Cosmology 1

2019/2020
Prof. Pierluigi Monaco

Third intermediate test
Topic: early universe. Deadline: June 12, 14:00

This is an extended version of proposed problem 22 (exercise 3.2 of Vittorio textbook), a reproposition of the argument proposed by Gamow, Alpher and Hermann in the 1940s to estimate the temperature of the CMB.

Start from the assumption that He is produced during the first minutes of life of the universe. You know that the present baryon density is $n_{b0} \sim 10^{-7}$ cm^{-3}, and that the cross section for deuterium formation is $\sigma \sim 10^{-29}$ cm^{-2}, but the CMB has not been detected yet. Deuterium forms when the age of the Universe is not much smaller than the interaction time-scale, and the temperature is low enough to limit the nuber of photons able to dissociate it. Follow this path:

(a) assume that nucleosynthesis happens when the deuterium bottleneck opens at $T_{\rm db}$,

(b) compute the age of a radiative universe t_u for that temperature,

(c) work out the thermal speed of baryons at that temperature,

(d) the timescale for deuterium formation is $t_d \sim 1/n_b \sigma v$, where n_b is the baryon number density at nucleosynthesis time,

(e) deuterium production starts when the age of the universe is $t_u \sim t_d$,

(f) CMB temperature evolves like $T = T_0\, a^{-1}$,

(g) baryon density evolves like $n = n_0\, a^{-3}$.

This allows to work out a relation between the CMB temperature today T_0 and the temperature of deuterium bottleneck opening $T_{\rm db}$. Another relation is obtained when the leading term of the Saha equation $\exp(-B_d/k_B T_{\rm db})$ is of the same order of the ratio η of baryon and photon number densities.

Use these two relations to find both T_0 (in K) and $T_{\rm db}$ (in keV). Try also to argue how a neutrino component would change the result. Two pages of text will be enough to present the result.

Solution

At nucleosynthesis time, energy density is dominated by radiation (photons plus neutrinos), so:

$$\rho = \frac{a_r T_{\rm db}^4}{c^2} \times (1 + 0.227 N_\nu)$$

Let's neglect neutrinos first, $N_\nu = 0$. The age of the universe is:

$$t_u = \sqrt{\frac{3}{32\pi G\rho}} = \frac{c}{T_{\rm db}^2}\sqrt{\frac{3}{32\pi G a_r (1 + 0.227 N_\nu)}}$$

The thermal velocity of protons is (up to order-of-unity factors) $v = \sqrt{k_B T_{\rm db}/m_p}$, so the timescale for deuterium formation is:

$$t_d = \frac{1}{n_b \sigma}\sqrt{\frac{m_p}{k_B T_{\rm db}}}$$

Setting $t_u = t_d$ provides a relation between $T_{\rm db}$ and n_b. These can be rescaled to T_0 and n_{b0} as follows:

$$T_0 = T_{\rm db}\left(\frac{n_{b0}}{n_b}\right)^{1/3}$$

and this allows to obtain the first relation between T_0 and $T_{\rm db}$:

$$T_0 = \left(\frac{32\pi G m_p a_r (1 + 0.227 N_\nu)}{3 k_B}\right)^{-1/6} (\sigma c n_{b0})^{1/3} T_{\rm db}^{1/2}$$

We have also that:

$$\eta = \frac{n_{b0}}{n_{\gamma 0}} = \frac{\pi^4}{30\zeta(3)}\frac{k_B}{a_r} n_{b0} T_0^{-3}$$

so the Saha equation $\exp(-Bd/k_B T_{\rm db}) = \eta$ translates to:

$$T_{\rm db} = -\frac{B_d}{k_B \ln(\eta)}$$

These two relations between T_0 and $T_{\rm db}$ can be solved to give, for $N_\nu = 0$:

$$T_0 = 4.2 \text{ K}, \qquad T_{\rm db} = 94 \text{ keV}.$$

Adding three neutrinos changes the numbers only marginally:

$$T_0 = 3.9 \text{ K}, \qquad T_{\rm db} = 95 \text{ keV}.$$