19.1 THE UNITY OF PHYSICS

Although we have discussed many topics and applications, we have covered only a small fraction of the possible computer simulations and models of natural phenomena. However, we know that the same physical principles can be applied to many kinds of phenomena. We express this point of view as *the same algorithms give the same results*. For example, the Monte Carlo methods that we applied to the simulation of classical liquids and to the analysis of quantum mechanical wave functions also were applied to the transport of neutrons and problems in chemical kinetics. Similar Monte Carlo methods are being used to analyze problems in quark confinement. Indeed, the increasing role of the computer in research is strengthening the interconnections of the various subfields of physics and the relation of physics to other disciplines.

The computer also has helped us think of natural phenomena in new ways that complement traditional methods. For example, consider a predator-prey model of the dynamics of minnows and sharks. Assume that the birth rate of the minnows is independent of the number of sharks, and that each shark kills a number of minnows proportional to their number. If we assume that $F(t)$, the number of minnows at time t, changes continuously, we can write

$$\frac{dF(t)}{dt} = [b_1 - d_1 S(t)]F(t), \tag{19.1}$$

where $S(t)$ is the number of sharks at time t, and b_1 and d_1 are constants independent of F and S. To obtain an equation for the rate of change of the sharks, we assume that the number of offspring produced by each shark is proportional to the number of minnows eaten by the shark. If we also assume that the death rate of the sharks is constant, we can write

$$\frac{dS(t)}{dt} = [b_2 F(t) - d_2]S(t). \tag{19.2}$$

Equations (19.1) and (19.2) are known as the Lotka-Volterra equations. They can be analyzed by standard methods and solved numerically using simple algorithms. Why is the dynamical behavior of (19.1) and (19.2) cyclic?

In the predator-prey model the numbers of predator and prey are assumed to change continuously and their spatial distribution is ignored. We now summarize an alternative model that can be most simply expressed as a computer algorithm. The model is a two-dimensional cellular automaton known as Wa-Tor.

1. For a desired concentration of minnows and sharks, minnows and sharks are placed at random on the sites of a lattice. The minnows and sharks are assigned random ages.
2. At time step t_n, consider each minnow sequentially. Determine the number of nearest neighbor sites that are unoccupied at time t_{n-1} and move the minnow at random to one of the unoccupied sites. If all the nearest neighbor sites are occupied, the minnow does not move.

3. If a minnow has survived for a multiple of `fbreed` iterations, the minnow has a single offspring. The new minnow is placed at the previous position of the parent minnow.
4. At time step t_n, consider each shark sequentially. If all the nearest neighbor sites of the shark at time t_{n-1} are unoccupied, move the shark at random to one of the four unoccupied sites. If one or more of the adjacent sites is occupied by a minnow, the shark moves at random to one of the occupied sites and eats the minnow.
5. If a shark moves `nstarve` times without eating, the shark dies. If a shark survives for a multiple of `sbreed` iterations, the shark has a single offspring. The new shark is placed at the previous position of the parent shark.

What is the dynamical behavior of Wa-Tor? Do Wa-Tor and the Lotka-Volterra equations exhibit similar behavior? Is the Wa-Tor model realistic? What are the advantages and disadvantages of each approach? See the references for suggestions for the numerical values of the parameters.

19.2 PERCOLATION AND GALAXIES

In addition to allowing us to investigate complex nonlinear problems and more realistic systems, the computer has reinforced one of the contemporary themes in physics, the unifying role of collective behavior. Systems composed of many individual constituents can exhibit common properties under certain conditions, even though there might be differences in the nature of the constituents and in their mutual interaction. The behavior of a system near a critical point is probably the best example of collective behavior in a familiar context. In the following, we discuss examples of collective behavior in the context of epidemiology and the structure of spiral galaxies. Our discussion follows closely the articles by Schulman and Seiden.

Consider an imaginary disease called *percolitis*. The disease conveys no immunity, and its incubation period and duration are both one day. The disease is so benign that its sufferers are able to come into contact with every member of the community so that every person comes into contact with every other person every day. At $t = 0$ one person contracts the disease from a source outside the community. Let N be the total population, t the time measured in days, p the transmission probability, and $n(t)$ the expected number of diseased individuals at time t. Convince yourself that for $N = 1000$ and $p = 0.0005$, the chance that there will be anyone suffering from the disease seven days later is vanishingly small. On the other hand, suppose that $p = 0.002$. Then $n(t = 1) = 2$, $n(t = 2) \approx 4$, and the odds are very high that after some time there will be an average number of approximately 800 victims. Do a simulation for various values of N and p and estimate the critical probability p_c such that for $p < p_c$ the average number of victims is zero, and for $p \geq p_c$ the average number of victims is nonzero. Note that no assumptions were made as to which individuals will become infected.

As its name suggests, the percolitis model has much in common with percolation. What are some of the connections? The model is sufficiently simple that an analytical solution is possible. What modifications of the model might make it more realistic