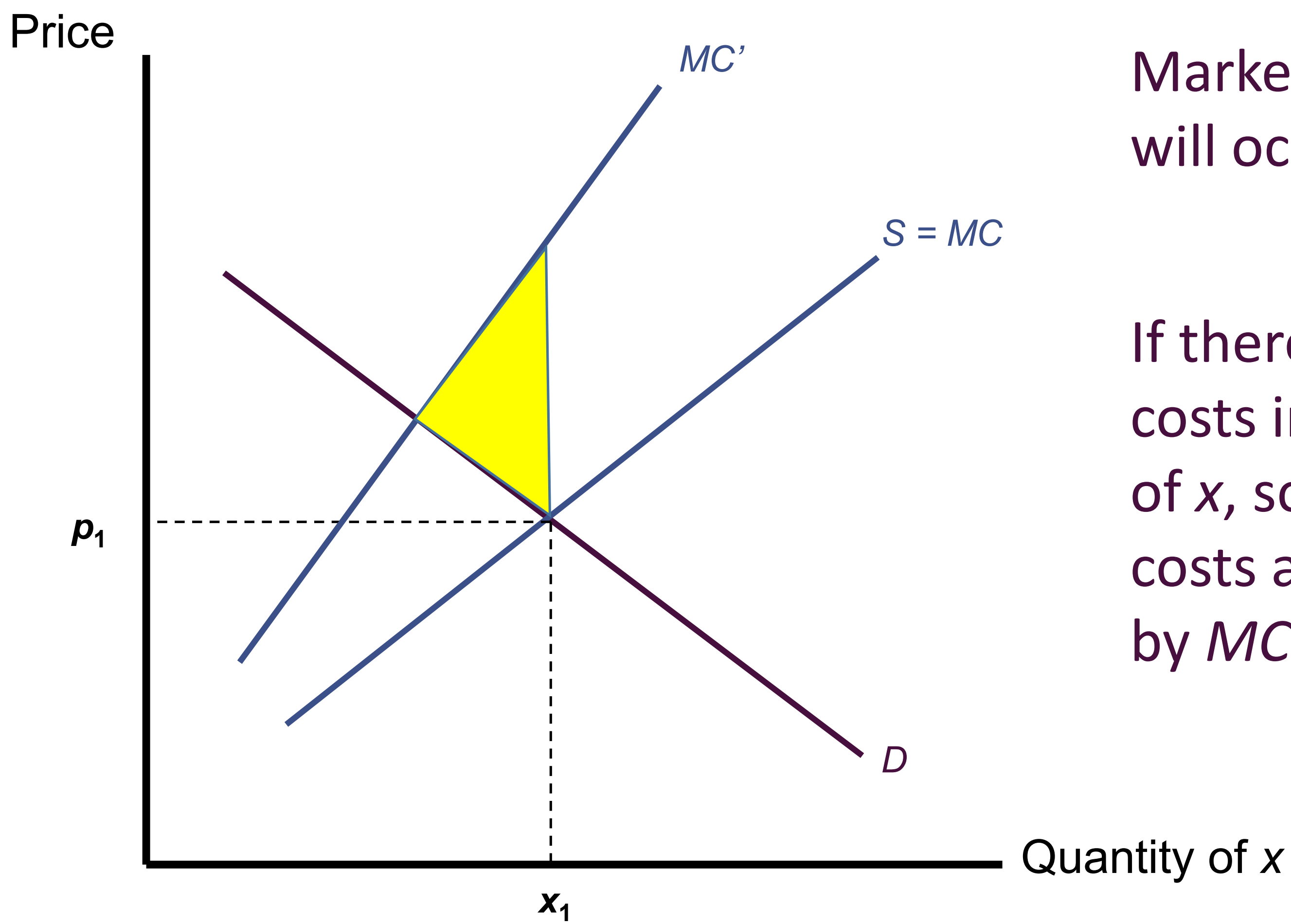
Total output increased with no change in total labor input

The earlier market-based allocation was inefficient because firm *x* did not take into account the effect of its hiring decisions on firm *y*

Solutions to the Externality Problem

The output of the externality-producing activity is too high under a market-determined equilibrium

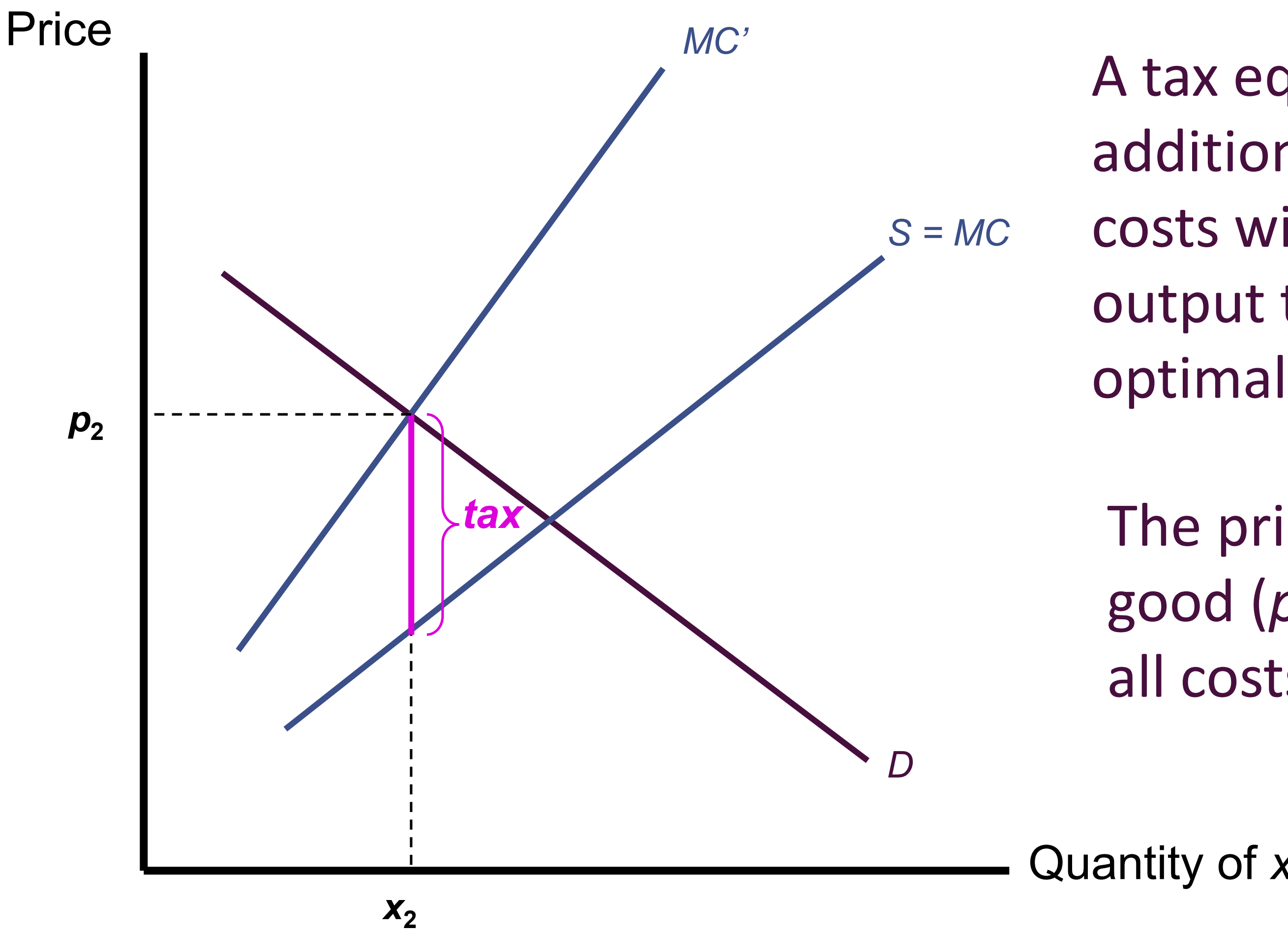
Incentive-based solutions to the externality problem originated with Pigou, who suggested that the most direct solution would be to tax the externality-creating entity



Market equilibrium will occur at p_1, x_1

If there are external costs in the production of x, social marginal costs are represented by *MC'*

In yellow the value of the externality, i.e. loss of efficiency



A tax equal to these additional marginal costs will reduce output to the socially optimal level (x_2)

The price paid for the good (p_2) now reflects all costs

Example 1: A Pigouvian Tax

Suppose a perfectly competitive market where the inverse supply (marginal costs) curve is

$$p_s = q + 0.01q^2$$

and the inverse demand is

$$p_d = 10000 - q$$

In the equilibrium $p_s = p_d$ → $q = 904$ and $p = 11096$

Suppose, that the production of this good produces an externality equal to 500 per each unit produced

The efficient quantity is computed using

$$p_s = q + 0.01q^2 + 500$$

Then $q = 880$

This can be obtained imposing a tax equal to 500 per unit, so the received price is $p_s - 500$

Suppose, that the production of this good produces an externality equal to 10% of the marginal cost

The efficient quantity is computed using

$$p_s = 1.1\, q + 0.011q^2$$

Then $q = 863$

This can be obtained imposing a tax equal to the difference between p_s and p_d at $q = 863$: $p_s = 8306$ and $p_d = 9137$, then $t = 830.6$ (10% of MC)

Example 2: A Pigouvian Tax on Newsprint

A suitably chosen tax on firm *x* can cause it to reduce its hiring to a level at which the externality vanishes

Because the river can handle pollutants with an output of $x = 38{,}000$, we might consider a tax that encourages the firm to produce at that level

Output of *x* will be 38,000 if $l_x = 361$

Thus, we can calculate *t* from the labor demand condition

$$(1 - t)MP_l = (1 - t)1{,}000(361)^{-0.5} = 50$$

$$t = 0.05$$

Therefore, a 5 percent tax on the price firm *x* receives would eliminate the externality

Remark: $MP_l = \frac{2{,}000\sqrt{l_x}}{dl_x}$ and $(1 - t)$ is price minus tax

Taxation in the General Equilibrium Model

The optimal Pigouvian tax in our general equilibrium model is to set

$$t = -p_y g_2$$

the per-unit tax on *x* should reflect the marginal harm that *x* does in reducing *y* output, valued at the price of good *y*

With the optimal tax, firm *x* now faces a net price of $(p_x - t)$ and will choose *y* input according to

$$p_y = (p_x - t) f_y$$

Replacing t we get that the resulting allocation of resources will achieve

$$MRS = \frac{p_x}{p_y} = \frac{1}{f_y} + \frac{t}{p_y} = \frac{1}{f_y} - g_2$$

The Pigouvian tax scheme requires that regulators have enough information to set the tax properly

in this case, they would need to know firm *y*'s production function

Pollution Rights

An innovation that would mitigate the informational requirements involved with Pigouvian taxation is the creation of a market for "pollution rights"

Suppose that firm *x* must purchase from firm *y* the rights to pollute the river they share

- *x*'s choice to purchase these rights is identical to its output choice

The net revenue that *x* receives per unit is given by p_x - *r*, where *r* is the payment the firm must make to firm *y* for each unit of *x* it produces

Firm *y* must decide how many rights to sell firm *x* by choosing firm *x* output to maximize its profits

$$\pi_y = p_y g(x_i, x_o) + r x_o - p_x x_i$$

The problem of firm y is

$$\max_{x_i, x_0} \pi_y = p_y g(x_i, x_0) + r x_o - p_x x_i$$

The first-order conditions for a maximum are

$$\frac{d\pi_y}{dx_i} = p_y g_1 - p_x = 0$$

$$\frac{d\pi_y}{dx_0} = p_y g_2 + r = 0$$

$$\rightarrow$$

$$r = -p_y g_2$$

The equilibrium solution is identical to that for the Pigouvian tax: i.e. it is possible to get the same effect as long as $r = t$

- from firm *x*'s point of view, it makes no difference whether it pays the fee to the government or to firm *y*

The Coase Theorem

The key feature of the pollution rights equilibrium is that the rights are well-defined and tradable with zero transactions costs

The initial assignment of rights is irrelevant

subsequent trading will always achieve the same, efficient equilibrium

Suppose that firm *x* is initially given x^T rights to produce (and to pollute)

it can choose to use these for its own production or it may sell some to firm *y*

Profits for firm *x* are given by

$$\pi_x = p_x x_o + r(x^T - x_o) - p_y y_i = (p_x - r)x_o + rx^T - p_y y_i$$

$$\pi_x = (p_x - r)f(y_i) + rx^T - p_y y_i$$

Profits for firm *y* are given by

$$\pi_y = p_y g(x_i, x_o) - r(x^T - x_o) - p_x x_i$$

Profit maximization in this case will lead to precisely the same solution as in the case where firm *y* was assigned the rights

The independence of initial rights assignment is usually referred to as the <u>Coase Theorem</u>

- in the absence of impediments to making bargains, all mutually beneficial transactions will be completed
- if transactions costs are involved or if information is asymmetric, initial rights assignments will matter

Attributes of Public Goods

A good is <u>exclusive</u> if it is relatively easy to exclude individuals from benefiting from the good once it is produced

A good is <u>nonexclusive</u> if it is impossible, or very costly, to exclude individuals from benefiting from the good

A good is <u>nonrival</u> if consumption of additional units of the good involves zero social marginal costs of production

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="2">Exclusive</td></tr>
<tr><td>Yes</td><td>No</td></tr>
<tr><td rowspan="2">Rival</td><td>Yes</td><td>Hot dogs, cars, houses</td><td>Fishing grounds, clean air</td></tr>
<tr><td>No</td><td>Bridges, swimming pools</td><td>National defense, mosquito control</td></tr>
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A good is a pure public good if, once produced,

- no one can be excluded from benefiting from its availability and
- if the good is nonrival – i.e. the marginal cost of an additional consumer is zero

Public Goods and Resource Allocation

General equilibrium model with two individuals (*A* and *B*)

There are only two goods, x and y:

- good y is an ordinary private good, each person begins with an allocation y^{A*}, y^{B*}.
- good *x* is a public good that is produced using *y*

$$x = f(y_s^A + y_s^B)$$

Resulting utilities for these individuals are:

$U^A(x, y^{A*} - y_s^A)$ and $U^B(x, y^{B*} - y_s^B)$

The level of *x* enters identically into each person's utility

it is nonexclusive and nonrival

each person's consumption is unrelated to what he contributes

each consumes the total amount of x produced

The necessary conditions for efficient resource allocation consist of choosing the levels of ${y_s}^A$ and ${y_s}^B$ that maximize the sum of the utilities of the two persons

The problem is:

$$\max_{y_s^A, y_s^b} U^A(x, y^{A*} - y_s^A) + U^B(x, y^{B*} - y_s^B)$$

$$\equiv$$

$$\max_{y_s^A, y_s^b} U^A(f(y_s^A + y_s^B), y^{A*} - y_s^A) + U^B(f(y_s^A + y_s^B), y^{B*} - y_s^B)$$
$$= TU$$

The first-order conditions for a maximum are

$$\frac{dTU}{dy_s^A} = U_1^A f' - U_2^A + U_1^B f' = 0$$

$$\frac{dTU}{dy_s^B} = U_1^B f' - U_2^B + U_1^A f' = 0$$

Comparing the two conditions we get $U_2^A = U_2^B$

Divide both sides of the initial first-order condition by U_2^A:

$$\frac{U_1^A}{U_2^A} f' - 1 + \frac{U_1^B}{U_2^A} f' = 0$$

Replacing $U_2^A = U_2^B$

$$\frac{U_1^A}{U_2^A} + \frac{U_1^B}{U_2^A} = \frac{1}{f'}$$

i.e.

$$MRS^A + MRS^B = 1/f'$$

The *MRS* must reflect all consumers because all will get the same benefits

Failure of a Competitive Market

Production of x and y in competitive markets will fail to achieve this allocation

- with perfectly competitive prices p_x and p_y, each individual will equate his MRS to p_x/p_y
- the producer will also set $1/f'$ equal to p_x/p_y to maximize profits
- the price ratio p_x/p_y will be too low
  - it would provide too little incentive to produce x

For public goods, the value of producing one more unit is the sum of each consumer's valuation of that output

- individual demand curves should be added vertically rather than horizontally

Thus, the usual market demand curve will not reflect the full marginal valuation

Inefficiency of a Nash Equilibrium

Suppose that individual A is thinking about contributing s_A of his initial y endowment to the production of x

The utility maximization problem for A is:

$$\max_{s_A} U^A(f(s_A + s_B), y^A - s_A)$$

The first-order condition for a maximum is

$$U_1^A f' - U_2^A = 0$$

$$U_1^A / U_2^A = MRS^A = 1/f'$$

Because a similar argument can be applied to B, the efficiency condition will fail to be achieved

each person considers only his own benefit

Example 1: The Roommates' Dilemma

Suppose two roommates with identical preferences derive utility from the number of paintings hung on their walls (x) and the number of granola bars they eat (y) with a utility function of

$$U_i(x,y_i) = x^{1/3}y_i^{2/3} \quad \text{(for } i\text{=1,2)}$$

Assume each roommate has \$300 to spend and that p_x = \$100 and p_y = \$0.20

We know from our earlier analysis of Cobb-Douglas utility functions that if each individual lived alone, he would spend 1/3 of his income on paintings (x = 1) and 2/3 on granola bars (y = 1,000)

When the roommates live together, each must consider what the other will do

if each assumed the other would buy paintings, x = 0 and utility = 0

If person 1 believes that person 2 will not buy any paintings, he could choose to purchase one and receive utility of

$$U_1(x,y_1) = 1^{1/3}(1{,}000)^{2/3} = 100$$

while person 2's utility will be

$$U_2(x,y_2) = 1^{1/3}(1{,}500)^{2/3} = 131$$

Person 2 has gained from his free-riding position

We show that this solution is inefficient by calculating each person's *MRS*

$$MRS_i = \frac{\frac{dU_i}{dx}}{\frac{dU_i}{dy_i}} = \frac{y_i}{2x}$$

At the allocations described,

$$MRS_1 = 1{,}000/2 = 500$$

$$MRS_2 = 1{,}500/2 = 750$$

Since $MRS_1 + MRS_2 = 1{,}250$, the roommates would be willing to sacrifice 1,250 granola bars to have one additional painting

- an additional painting would only cost them 500 granola bars
- too few paintings are bought

To calculate the efficient level of x, we must set the sum of each person's MRS equal to the price ratio

$$MRS_1 + MRS_2 = \frac{y_1}{2x} + \frac{y_2}{2x} = \frac{y_1 + y_2}{2x} = \frac{p_x}{p_y} = \frac{100}{0.2}$$

This means that

$$y_1 + y_2 = 1{,}000x$$

Substituting into the budget constraint,

$$0.20(y_1 + y_2) + 100x = 600$$

we get

$$0.20(\mathit{1000x}) + 100x = 600$$

$$x = 2 \text{ and } y_1 + y_2 = 2{,}000$$

The allocation of the cost of the paintings depends on how each roommate plays the strategic financing game

Note that in this example the production function of x has to be written in terms of quantity of y, i.e.

$x = f(y) = \frac{0.2\,y}{100}$ (i.e. $0.2\,y$ is the value of y units, 100 is the price of x)

Then $f' = \frac{0.2}{100}$

So we can apply the formula $MRS^A + MRS^B = 1/f'$

Solution (Nash equilibrium) of **The Roommates' Dilemma** by computing best responses.

$$U_i(x,y_i) = x^{1/3}y_i^{2/3} \quad \text{(for } i\text{=1,2)}$$

Assume each roommate has \$300 to spend and that p_x = \$100 and p_y = \$0.20

Let be x_1 and x_2 the quantity f x provided respectively from person 1 and person 2.

Then problem of person 1 is:

$$\max_{x_1,y_1}(x_1+x_2)^{\frac{1}{3}}\,y_1^{\frac{2}{3}}$$

subject to

$$100\,x_1 + 0.2y_1 = 300$$

From the constraint we get

$$y_1 = 1500 - 500\,x_1$$

Replacing y_1 into the objective function the problem becomes a simply unconstrained problem in one variable:

$$\max_{x_1}(x_1 + x_2)^{\frac{1}{3}}(1500 - 500\, x_1)^{\frac{2}{3}}$$

FOC is:

$$\frac{1}{3}(x_1 + x_2)^{-\frac{2}{3}}(1500 - 500\, x_1)^{\frac{2}{3}} - \frac{1000}{3}(1500 - 500\, x_1)^{-\frac{1}{3}}(x_1 + x_2)^{\frac{1}{3}} = 0$$

Simplifying we get the best response of person 1

$$x_1 = 1 - \frac{2}{3}x_2$$

Solving the problem of person 2 we get his best response:

$$x_2 = 1 - \frac{2}{3}x_1$$

Solving the system composed by the two best responses we get

$$x_1 = x_2 = \frac{3}{5} \quad \rightarrow \; x_1 + x_2 = \frac{6}{5}$$

$$y_1 = y_2 = 1500 - 500\frac{6}{5} = 900$$

Public good with linear utility function

This is an example of a game with continuous strategies

Two individuals are endowed with 10 pounds.

They can contribute to a public good by delivering any amount of money out of their endowment (c_1, c_2)

For each individual the value of the public good is given by the sum of the contributions multiplied by 0.7

$$v_p = 0.7(c_1 + c_2)$$

Payoff of player 1 = Endowment – contribution + value of the public good

$$\pi_1 = 10 - c_1 + v_p = 10 - c_1 + 0.7(c_1 + c_2)$$

Payoff of player 2 = Endowment – contribution + value of the public good

$$\pi_2 = 10 - c_2 + v_p = 10 - c_2 + 0.7(c_1 + c_2)$$

The best response of player 1 is given by the solution of the problem:

$$\max_{c_1} \pi_1 = 10 - c_1 + 0.7(c_1 + c_2)$$

FOCs are:

$$-1 + 0.7 < 0$$

Then player 1's best response is $c_1 = 0$

The best response of player 2 is given by the solution of the problem:

$$\max_{c_2} \pi_2 = 10 - c_2 + 0.7(c_1 + c_2)$$

FOCs are:

$$-1 + 0.7 < 0$$

Then player 2's best response is $c_2 = 0$

Unique Nash equilibrium: both individuals contribute by 0

But the efficient outcome is both contributing by 10