Site percolation on a triangular lattice - adapted from

Problem 13.2 of Gould-Tobochnik II ed

OR
Problem 12.3 in the III edition

http://www.opensourcephysics.org/items/detail.cfm?ID=7375

PROBLEM

The value of p_c depends on the symmetry of the lattice and on its dimension. In addition to the square lattice, the most common two-dimensional lattice is the triangular (hexagonal) lattice. The essential difference between the square and triangular lattices is in the number of nearest neighbors.

Write a program to simulate random site percolation on a triangular lattice. Assume that

a connected path connects the top and bottom sides of the lattice (see Fig. 13.5). Do you expect p_c for the triangular lattice to be smaller or larger than the value of p_c for the square lattice? Estimate $p_c(L)$ for $L = 4, 16$, and 32. Are your results for p_c consistent with your expectations?

(How do you define the existence of a spanning cluster? Describe your visual "algorithm" for determining if a spanning cluster exists.)

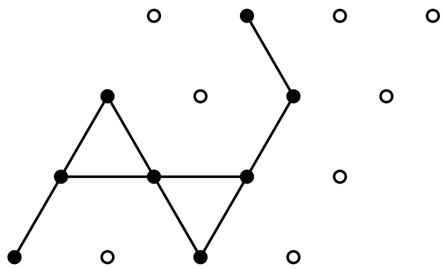

Figure 13.5: Example of a spanning cluster on a $L = 4$ triangular lattice. The bonds between the occupied sites are drawn to clarify the symmetry of the lattice.

For bond percolation on the square lattice, the exact value of p_c can be obtained by introducing the *dual* lattice. The nodes of the dual lattice are the centers of the squares between the nodes in the original lattice (see Figure 12.7). The occupied bonds of the dual lattice are those that do not cross an occupied bond of the original lattice. Because every occupied bond on the dual lattice crosses exactly one unoccupied bond of the original lattice, the probability $\tilde{p}$ of an occupied bond on the dual lattice is $1 - p$, where p is the probability of an occupied bond on the original lattice. If we assume that the dual lattice percolates if and only if the original lattice does not and vice versa, then $p_c = 1 - p_c$ or $p_c = 1/2$. This assumption holds for bond percolation on a square lattice because if a cluster spans in the original lattice in both directions, then because the occupied dual lattice bonds can only cross unoccupied bonds of the original lattice, the dual lattice clusters are blocked from spanning. An example is shown in Figure 12.7. This argument does not work for cubic lattices in three dimensions, but it can be used for site percolation on a triangular lattice to yield $p_c = 1/2$.