FLUIDODINAMICA E TRASPORTO DEI SEDIMENTI

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Introduction to Clastic Sedimentology

Chapter 4

http://spartan.ac.brocku.ca/~rcheel/teaching/sedimentology/

Chapter 4: Fluid Flow and Sediment Transport

Affronteremo ora il tema di come i sedimenti su muovino in risposta ad un flusso fluido (in una direzione).

Perchè?

La conoscenza delle condizioni di trasporto dei sedimenti è necessaria per la risoluzione di problemi ingegneristici, come ad es. la costruzione di un canale e il mantenimento funzionale dello stesso.

Nell'interpretazione dei sedimenti fossili: la gran parte dei sedimenti hanno subito processi associati ad acque in movimento (fiumi, correnti oceaniche, maree).

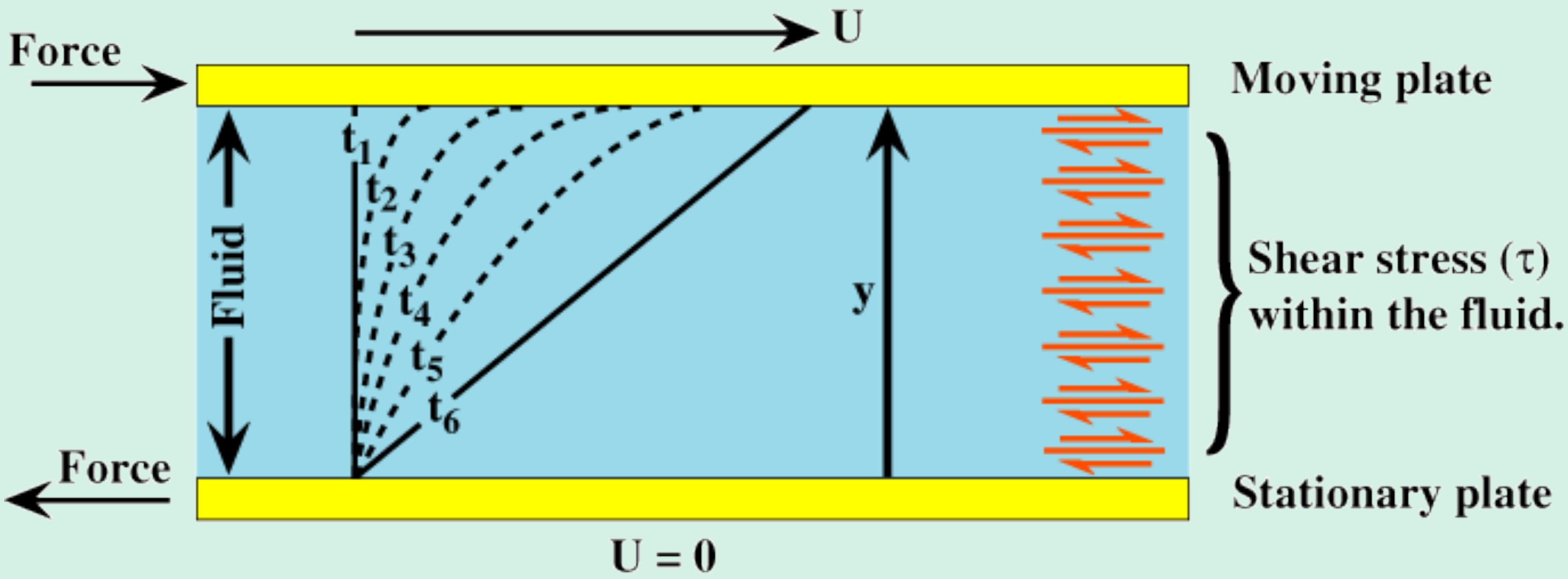
Cinematica dei fluidi e trasporto dei sedimenti sono ovviamente collegati nella formazione delle strutture sedimentarie primarie.

Flusso fluido tra due piastre parallele

La piastra al fondo è fissa e quella superiore viene accelerata applicando una determinata forza che agisce da sinistra a destra.

La piastra superiore accelererà fino a raggiungere una certa velocità terminale e il fluido tra le due piastre si metterà in movimento.

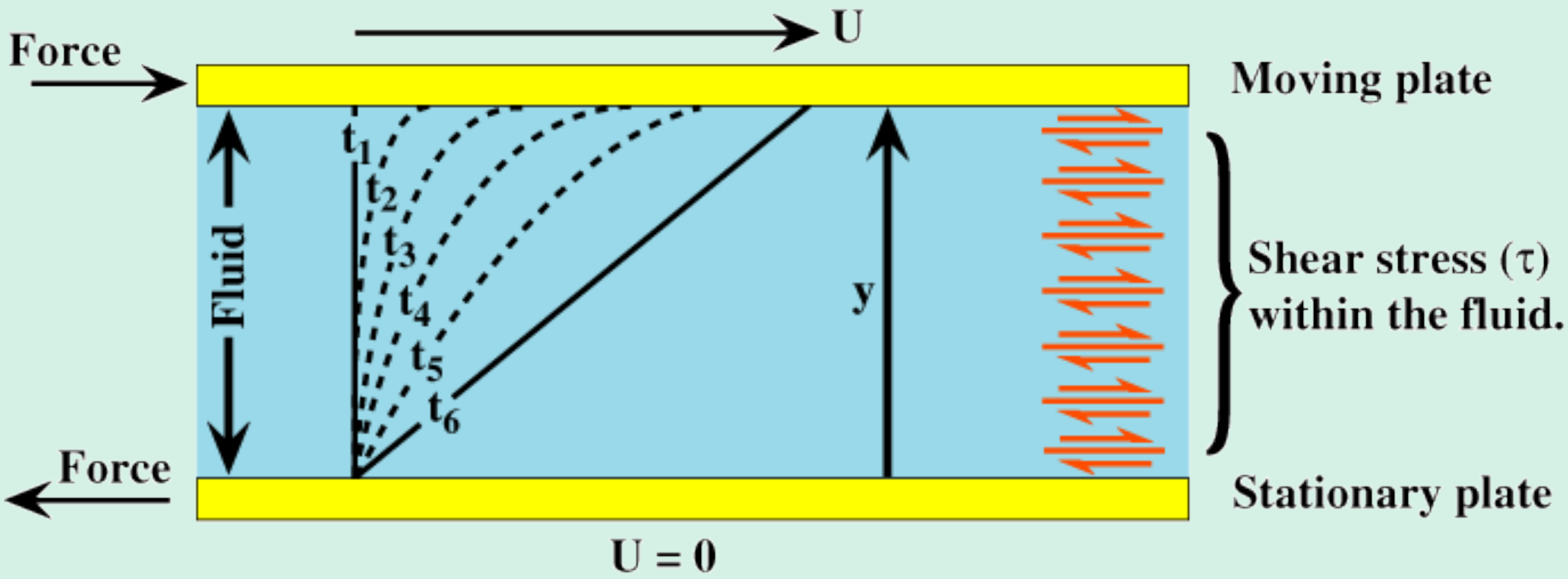
La velocità terminale viene raggiunta quando la forza applicata è bilanciata dalla forza resistente (qui rappresentata come una forza uguale e opposta applicata sulla piastra stazionaria al fondo).



Quando la piastra superiore inizia ad accelerare, la velocità del fluido in contatto con la piastra è uguale alla velocità della piastra (cioè vi è una *no slip condition* tra la piastra e il fluido).

Le molecole del fluido in contatto con quelle contro la piastra saranno quindi accelerate a causa dell'attrazione viscosa tra loro .... e così avanti lungo tutta la colonna del fluido.

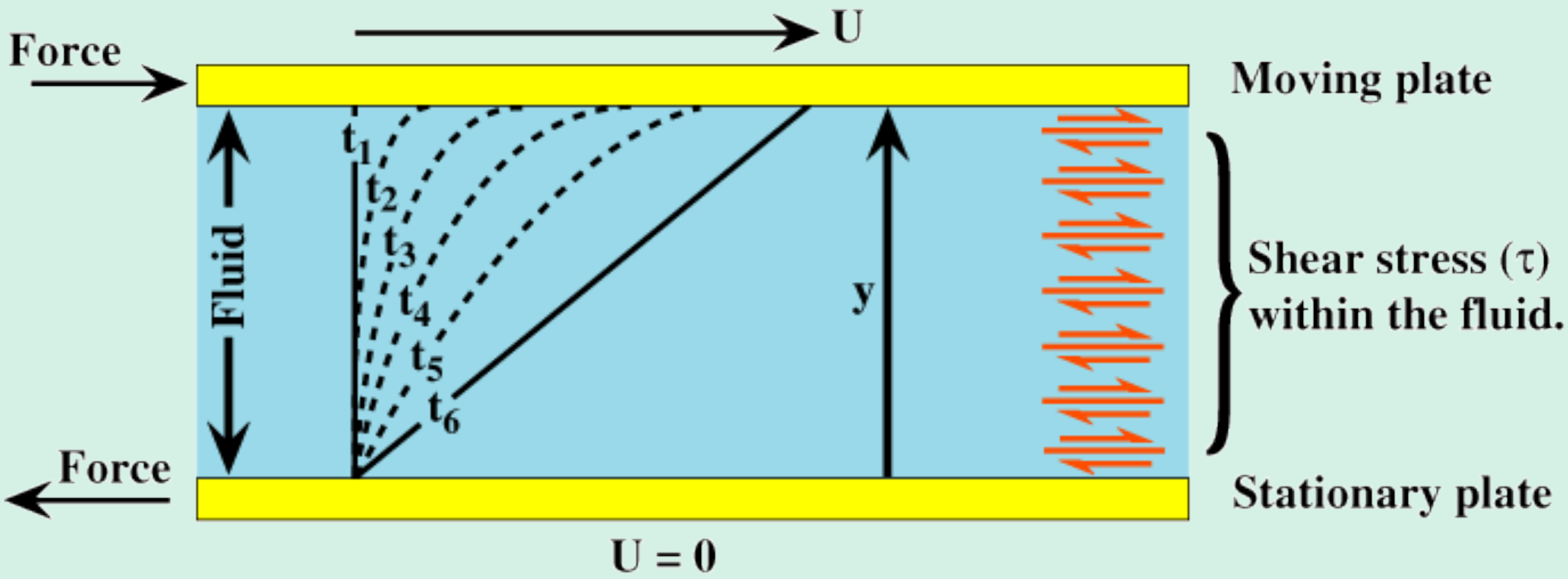
La viscosità del fluido (**μ**, l'attrazione tra le molecole del fluido) comporta quindi una dislocazione progressiva degli strati del fluido a partire dalla piastra in movimento.



La piastra al fondo è stazionaria (U=0) e quindi il gradiente di velocità varierà da zero al fondo a U_{term} alla sommità, che è uguale alla velocità terminale della piastra superiore.

Il gradiente di velocità (cambio di velocità tra le due piastre; du/dy) varierà linearmente da zero a U_{term} .

La velocità terminale è raggiunta quando la forza resistente (riferita alla piastra stazionaria di fondo) è uguale alla forza applicata alla piastra superiore (le forze si eguagliano quando non c'è cambio di velocità nel tempo).

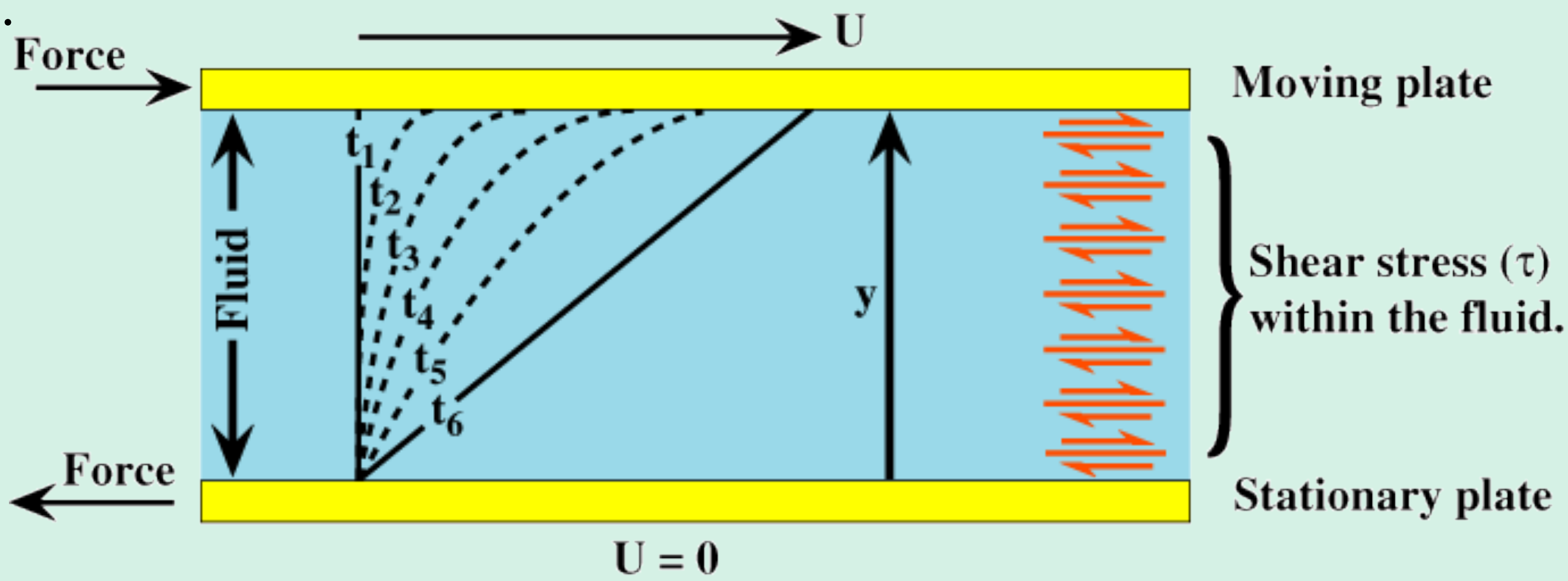


Questa forza resistente è la “resistenza del fluido” piuttosto che una forza applicata sulla piastra di fondo.

Man mano che la velocità incrementa verso l’alto lungo la colonna di fluido, ci deve essere uno slittamento attraverso ciascun piano del fluido che è parallelo alle piastre.

Al contempo, ci deve essere una resistenza allo slittamento, altrimenti la piastra superiore accelererebbe all’infinito.

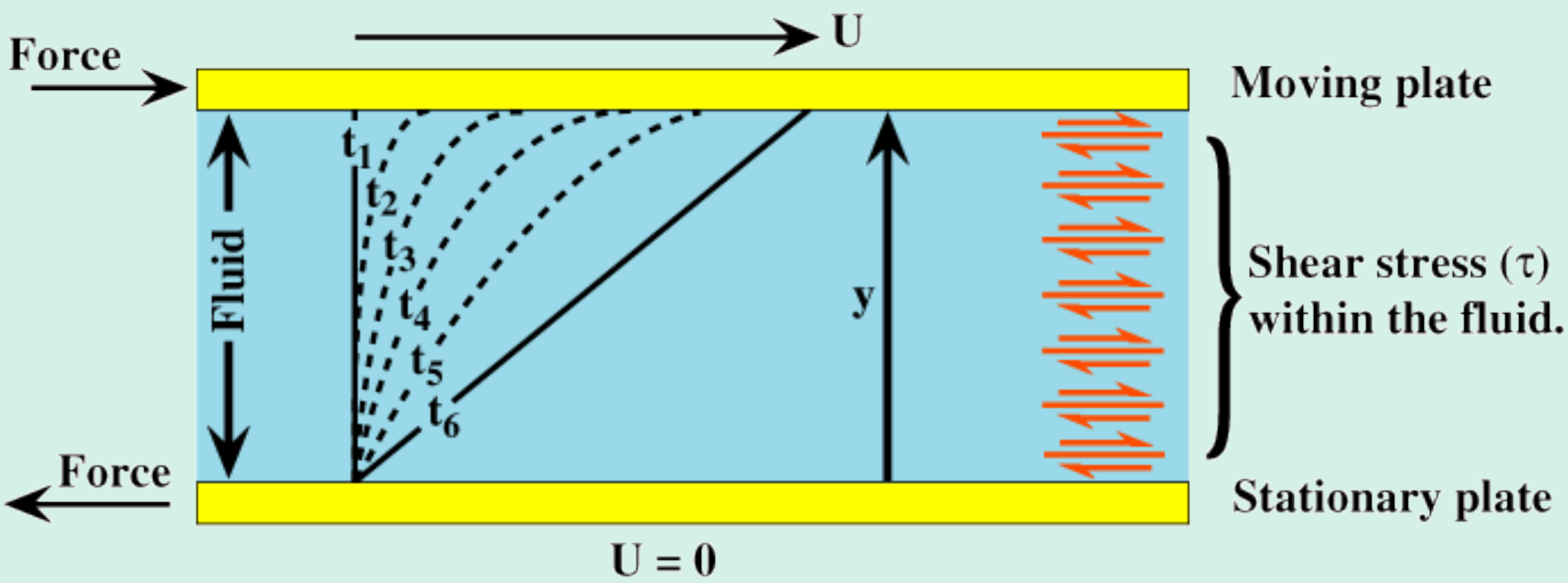
Questa stessa resistenza comporta un’accelerazione iniziale di ciascun strato fluido fino alla propria velocità terminale (che diminuisce verso il basso).



Fluid viscosity is the cause of fluid resistance and the total viscous resistance equals the applied force when the terminal velocity is achieved.

The viscous resistance results in the transfer of the force applied to the top plate through the column of fluid.

Within the fluid this force is applied as a shear stress (τ, the lower case Greek letter tau; a force per unit area) across an infinite number of planes between the top and bottom plates.

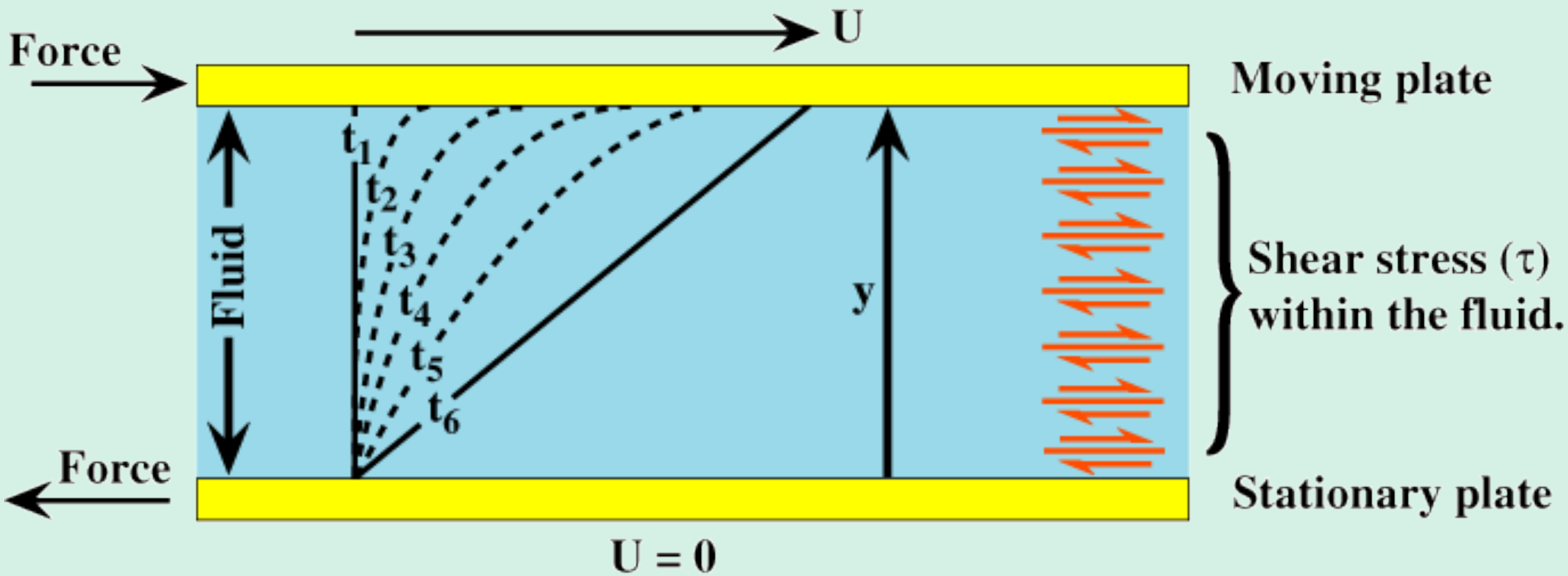


The shear stress transfers momentum (mass times velocity) through the fluid to maintain the linear velocity profile.

The magnitude of the shear stress is equal to the force that is applied to the top plate.

The relationship between the shear stress, the fluid viscosity and the velocity gradient is given by:

$$\tau = \mu \frac{du}{dy}$$



$$\tau = \mu \frac{du}{dy}$$

From this relationship we can determine the velocity at any point within the column of fluid.

Rearranging the terms: $$\frac{\tau}{\mu} = \frac{du}{dy}$$

We can solve for u at any height y above the bottom plate by integrating with respect to y.

$$u_y = \int \frac{du}{dy} dy \quad = \frac{\tau}{\mu} \int dy + c$$

$$= \frac{\tau}{\mu} y + c$$

Where c (the constant of integration) is the velocity at y=0 (where u=0) such that:

$$u_y = \frac{\tau}{\mu} y$$

$$u_y = \frac{\tau}{\mu} y$$

From this relationship we can see the following:

1. That the velocity varies in a linear fashion from 0 at the bottom plate (y=0) to some maximum at the highest position (i.e., at the top plate).

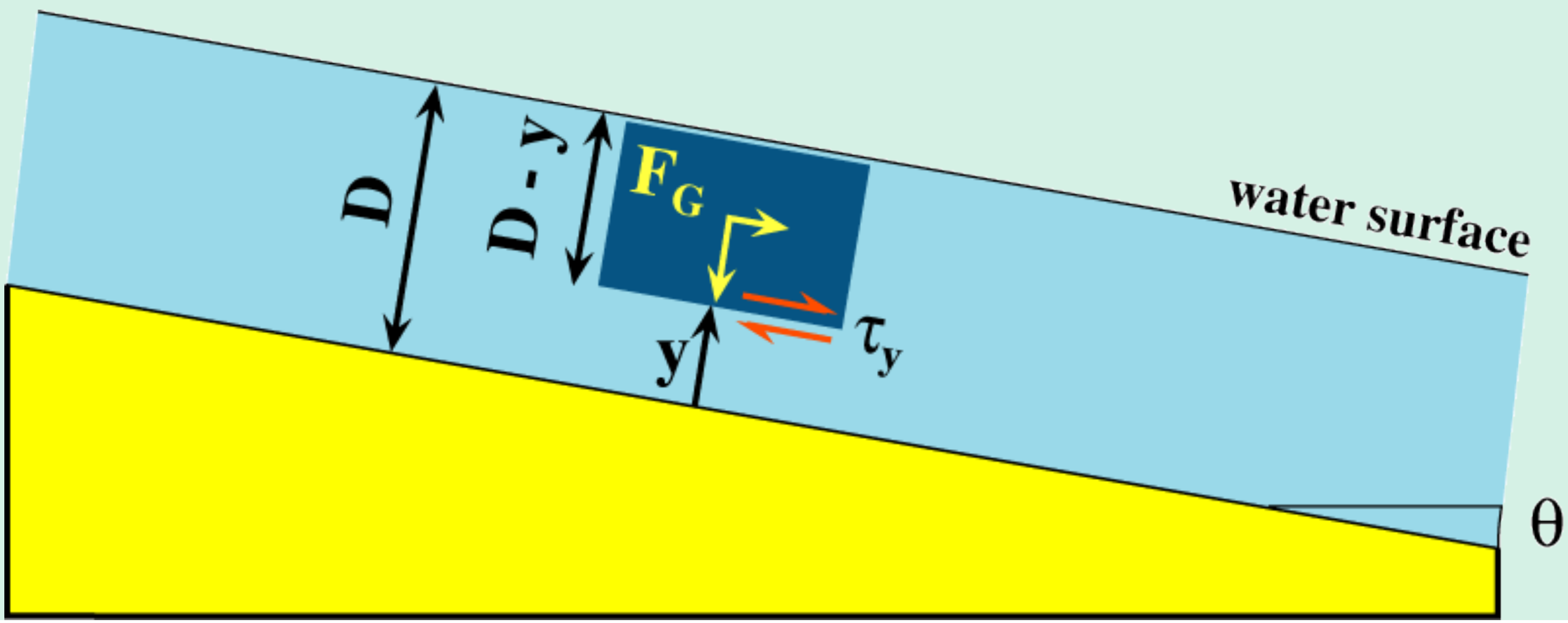
2. That as the applied force (equal to τ) increases so does the velocity at every point above the lower plate.

3. That as the viscosity increases the velocity at any point above the lower plate decreases.

Fluid Gravity Flows

Water flowing down a slope in response to gravity (e.g., rivers).

In this case, the driving force is the down slope component of gravity acting on the mass of fluid; more complicated because the deeper into the flow the greater the weight of overlying fluid.

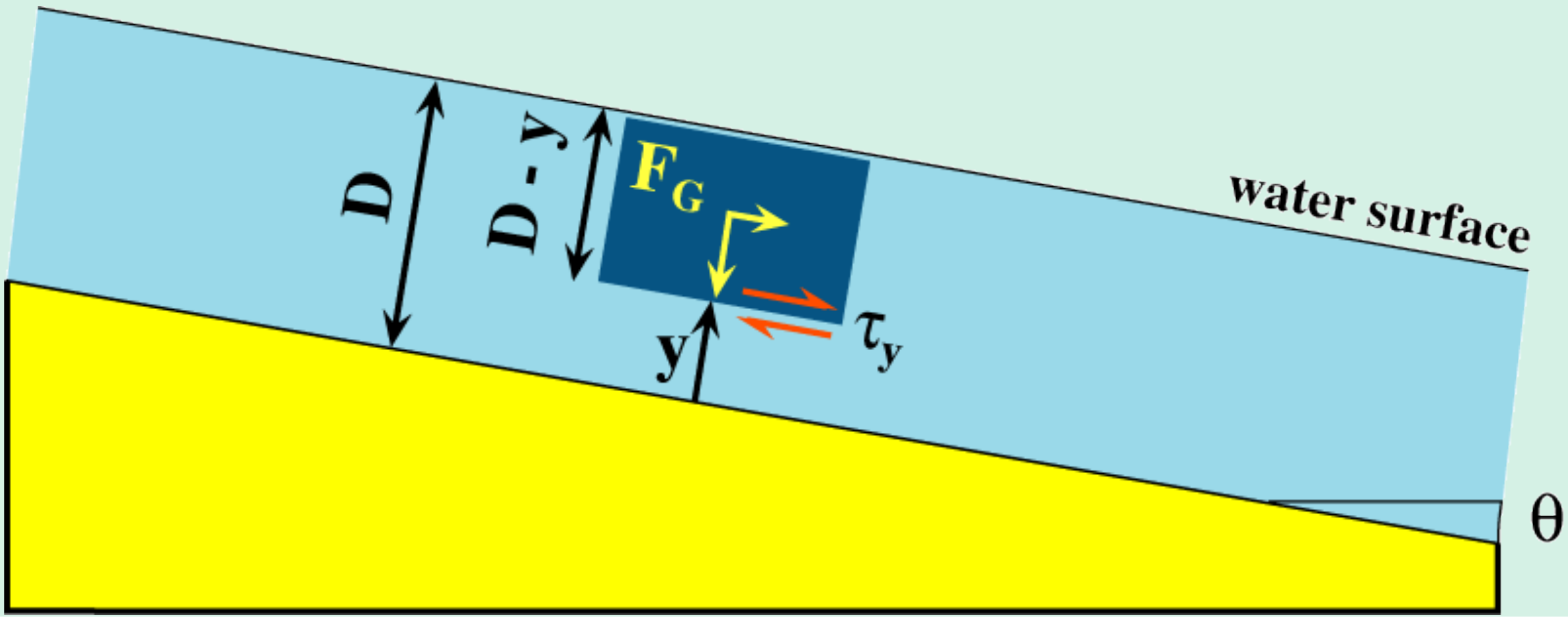


D is the flow depth.

F_G is the force of gravity acting on a cube of fluid with dimensions (D-y) x 1 x 1; note that *y* is the height above the lower boundary.

θ is the slope of the water surface (note that depth is uniform so that this is also the slope of the lower boundary).

τ_y is the shear stress that is acting across the bottom of the block of fluid (it is the downslope component of the weight of fluid in the block).



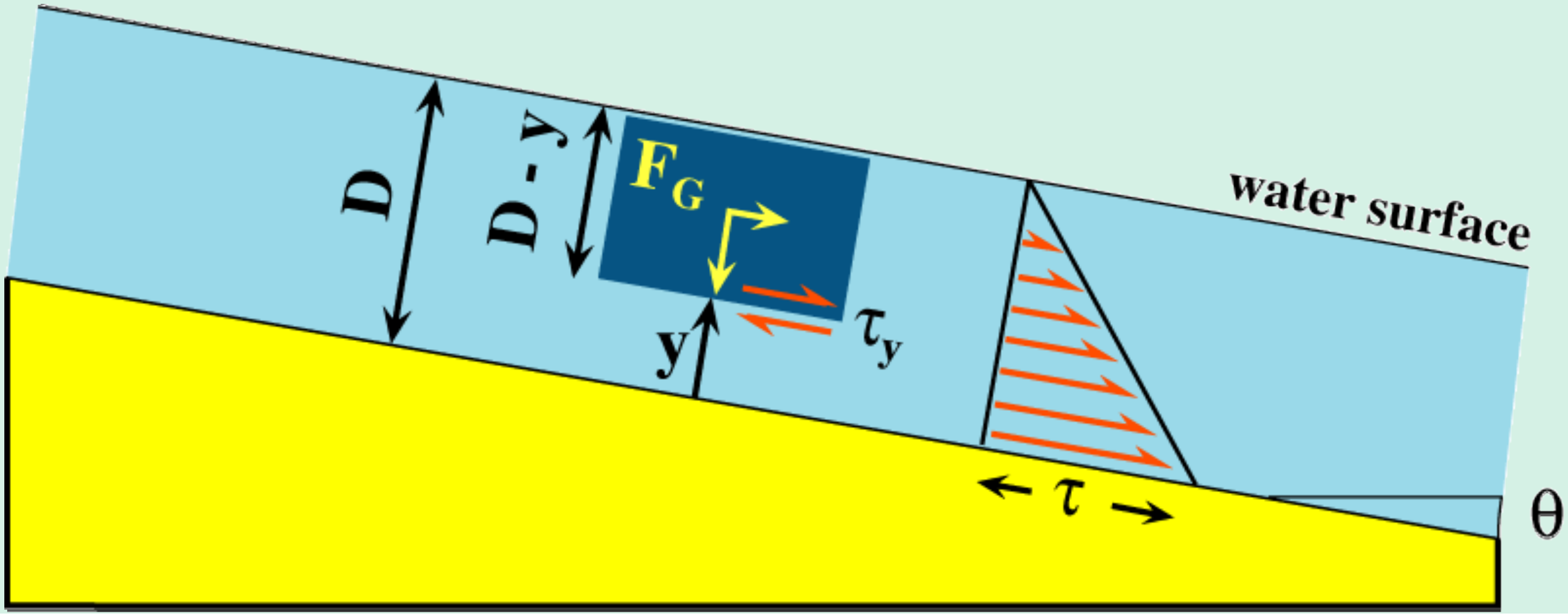
For this general situation, τ_y, the shear stress acting on the bottom of such a block of fluid that is *y* above the bed:

$$\tau_y = \boxed{\rho g(D-y)\times 1\times 1}\times\boxed{\sin\theta}$$

Weight of water in the block

The proportion of that weight that is acting down the slope.

Clearly, the deeper within the water (i.e., with decreasing *y*) the greater the shear stress acting across any plane within the flow.

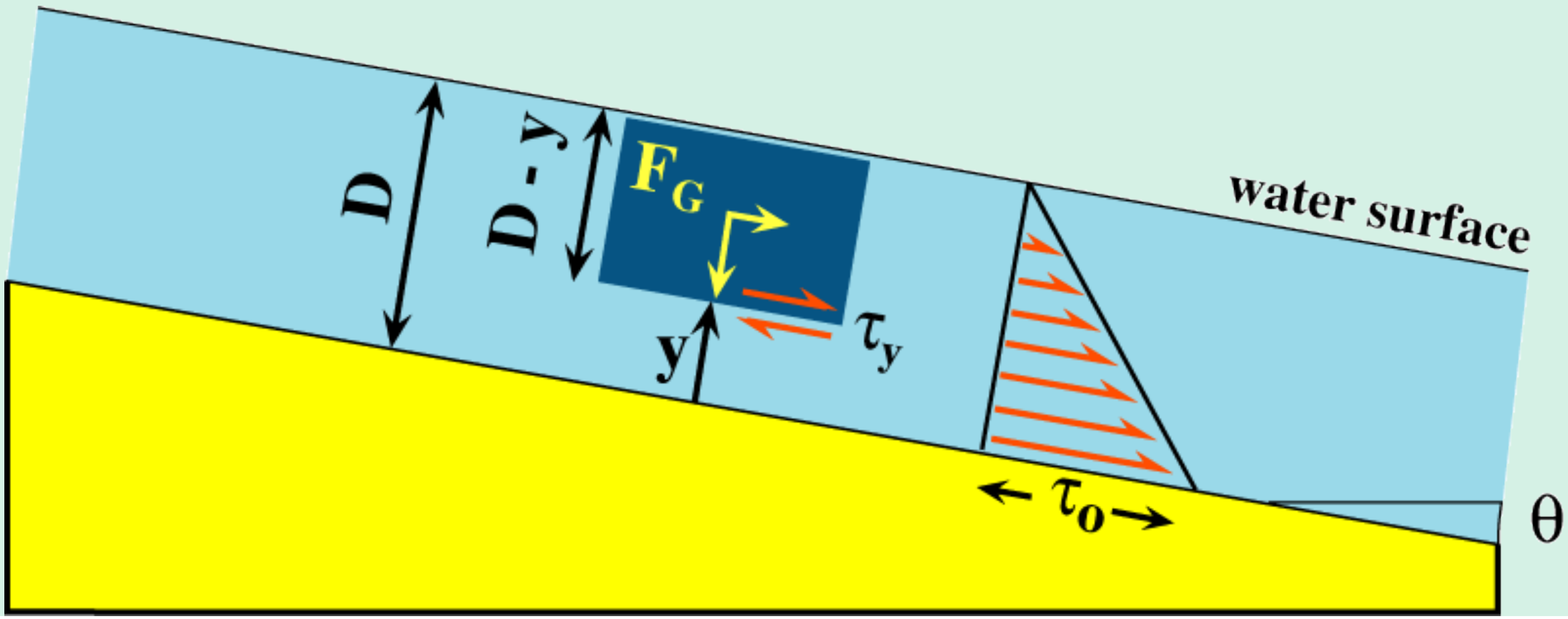


$$\tau_y = \rho g(D - y)\sin\theta$$

At the boundary (y=0) the shear stress is greatest and is referred to as the *boundary shear stress* (τ_o); this is the force per unit area acting on the bed which is available to move sediment.

Setting y=0:

$$\tau_o = \rho g D \sin\theta$$



Given that:

$$\tau_y = \rho g (D - y) \sin\theta$$

and

$$\tau_y = \mu \frac{du}{dy}$$

or $$\frac{\tau_y}{\mu} = \frac{du}{dy}$$

We can calculate the velocity distribution for such flows by substituting:

$$\frac{du}{dy} = \frac{\rho g \sin\theta (D - y)}{\mu}$$

$$\frac{du}{dy} = \frac{\rho g \sin\theta (D - y)}{\mu}$$

Integrating with respect to *y*: $$u_y = \int \frac{du}{dy} dy$$

$$= \frac{\rho g \sin\theta}{\mu} \int (D - y) dy + c$$

$$= \frac{\rho g \sin\theta}{\mu} (yD - \frac{y^2}{2}) + c$$

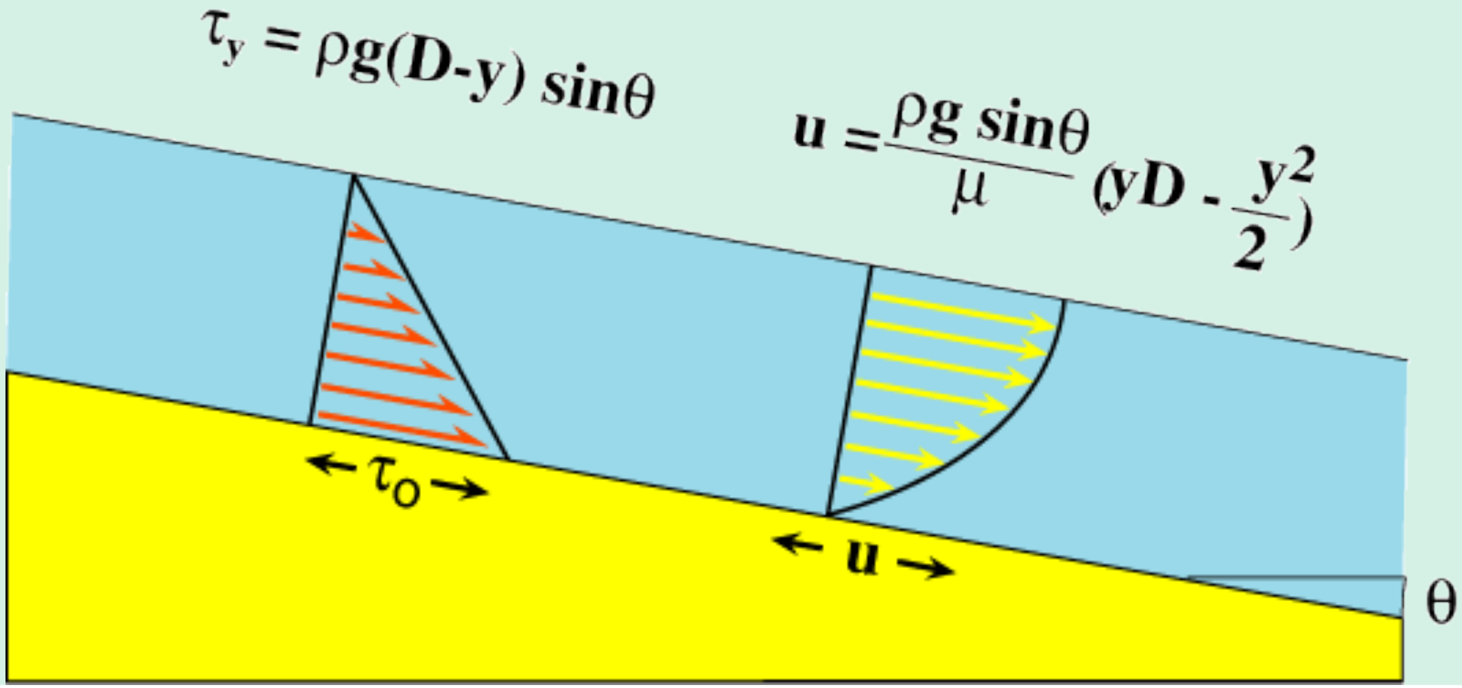
Where c is the constant of integration and equal to the velocity at the boundary (u_y=0) such that:

$$u_y = \frac{\rho g \sin\theta}{\mu} \left(yD - \frac{y^2}{2} \right)$$

$$u_y = \frac{\rho g \sin\theta}{\mu}\left(yD - \frac{y^2}{2}\right)$$

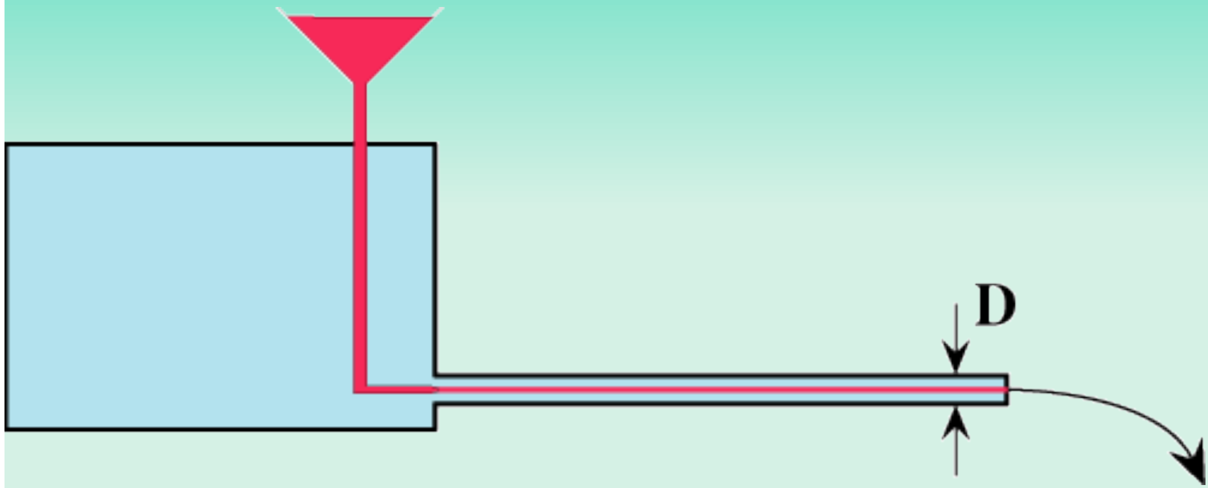
Velocity varies as an exponential function from 0 at the boundary to some maximum at the water surface; this relationship applies to:

Steady flows: not varying in velocity or depth over time.
Uniform flows: not varying in velocity or depth along the channel.
Laminar flows: see next section.



I. The classification of fluid gravity flows

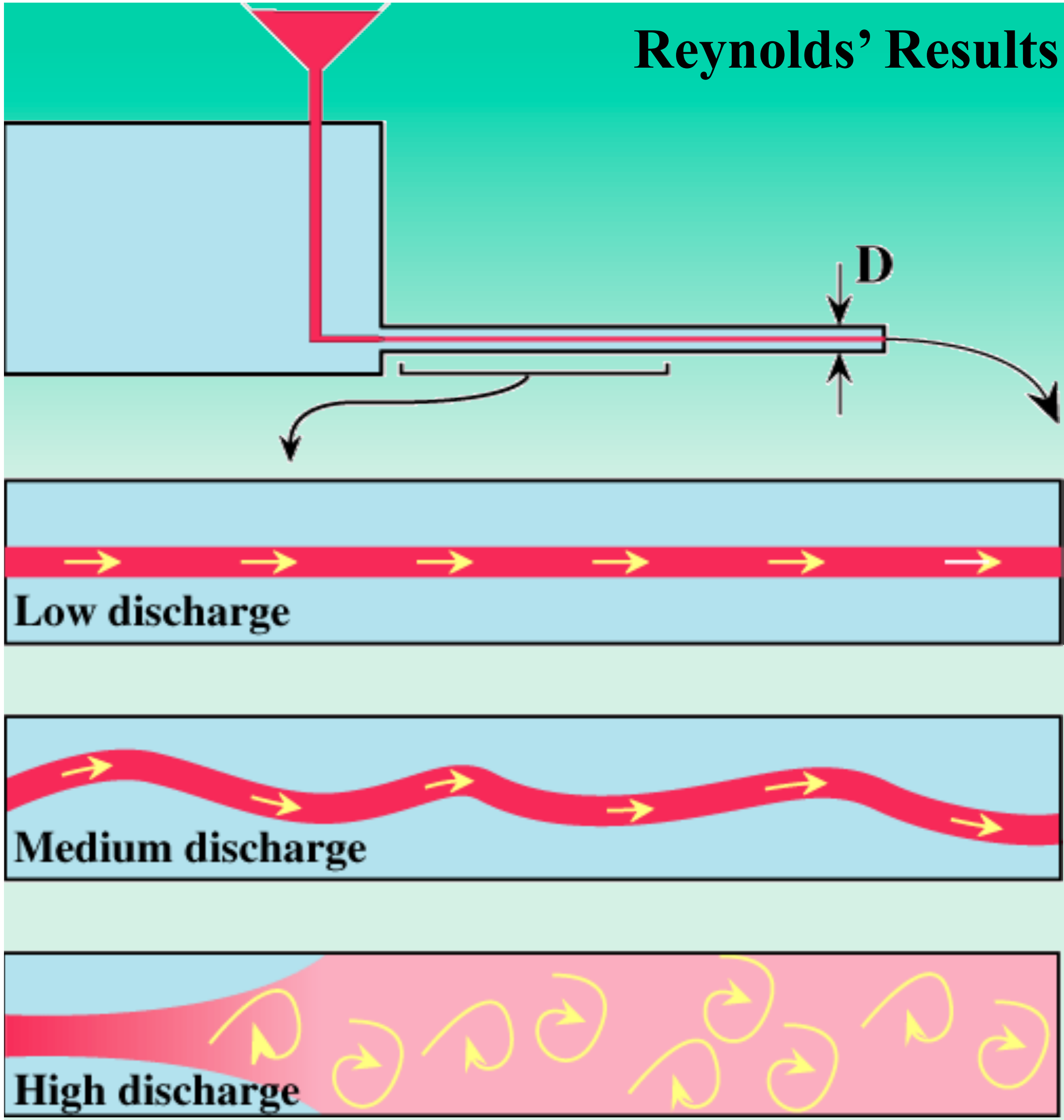
a) Flow Reynolds' Number (R).



Reynold's experiments involved injecting a dye streak into fluid moving at constant velocity through a transparent tube.

Fluid type, tube diameter and the velocity of the flow through the tube were varied.

Reynolds' Results

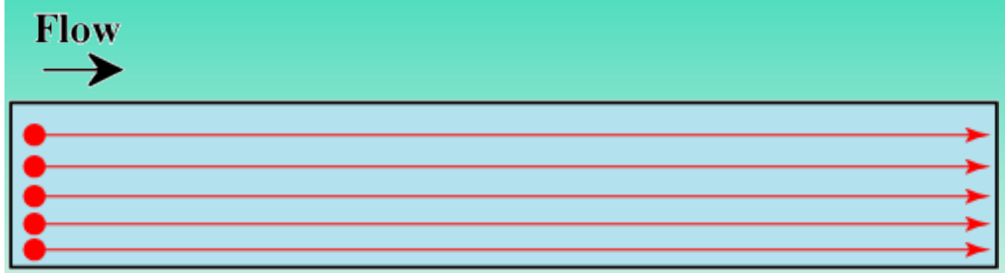


Dye followed a straight path.

Dye followed a wavy path with streak intact.

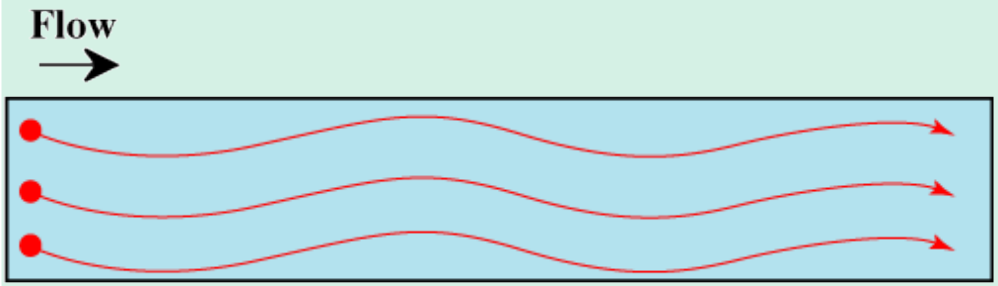
Dye rapidly mixed through the fluid in the tube

Reynolds classified the flow type according to the motion of the fluid.



Low discharge

Laminar Flow: every fluid molecule followed a straight path that was parallel to the boundaries of the tube.



Medium discharge

Transitional Flow: every fluid molecule followed wavy but parallel path that was not parallel to the boundaries of the tube.

High discharge

Turbulent Flow: every fluid molecule followed very complex path that led to a mixing of the dye.

Reynolds found that conditions for each of the flow types depended on:

1. The velocity of the flow (U)
2. The diameter of the tube (D)
3. The density of the fluid (ρ).
4. The fluid's dynamic viscosity (μ).

He combined these variables into a dimensionless combination now known as the Flow Reynolds' Number (**R**) where:

$$\mathbf{R} = \frac{\rho U D}{\mu}$$

$$\mathbf{R} = \frac{\rho U D}{\mu}$$

Flow Reynolds' number is often expressed in terms of the fluids *kinematic viscosity* (ν; lower case Greek letter nu), where:

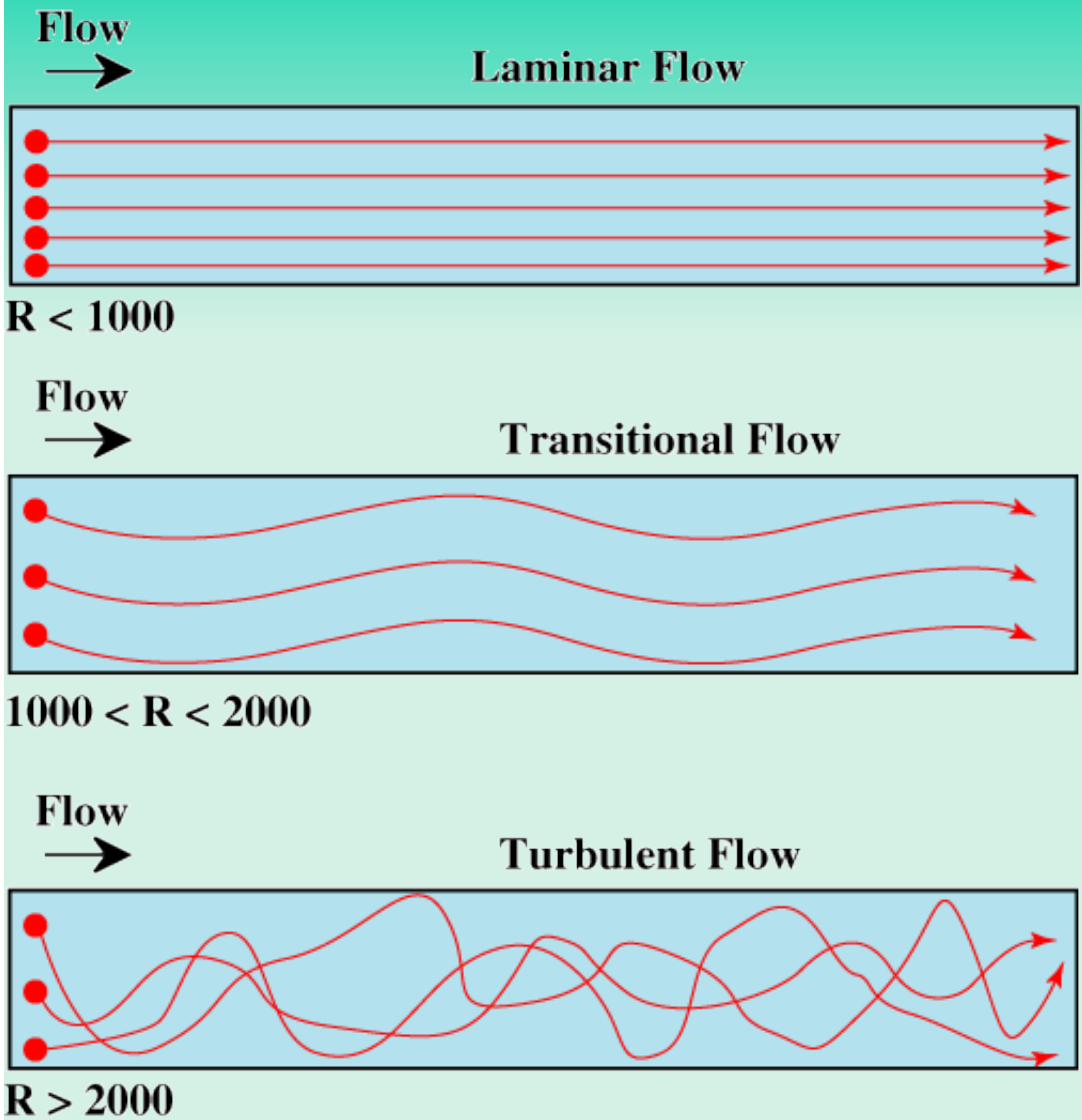
$$\nu = \frac{\mu}{\rho} \qquad \text{(units are m}^2\text{/s)}$$

Such that:

$$\mathbf{R} = \frac{U D}{\nu}$$

The value of **R** determined the type of flow in the experimental tubes:

$$\mathbf{R} = \frac{UD}{\nu}$$

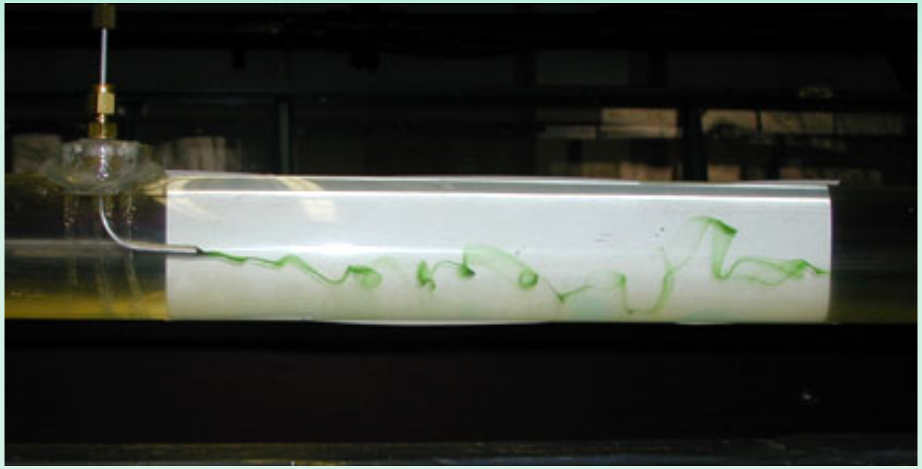


< 1000

1000 - 2000

> 2000

Laminar



Transitional

Turbulent

Example: Given two pipes, one with a diameter of 10 cm and the other with a diameter of 1 m, at what velocities will the flows in each pipe become turbulent?

What is the critical velocity for **R** = 2000?

$$\mathbf{R} = \frac{UD}{\nu} = 2000$$

Solve for U: $$U = \frac{2000\nu}{D}$$

Given: $$\nu = \frac{\mu}{\rho} = \frac{1.005\times10^{-3}}{998.2} = 1.007\times10^{-6}\ \mathrm{m^2/s}$$

Distilled water at 20°C.

Solve for D = 0.1 m and D = 1.0 m.

For a 0.1 m diameter pipe:

$$U = \frac{2000 \times 1.007 \times 10^{-6}}{0.1}$$

$$U = 0.02\text{m/s} = 2\text{cm/s}$$

For a 1.0 m diameter pipe:

$$U = \frac{2000 \times 1.007 \times 10^{-6}}{1}$$

$$U = 0.002\text{m/s} = 2\text{mm/s}$$

b) Flow Froude Number (F).

Classification of flows according to their water surface behaviour.

An important part of the basis for classification of flow regime.

$$\mathbf{F} = \frac{U}{\sqrt{gD}}$$

Where g is the acceleration due to gravity.

$\mathbf{F} < 1$ subcritical flow (tranquil flow)

$\mathbf{F} = 1$ critical flow

$\mathbf{F} > 1$ supercritical flow (shooting flow)

Formulazione matematica

Per ricavare l'espressione precedente che definisce il numero di Froude, bisogna anzitutto esprimere il rapporto tra **forza d'inerzia** e **forza peso** in termini generali.
La forza d'inerzia (F) può essere scritta, in base al secondo principio della dinamica classica, come prodotto tra massa (m) e accelerazione (a):

$$F = ma$$

In una situazione generica, si considera una massa di riferimento m_0 mentre l'accelerazione a può essere espressa come il rapporto tra una lunghezza di riferimento L_0 e il quadrato di un tempo di riferimento t_0, cioè:

$$F = \frac{m_0\, L_0}{t_0^2}$$

moltiplicando e dividendo per L_0, si ottiene:

$$F = \frac{m_0\, L_0^2}{t_0^2\, L_0}$$

Si pone $\dfrac{L_0}{t_0}$ pari ad una velocità di riferimento V_0, per cui:

$$F = \frac{m_0\, V_0^2}{L_0}$$

La forza peso (P) risulta essere il prodotto tra massa di un corpo e accelerazione di gravità agente su di esso, ovvero:

$$P = mg$$

Ricorrendo a delle grandezze di riferimento, possiamo scrivere:

$$P = m_0\, g$$

membro a membro le espressioni delle due forze in termini di grandezze di riferimento, abbiamo:

$$\frac{F}{P} = \frac{\left(\frac{m_0\, V_0^2}{L_0}\right)}{m_0\, g} = \frac{V_0^2}{g\, L_0}$$

A questo punto, mettendo il rapporto delle forze sotto radice, si ottiene l'espressione del numero di Froude:

$$\sqrt{\frac{F}{P}} = \sqrt{\frac{V_0^2}{g\,L_0}} = \mathrm{Fr}$$

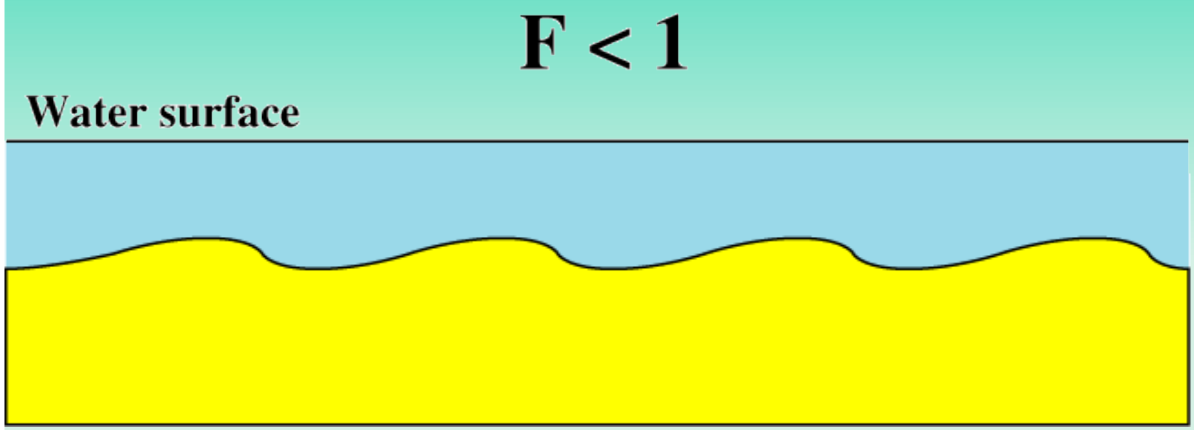
$$\mathbf{F} = \frac{U}{\sqrt{gD}}$$

$\sqrt{gD}$ = the celerity (speed of propagation) of gravity waves on a water surface.

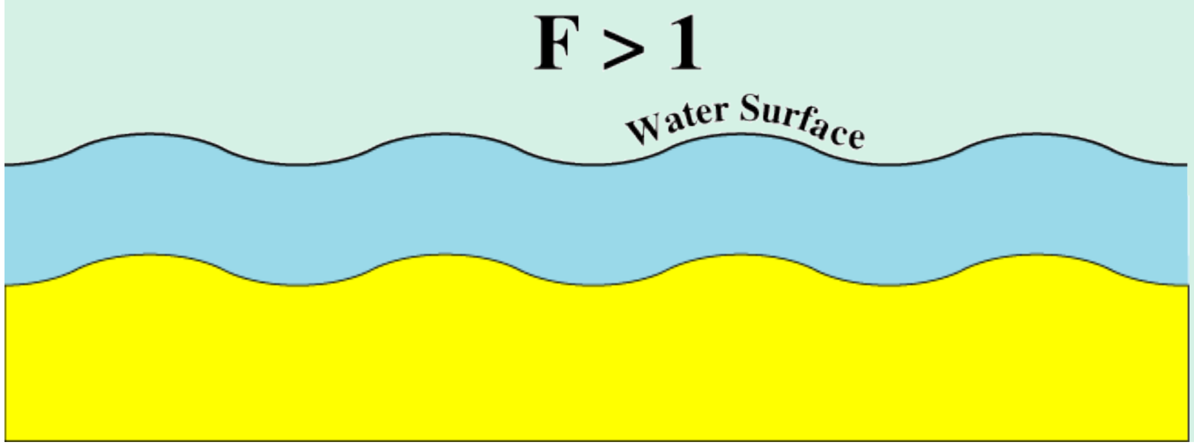
F < 1, U $< \sqrt{gD}$ water surface waves will propagate upstream because they move faster than the current.

F > 1, U $> \sqrt{gD}$ water surface waves will be swept downstream because the current is moving faster than they can propagate upstream.

In sedimentology the Froude number is important to predict the type of bed form that will develop on a bed of mobile sediment.



Bed forms are not in phase with the water surface.



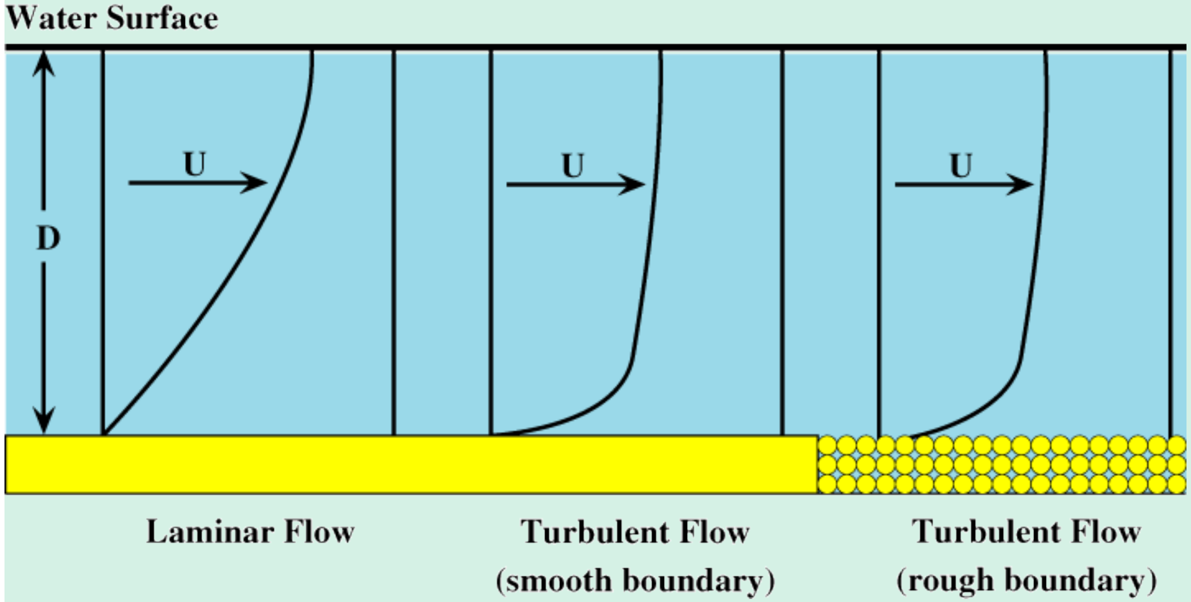
Bed forms are in phase with the water surface.

II. Velocity distribution in turbulent flows

Earlier we saw that for laminar flows the velocity distribution could be determined from:

$$u_y = \frac{\rho g \sin\theta}{\mu}\left(yD - \frac{y^2}{2}\right)$$

In laminar flows the fluid momentum is transferred only by viscous shear; a moving layer of fluid drags the underlying fluid along due to viscosity (see the left diagram, below).



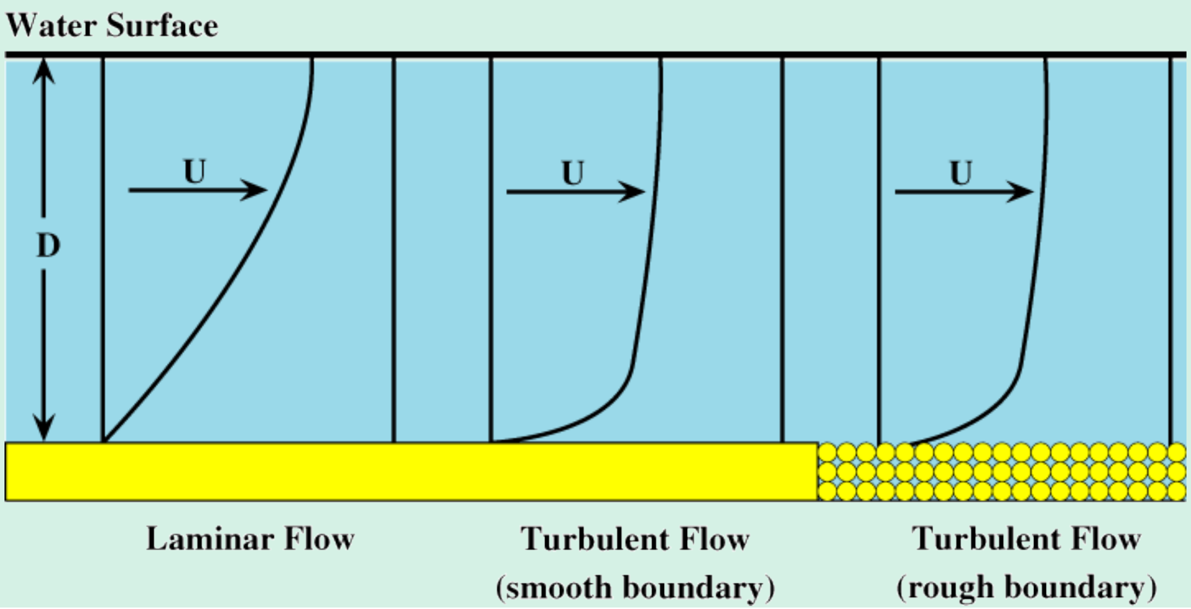
The velocity distribution in turbulent flows has a strong velocity gradient near the boundary and more uniform velocity (on average) well above the boundary.

The more uniform distribution well above the boundary reflects the fact that fluid momentum is being transferred not only by viscous shear.

The chaotic mixing that takes place also transfers momentum through the flow.

The movement of fluid up and down in the flow more evenly distributes the velocity: low speed fluid moves upward from the boundary and high speed fluid in the outer layer moves upward and downward.

This leads to a redistribution of fluid momentum.



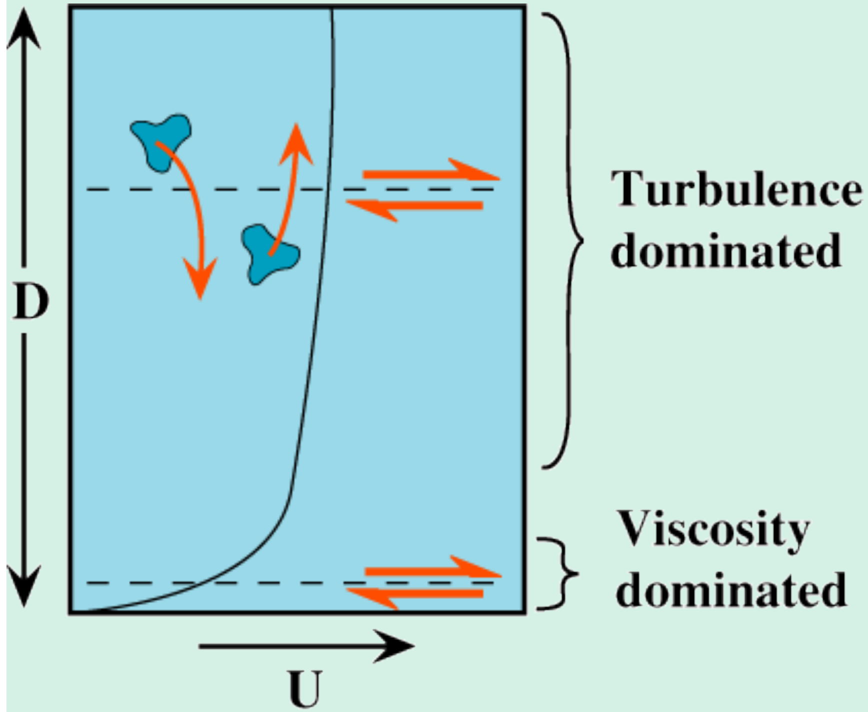
Turbulent flows are made up of two regions:

An inner region near the boundary that is dominated by viscous shear, i.e.,

$$\tau_y = \mu \frac{du}{dy}$$

An outer region that is dominated by turbulent shear (transfer of fluid momentum by the movement of the fluid up and down in the flow).

i.e.,



$$\tau_y = \eta \frac{du}{dy} + \mu \frac{du}{dy}$$

Where η (lower case Greek letter eta) is the *eddy viscosity* which reflects the efficiency by which turbulence transfers momentum through the flow.

As a result, the formula for determining the velocity distribution of a laminar flow cannot be used to determine the distribution for a turbulent flow (it neglects the transfer of momentum by turbulence).

Experimentally determined formulae are used to determine the velocity distribution in turbulent flows.

E.g. the Law of the Wall for rough boundaries under turbulent flows:

$$\frac{u_y}{U_*} = 8.5 + \frac{2.3}{\kappa} \log \frac{y}{y_o}$$

Where κ (lower case Greek letter kappa) is Von Karman's constant (0.41 for clear water flows lacking sediment).

y is the height above the boundary.

$$y_o = \frac{d}{30}$$

Where d is grain size.

U_* is the shear velocity of the flow where: $U_* = \sqrt{\frac{\tau_o}{\rho}}$

If the flow depth and shear velocity are known, as well as the bed roughness, this formula can be used to determine the velocity at any height *y* above the boundary.

$$u_y = U_*\left(8.5 + \frac{2.3}{\kappa}\log\frac{y}{y_o}\right)$$

This formula may be used to estimate the average velocity of a turbulent flow by setting y to 0.4 times the depth of the flow (i.e., y = 0.4D).

Experiments have shown that the average velocity is at 40% of the depth of the flow above the boundary.

$$\frac{u_y}{U_*} = 8.5 + \frac{2.3}{\kappa}\log\frac{y}{y_o}$$

Set y = 0.4D

$$\bar{u} = u_{0.4D} = U_*\left(8.5 + \frac{2.3}{\kappa}\log\frac{0.4D}{y_o}\right)$$

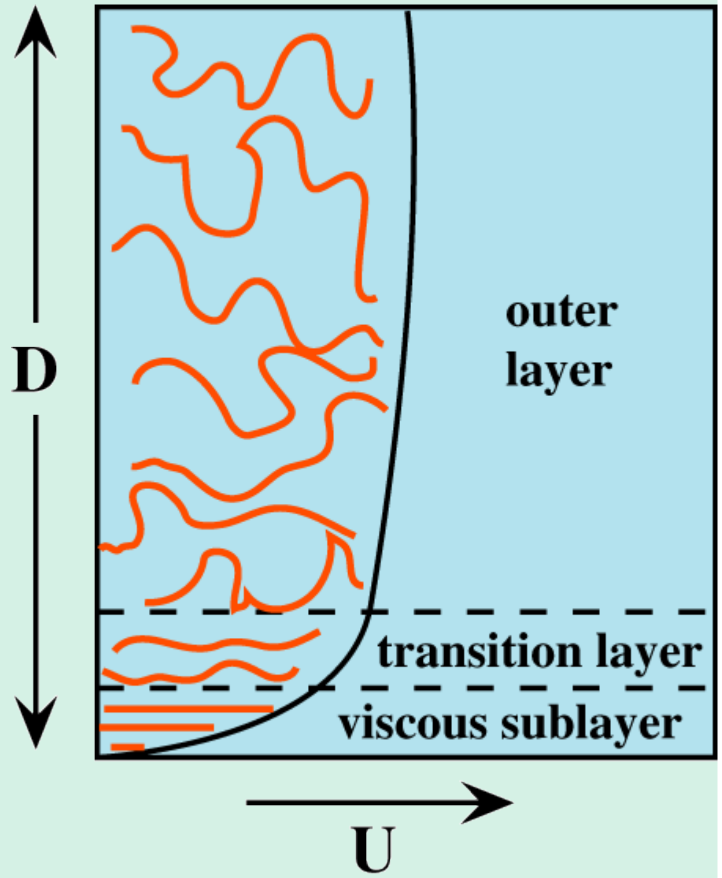
III. Subdivisions of turbulent flows

Turbulent flows can be divided into three layers:

Viscous Sublayer: the region near the boundary that is dominated by viscous shear and quasilaminar flow (also referred to, inaccurately, as the "laminar layer").

Transition Layer: intermediate between quasilaminar and fully turbulent flow.

Outer Layer: fully turbulent and momentum transfer is dominated by turbulent shear.



i) Viscous Sublayer (VSL)

The thickness of the VSL (δ the lower case Greek letter delta) is known from experiments to be related to the kinematic viscosity and the shear velocity of the flow by:

$$\delta = \frac{12\nu}{U_*}$$

It ranges from a fraction of a millimetre to several millimetres thick.

The thickness of the VSL is particularly important in comparison to size of grains (d) on the bed (we'll see later that the forces that act on the grains vary with this relationship).

The *Boundary Reynolds' Number* (**R_***)is used to determine the relationship between δ and d:

$$\mathbf{R}_* = \frac{U_* d}{\nu}$$

A key question is "at what value of **R*** is the diameter of the grains on the bed equal to the thickness of the VSL?"

Given that: $$\delta = \frac{12\nu}{U_*}$$ The condition exists when δ = d.

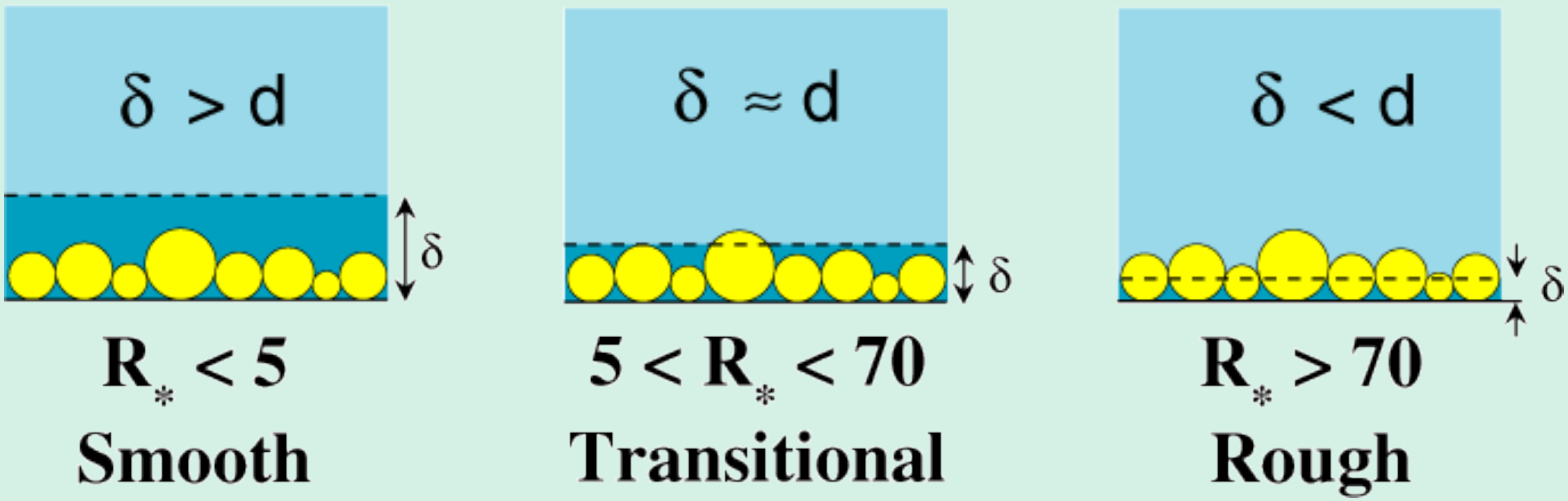
Substituting: $$\mathbf{R}_* = \frac{\cancel{U_*}}{\cancel{\nu}} \times \frac{12\cancel{\nu}}{\cancel{U_*}} = 12$$

$R_* < 12$	$\delta > d$
$R_* = 12$	$\delta = d$
$R_* > 12$	$\delta < d$

Turbulent boundaries are classified on the basis of the relationship between thickness of the VSL and the size of the bed material.

Given that there is normally a range in grain size on the boundary, the following shows the classification:

Classification of turbulent boundaries

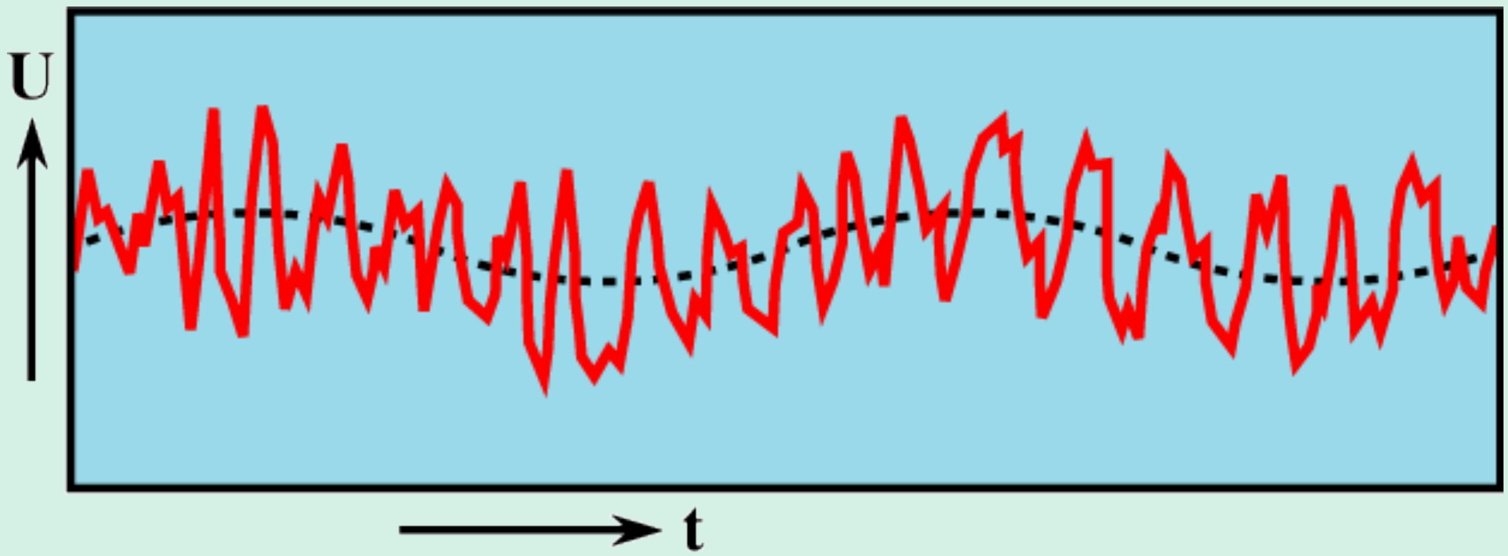


IV. Organized structure of turbulent flows

We characterized turbulent flows as being of a “chaotic” nature marked by random fluid motion.

More accurately, turbulence consists of organized structures of various scale with randomness likely superimposed.

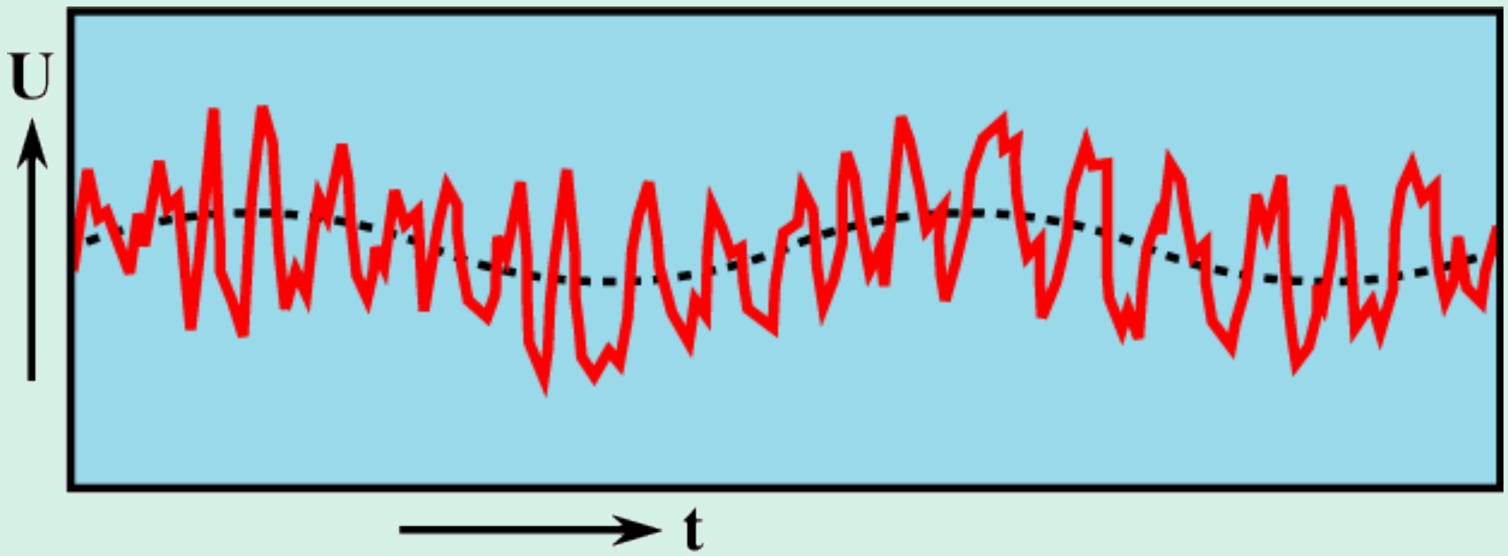
The following illustration shows a hypothetical record of changing flow velocity at a point in a flow.



Note that there are short duration, relatively large magnitude fluctuations that are superimposed on a longer duration, lower magnitude, regular variation in velocity.

Such a pattern of velocity fluctuations is due to large and small scale organized structures.

Note that a similar pattern of variation would be apparent if boundary shear stress were plotted instead of velocity.



Note on boundary shear stress, erosion and deposition

At the boundary of a turbulent flow the boundary shear stress (τ_o) can be determined using the same relationship as for a laminar flow.

In the viscous sublayer viscous shear predominates so that the same relationship exists:

$$\tau_o = \rho g D \sin\theta$$

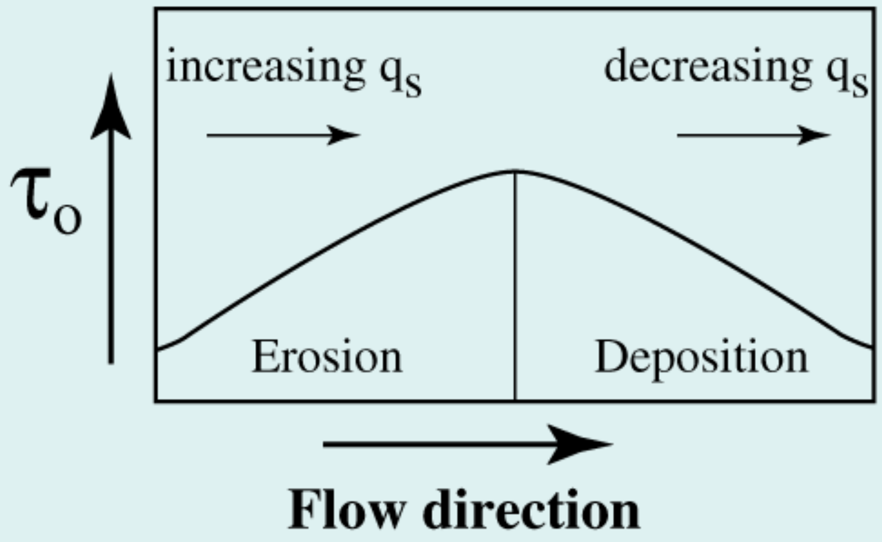
This applies to steady, uniform turbulent flows.

Boundary shear stress governs the power of the current to move sediment; specifically, erosion and deposition depend on the change in boundary shear stress in the downstream direction.

In general, sediment transport rate (q_s; the amount of sediment that is moved by a current) increases with increasing boundary shear stress.

When τ_o increases downstream, so does the sediment transport rate; this leads to erosion of the bed (providing that τ_o is sufficient to move the sediment).

When τ_o decreases downstream, so does the sediment transport rate; this leads to deposition of sediment on the bed



Variation in τ_o along and across the flow due to turbulence leads to a pattern of erosion and deposition on the bed of a mobile sediment.

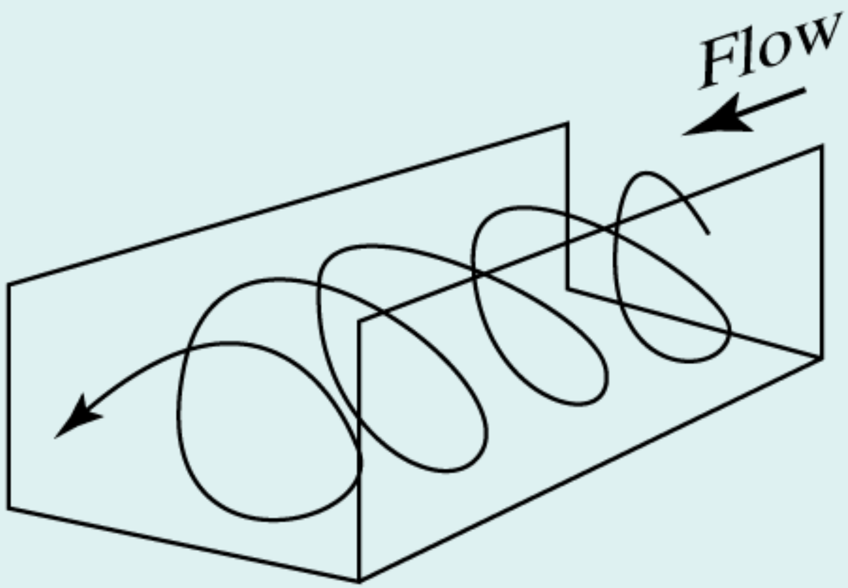
a) Large scale structures of the outer layer

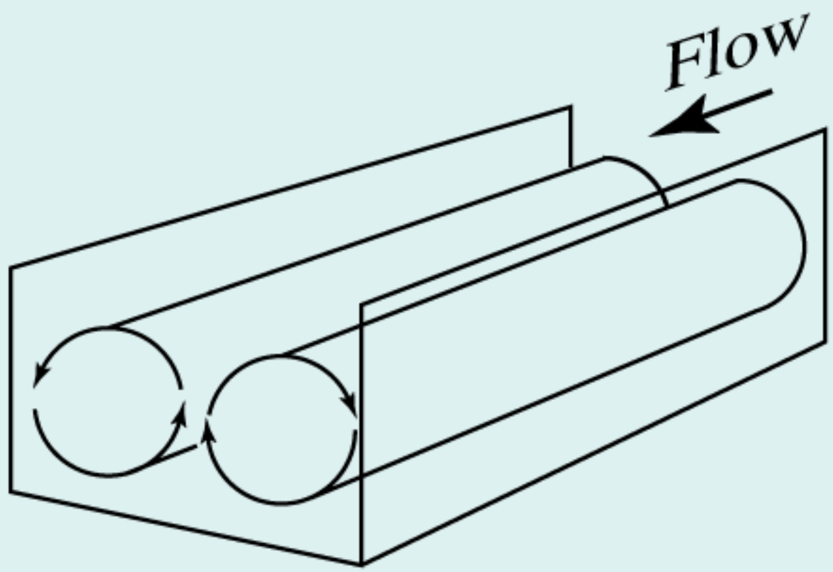
Rotational structures in the outer layer of a turbulent flow.

i) Secondary flows.

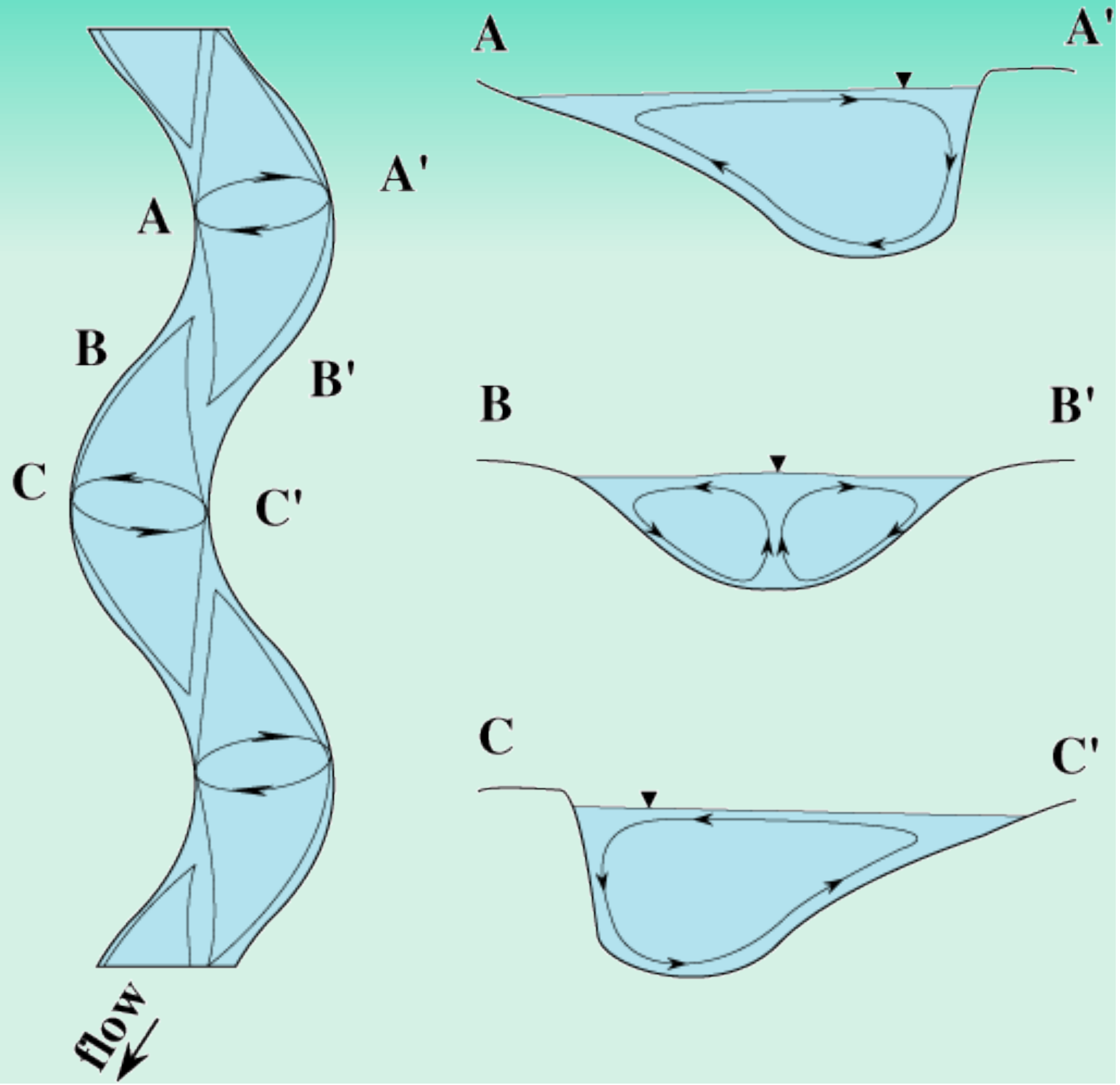
Involves a rotating component of the motion of fluid about an axis that is parallel to the mean flow direction.

Commonly there are two or more such rotating structures extending parallel to each other.





In meandering channels, characterized by a sinusoidal channel form, counter-rotating spiral cells alternate from side to side along the channel.

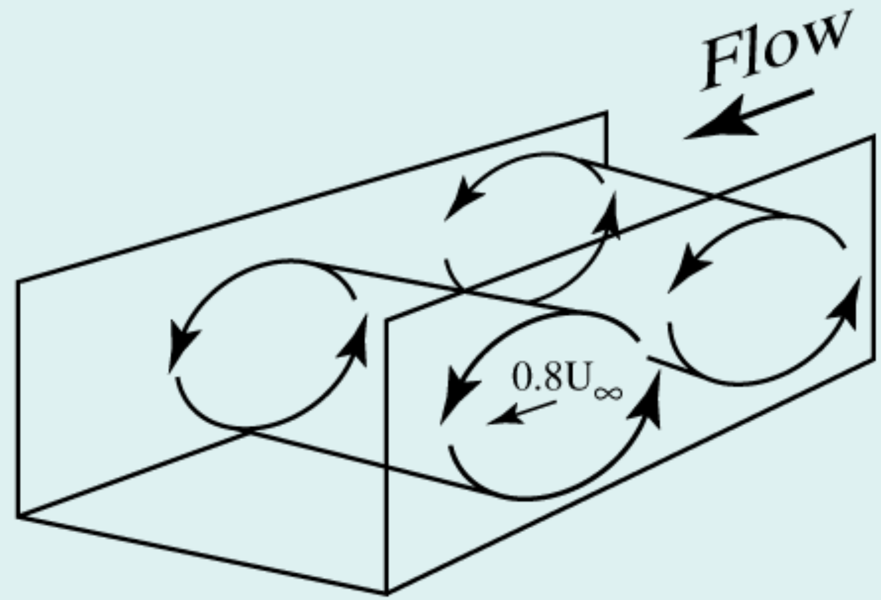


ii) Eddies (vortici)

Components of turbulence that rotate about axes that are perpendicular to the mean flow direction.

Smaller scale than secondary flows and move downstream with the current at a speed of approximately 80% of the water surface velocity (U_∞).

Eddies move up and down within the flow as the travel downstream and lead to variation in boundary shear stress over time and along the flow direction.

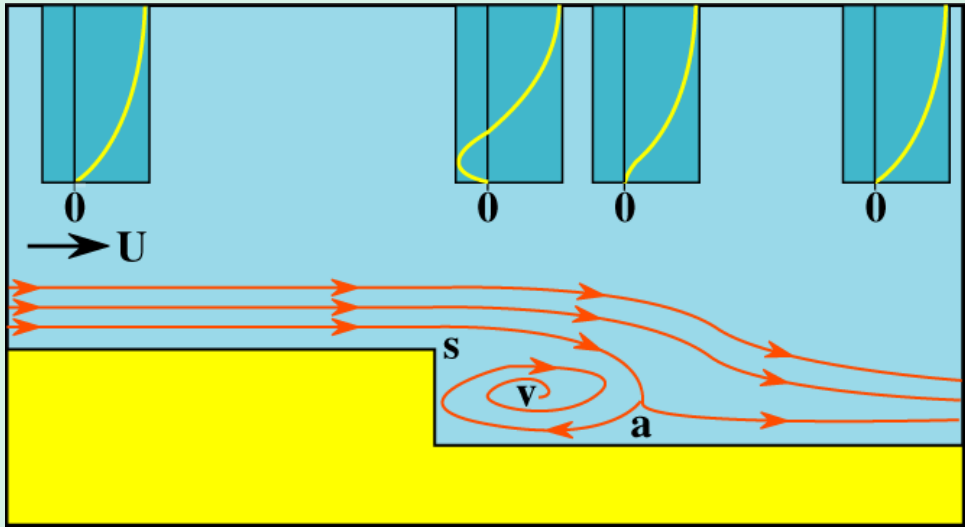


Some eddies are created by the topography of the bed.

In the lee of a negative step on the bed (see figure below) the flow separates from the boundary (“s” in the figure) and reattaches downstream (“a” in the figure).

A *roller eddy* develops between the point of separation and the point of attachment.

Asymmetric bed forms (see next chapter) develop similar eddies.



b) Small scale structures of the viscous sublayer.

i) Streaks

Alternating lanes of high and low speed fluid within the VSL.

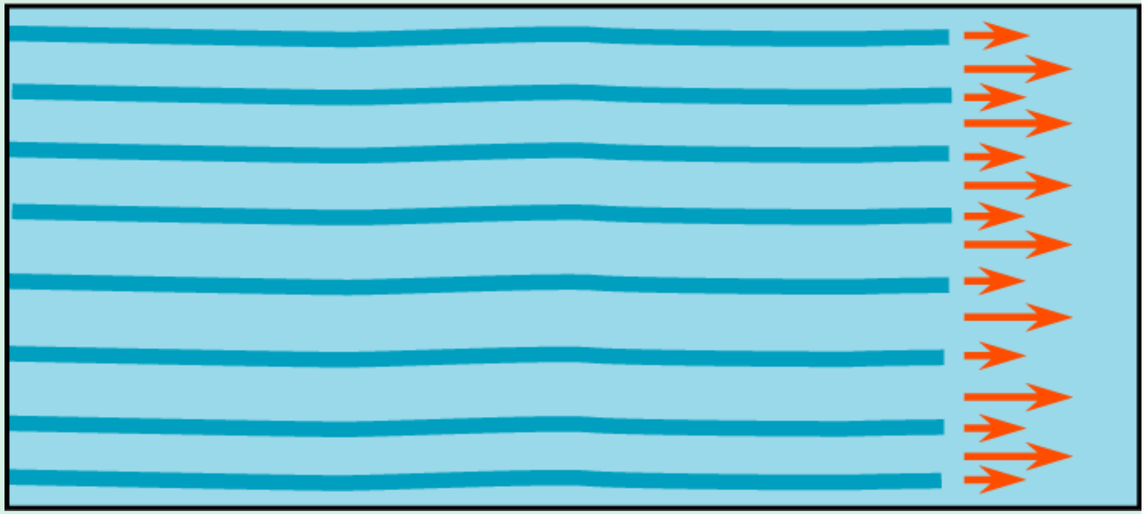
Associated with counter-rotating, flow parallel vortices within the VSL.

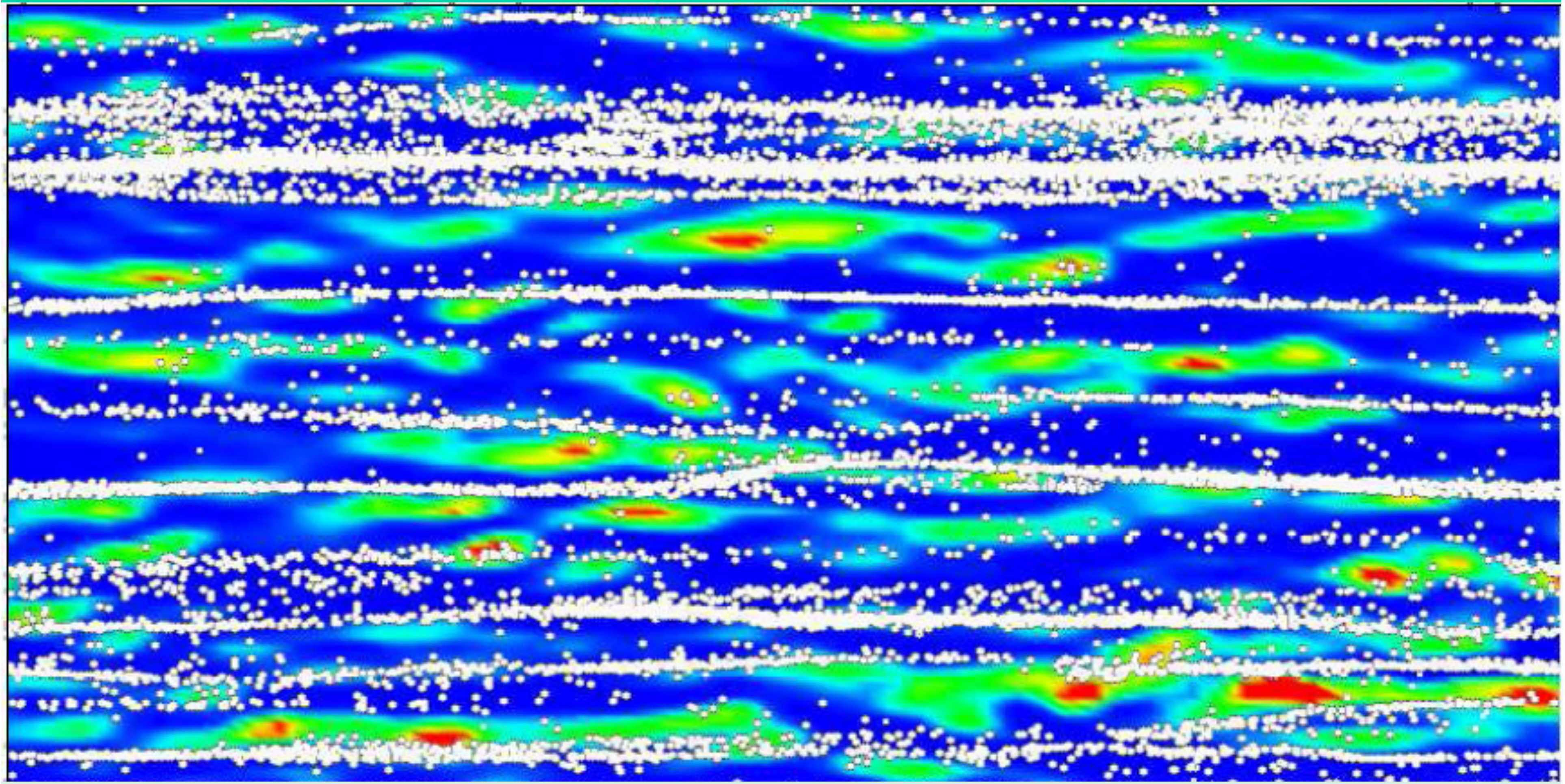
Streak spacing (λ) varies with the shear velocity and the kinematic viscosity of the fluid; λ ranges from millimetres to centimetres.

$$\lambda \approx \frac{100\nu}{U_*}$$

λ increases when sediment is present.

View looking down onto alternating high-speed and low-speed streaks in the viscous sublayer





Red = high velocity
Blue = low velocity

ii) Bursts and sweeps (lett. scoppi e spazzate)

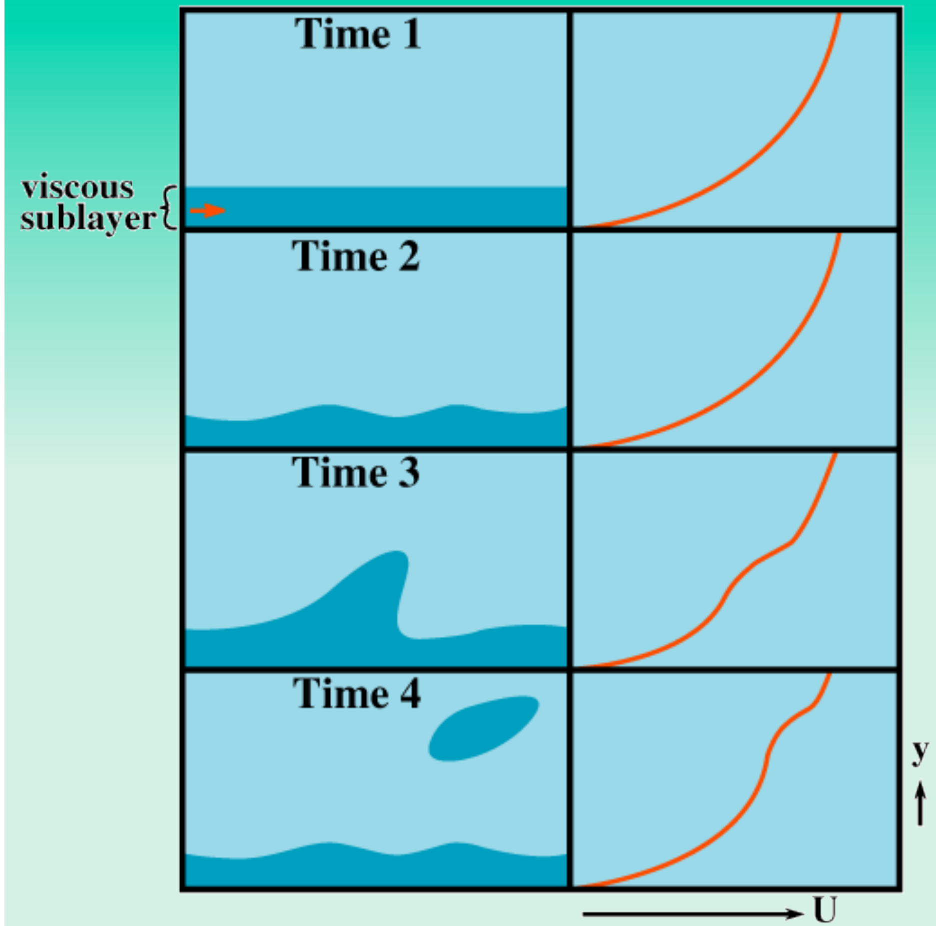
Burst: ejection of low speed fluid from the VSL into the outer layer.

Sweep: injection of low speed fluid from the outer layer into the VSL.

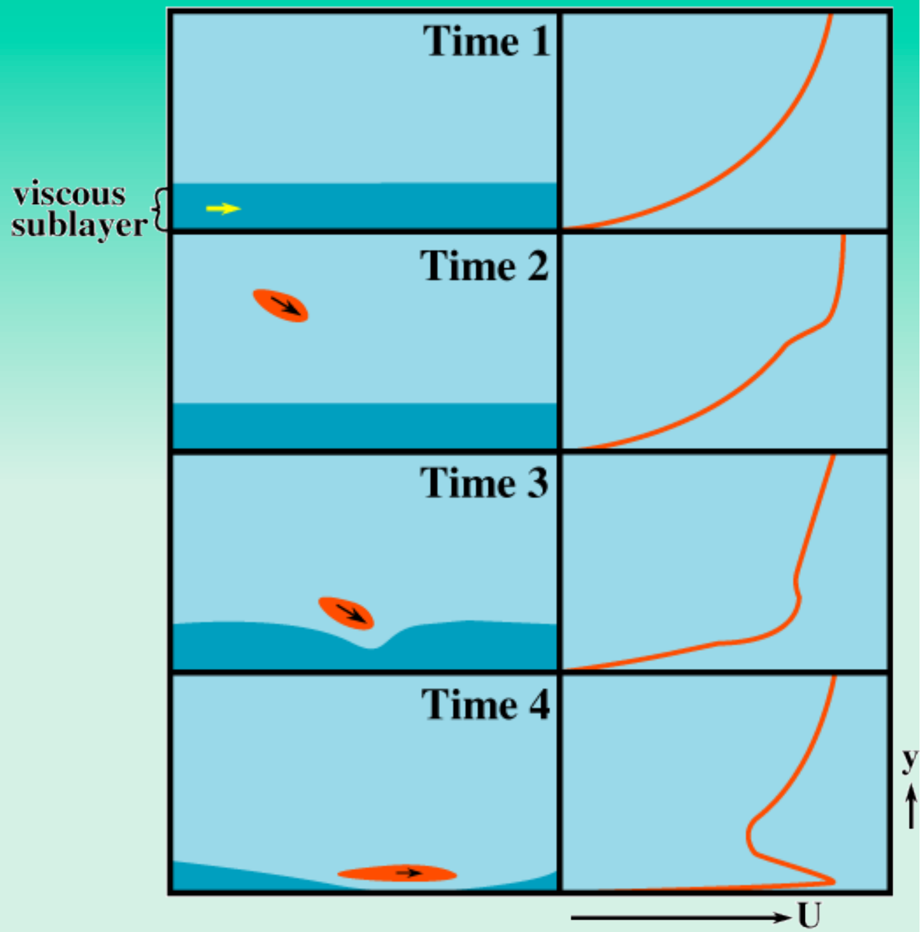
Often referred to as the “bursting cycle” but not every sweep causes a burst and vise versa.

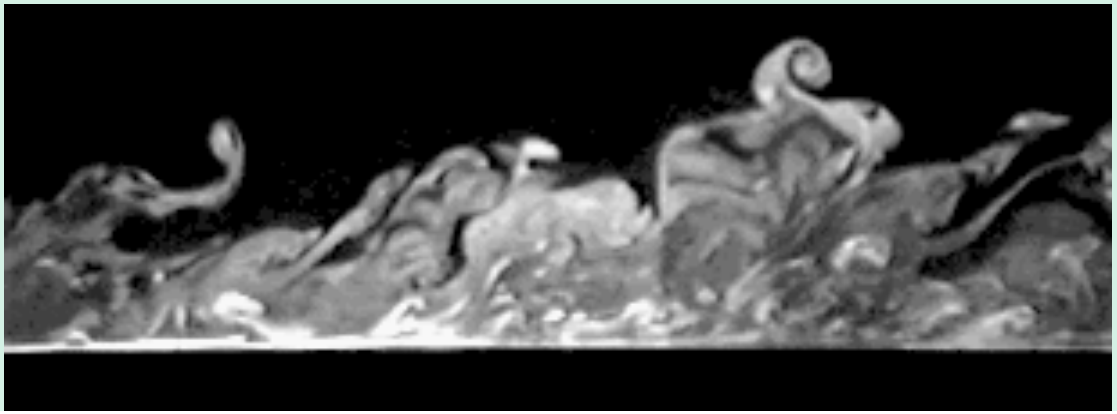
However, the frequency of bursting and sweeps are approximately equal.

A burst



A Sweep





Sediment transport under unidirectional flows

I. Classification of sediment load

The sediment that is transported by a current.

Two main classes:

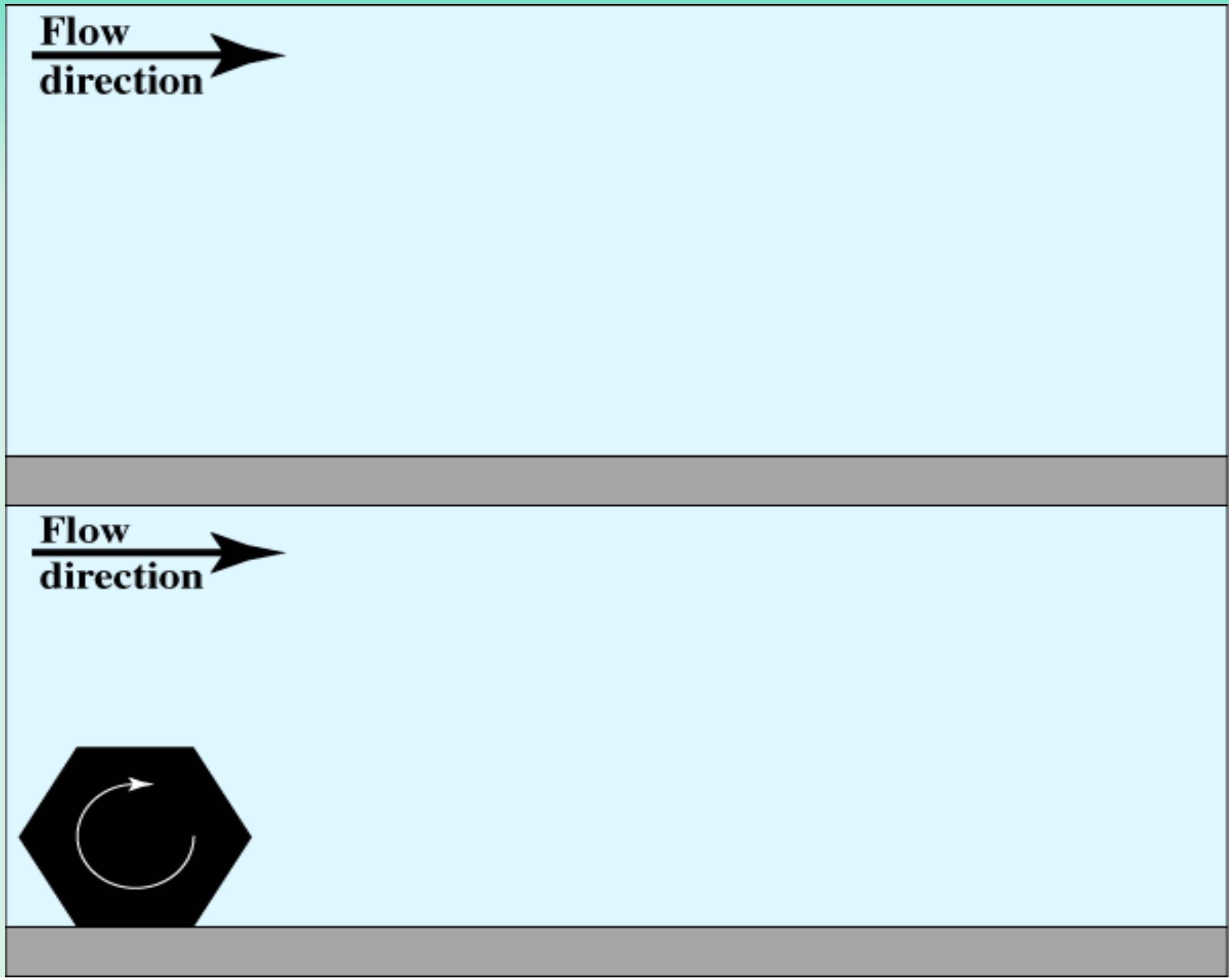
Wash load: silt and clay size material that remains in suspension even during low flow events in a river.

Bed material load: sediment (sand and gravel size) that resides in the bed but goes into transport during high flow events (e.g., floods).

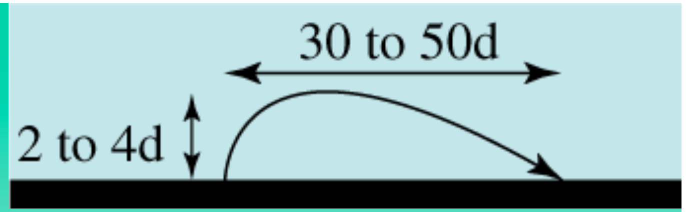
Bed material load makes up many arenites and rudites in the geological record.

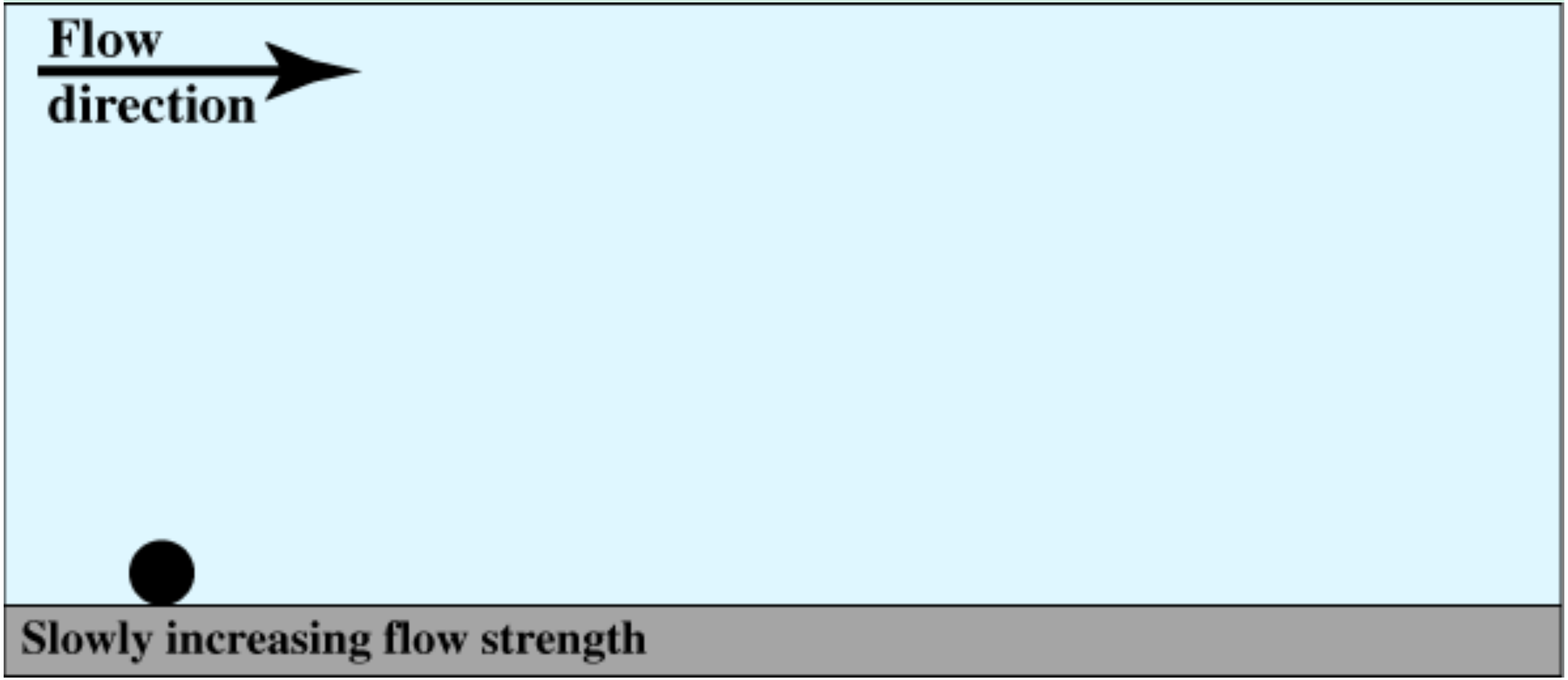
Three main components of bed material load.

Contact load: particles that move in contact with the bed by sliding or rolling over it.

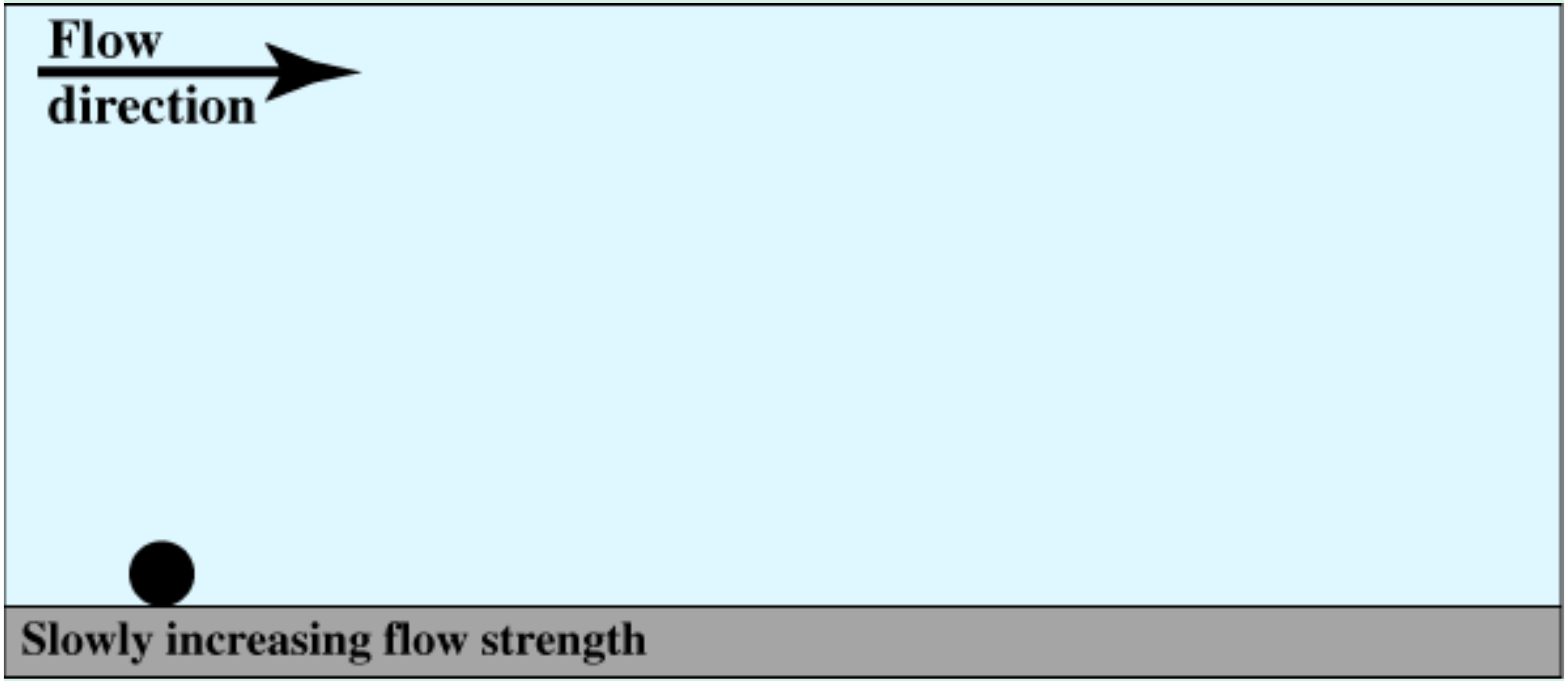


Saltation load: movement as a series of "hops" along the bed, each hop following a ballistic trajectory.





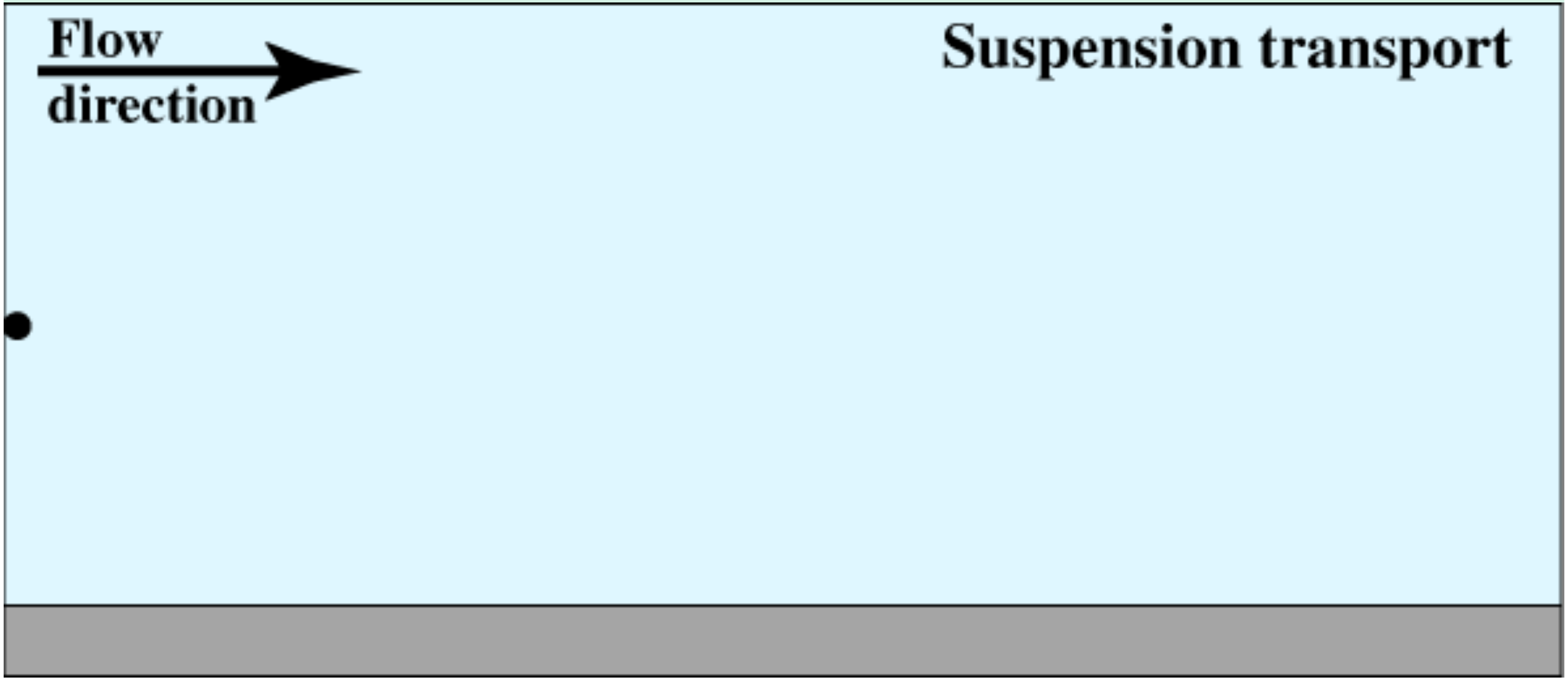
When the ballistic trajectory is disturbed by turbulence the motion is referred to as *Suspensive saltation*.



Intermittent suspension load: carried in suspension by turbulence in the flow.

“Intermittent” because it is in suspension only during high flow events and otherwise resides in the deposits of the bed.

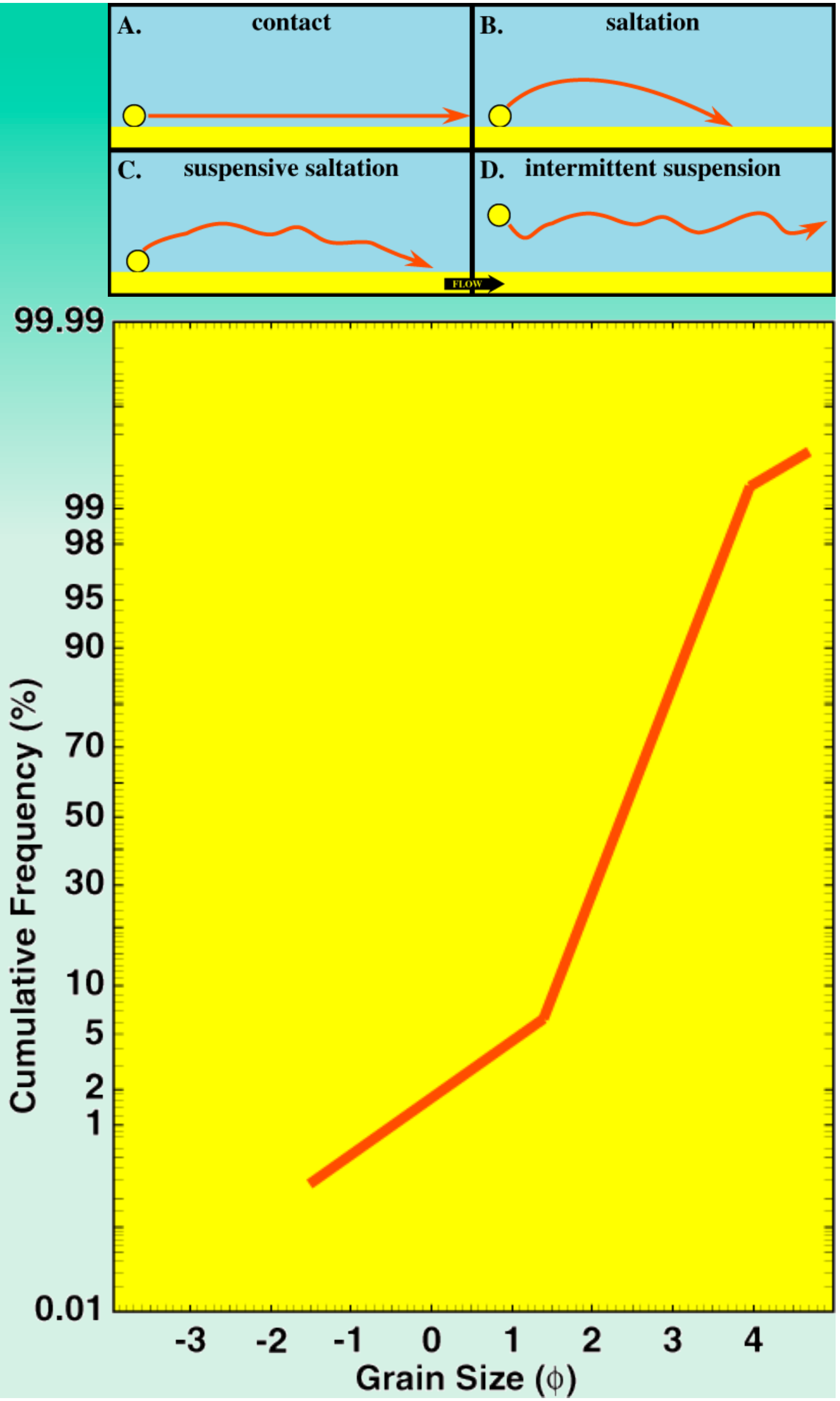
Bursting is an important process in initiating suspension transport.



II. Hydraulic interpretation of grain size distributions

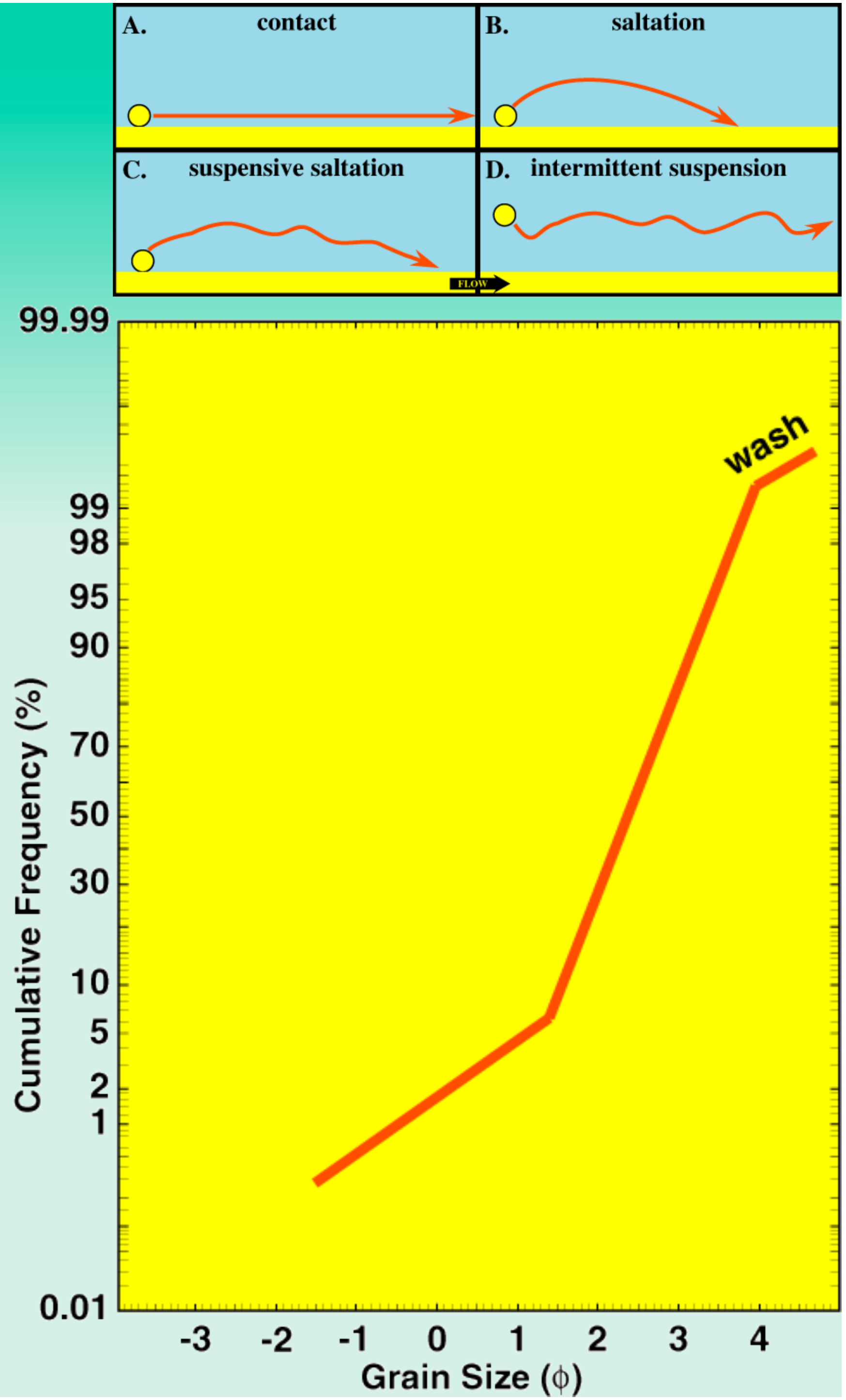
In the section on grain size distributions we saw that some sands are made up of several normally distributed subpopulations.

These subpopulations can be interpreted in terms of the modes of transport that they underwent prior to deposition.



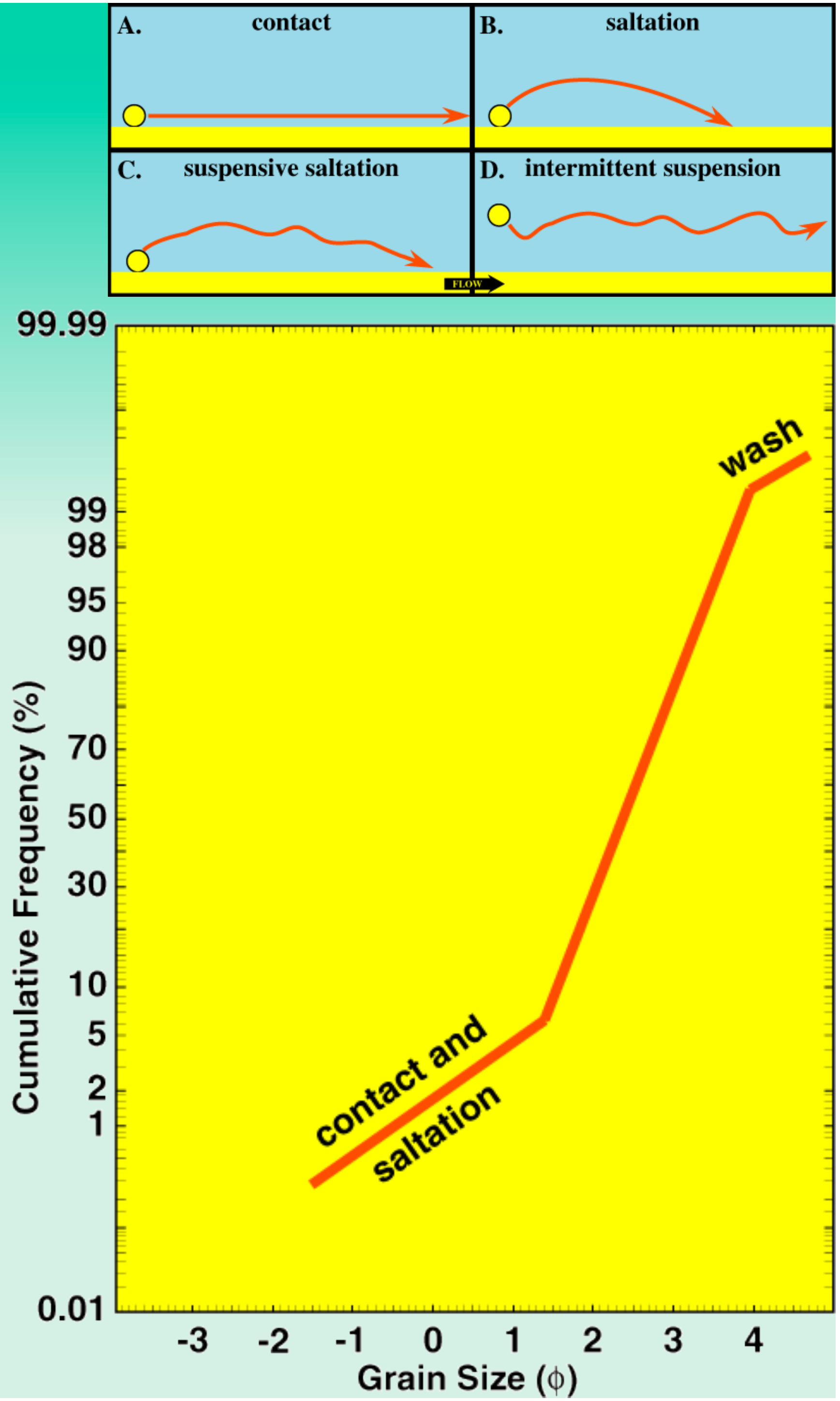
The finest subpopulation represents the wash load.

Only a very small amount of wash load is ever stored within the bed material so that it makes up a very small proportion of these deposits.



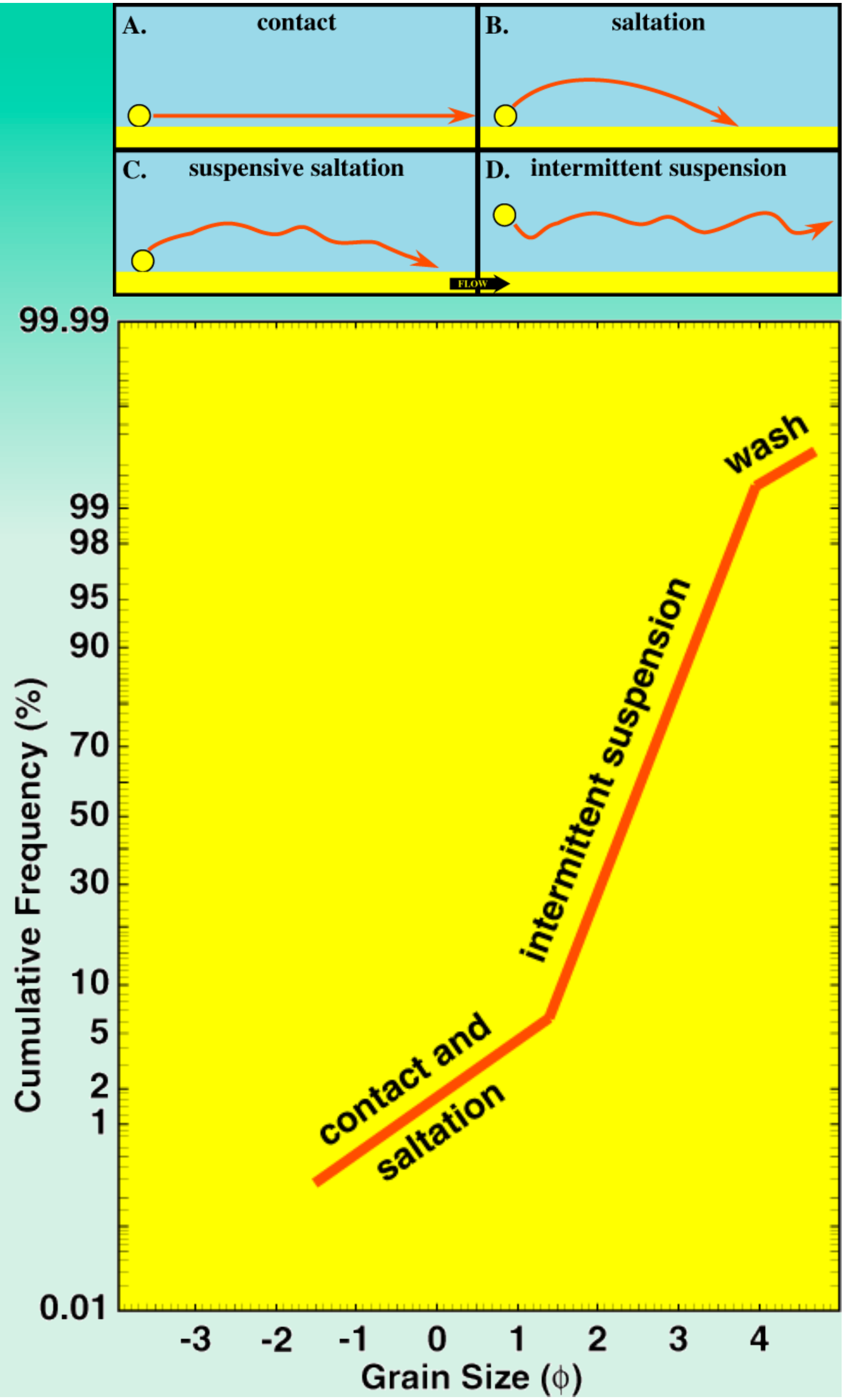
The coarsest subpopulation represents the contact and saltation loads.

In some cases they make up two subpopulations (only one is shown in the figure.



The remainder of the distribution, normally making up the largest proportion, is the intermittent suspension load.

This interpretation of the subpopulations gives us two bases for quantitatively determining the strength of the currents that transported the deposits.



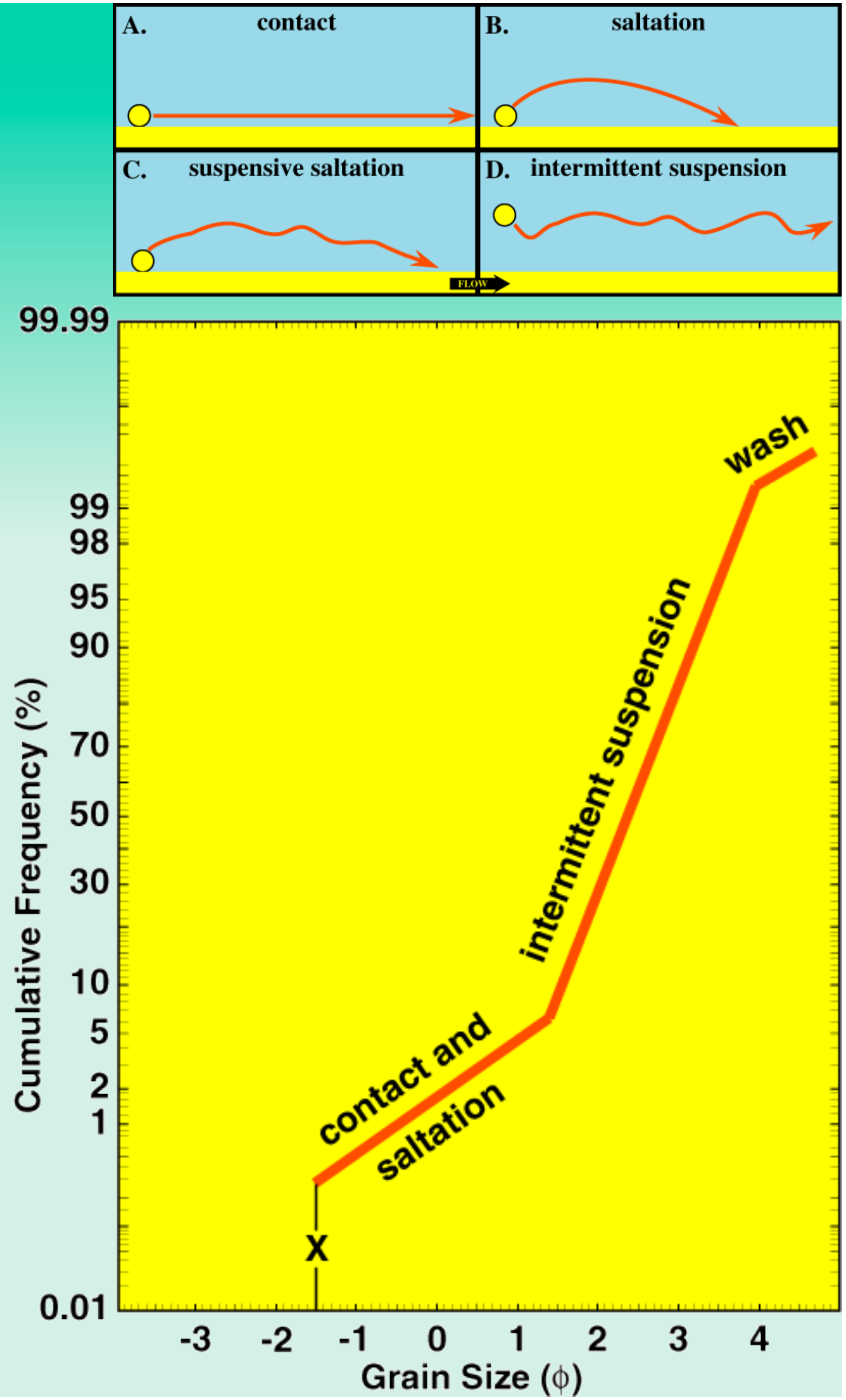
The grain size "X" is the coarsest sediment that the currents could move on the bed.

In this case, X = -1.5 ϕ or approximately 2.8 mm.

If the currents were weaker, that grain size would not be present.

If the currents were stronger, coarser material would be present.

This assumes that there were no limitations to the size of grains available in the system.



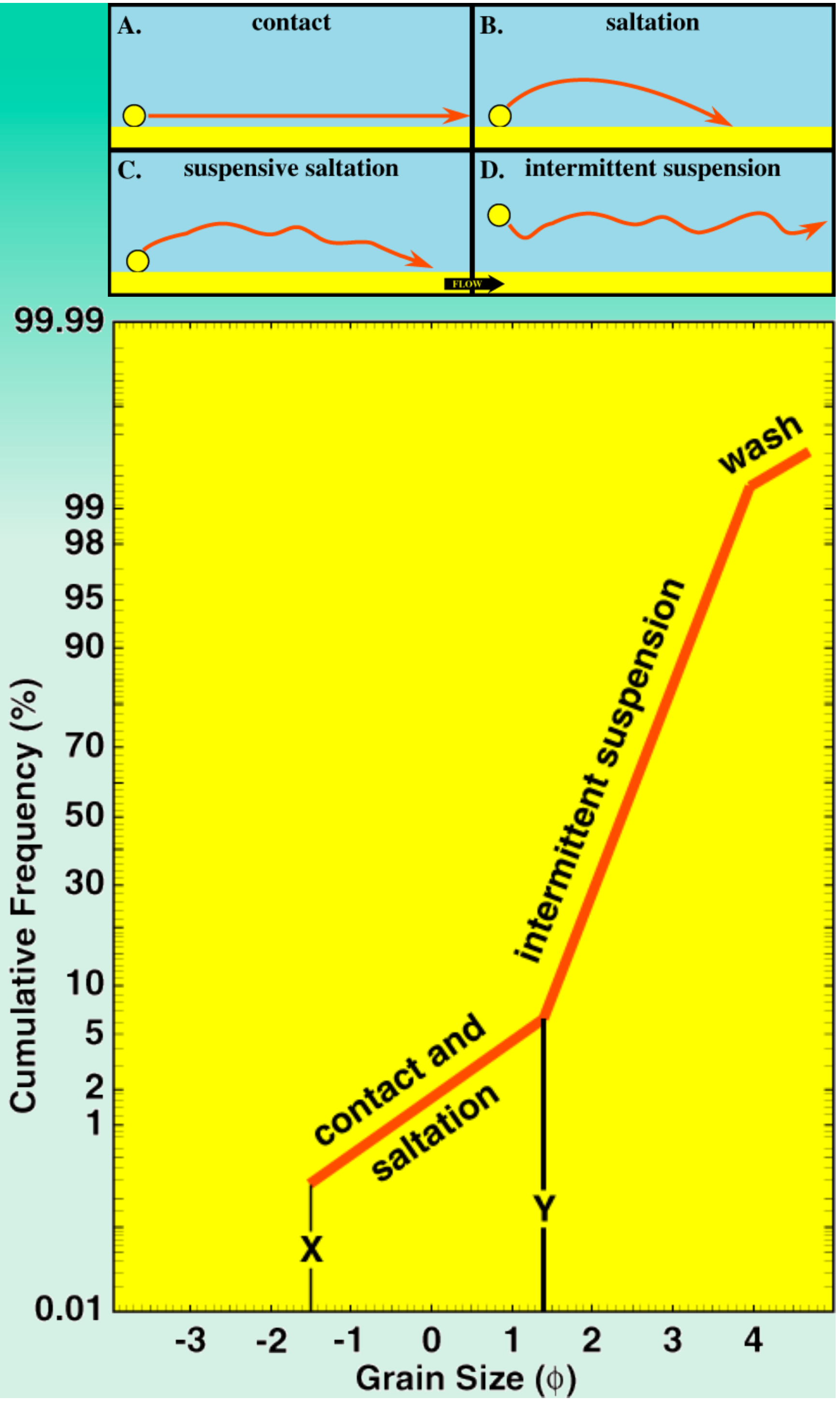
The grain size “Y” is the coarsest sediment that the currents could take into suspension.

In this case, Y = 1.3 ϕ or approximately 0.41 mm.

Therefore the currents must have been just powerful enough to take the 0.41 mm particles into suspension.

If the currents were stronger the coarsest grain size would be larger.

This assumes that there were no limitations to the size of grains available in the system.

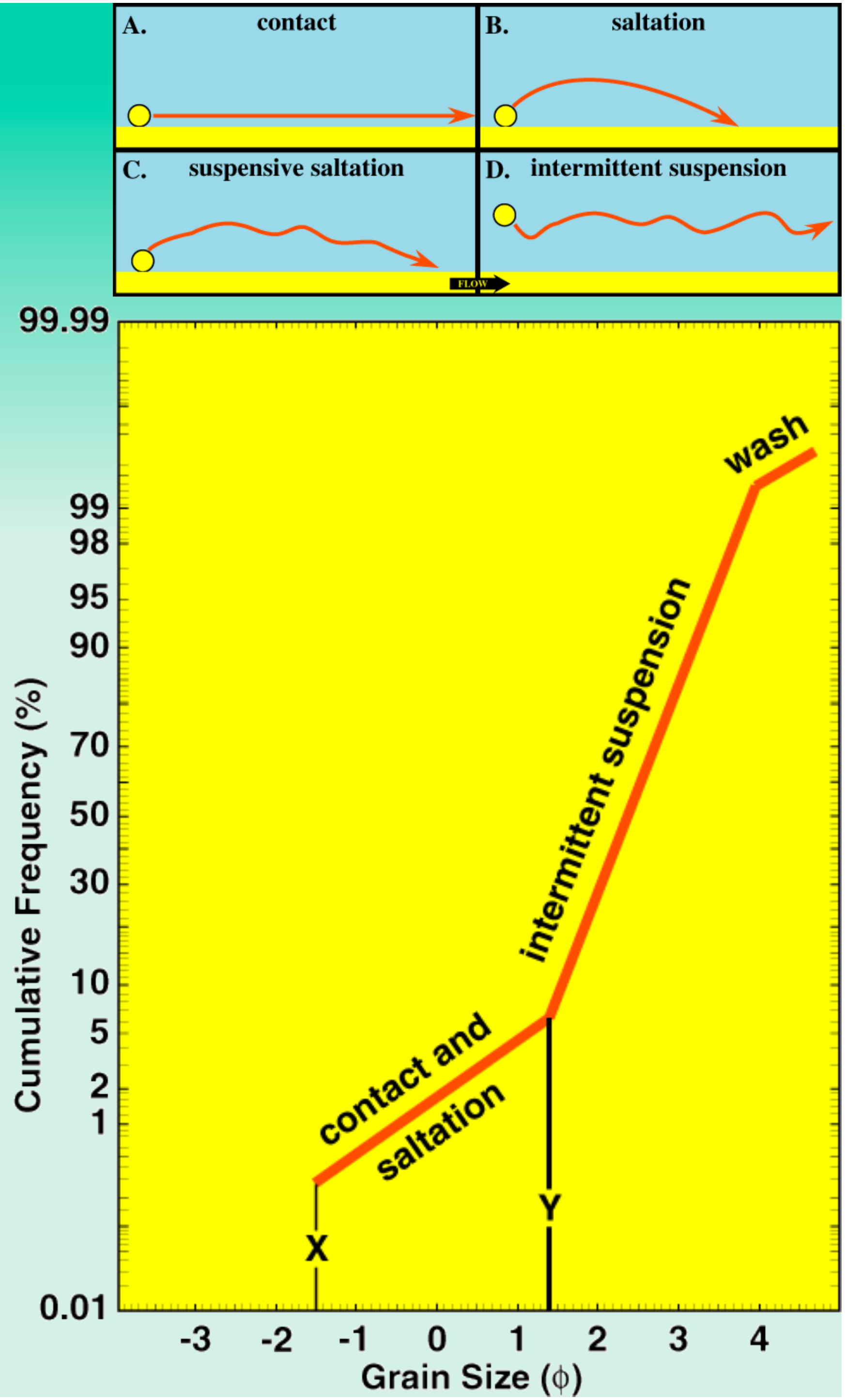


To quantitatively interpret “X” we need to know the hydraulic conditions needed to just begin to move of that size.

This condition is the “threshold for sediment movement”.

To quantitatively interpret “Y” we need to know the hydraulic conditions needed to just begin carry that grain size in suspension.

This condition is the “threshold for suspension”.



a) The threshold for grain movement on the bed.

Grain size "X" can be interpreted if we know what flow strength is required to just move a particle of that size.

That flow strength will have transported sediment with that maximum grain size.

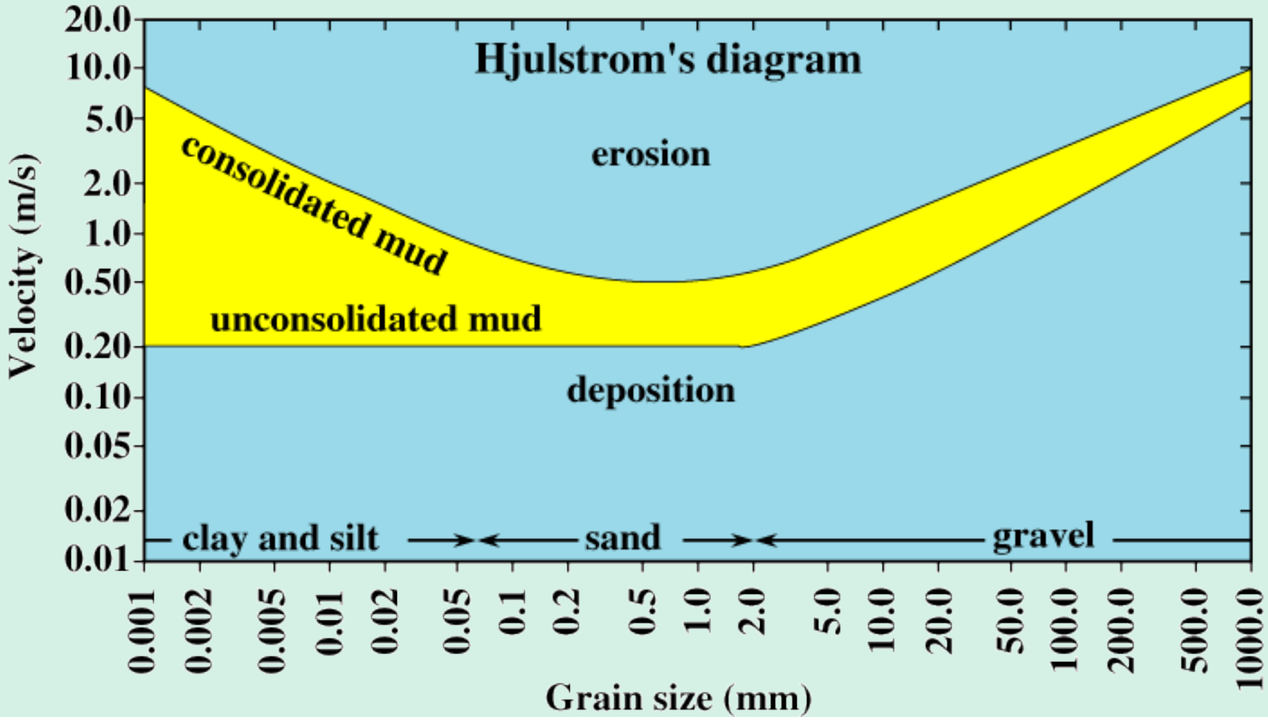
Several approaches have been taken to determine the critical flow strength to initiate motion on the bed.

i) Hjulstrom's Diagram

Based on a series of experiments using unidirectional currents with a flow depth of 1 m.

The diagram (below) shows the critical velocity that is required to just begin to move sediment of a given size (the top of the yellow field).

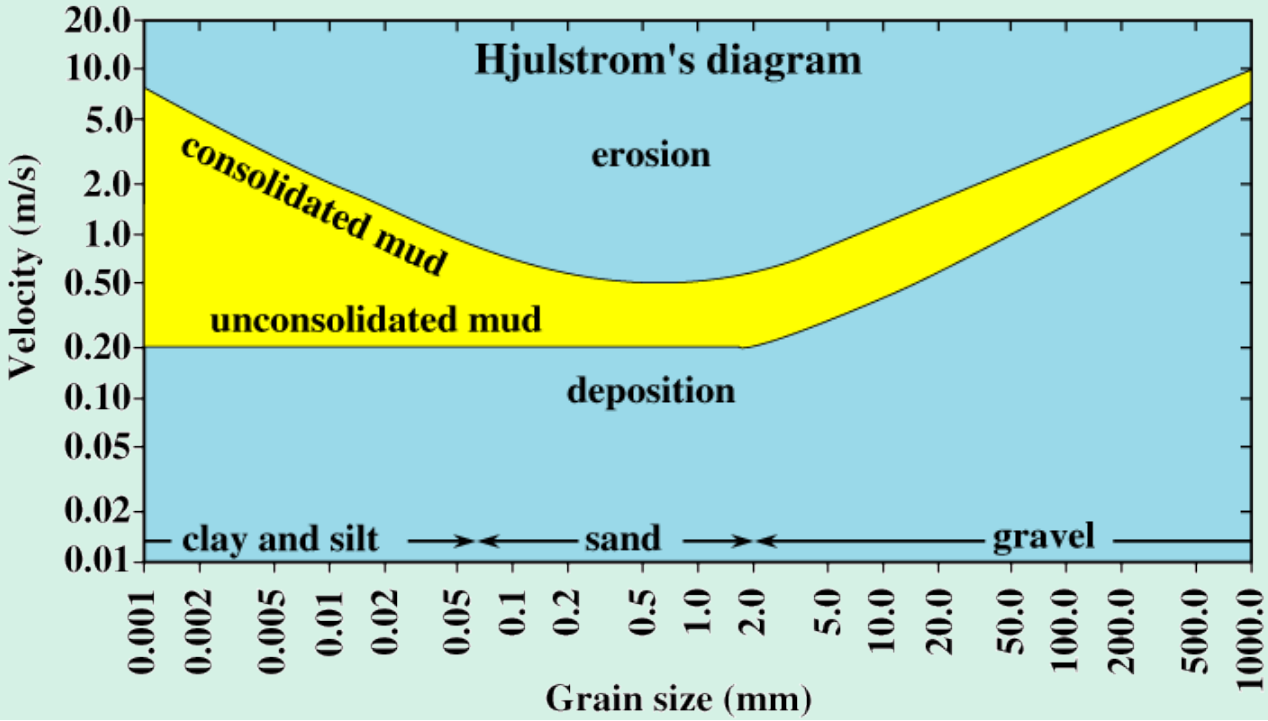
It also shows the critical velocity for deposition of sediment of a given size (the bottom of the yellow field).



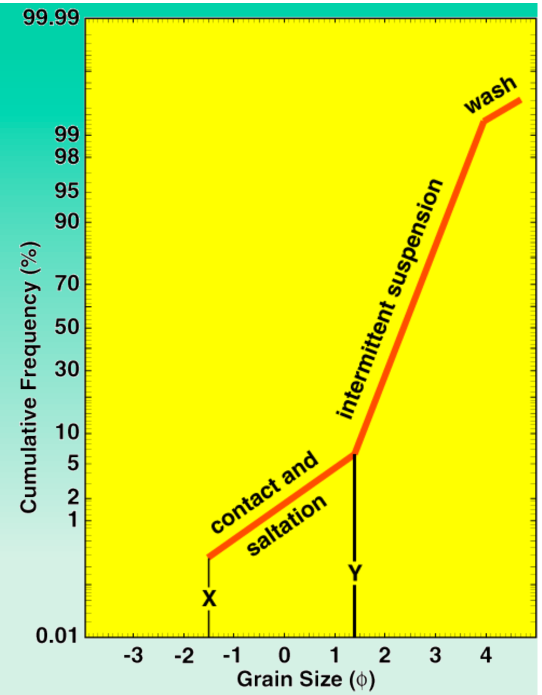
Note that for grain sizes coarser than 0.5 mm the velocity that is required for transport increases with grain size; the larger the particles the higher velocity that is required for transport.

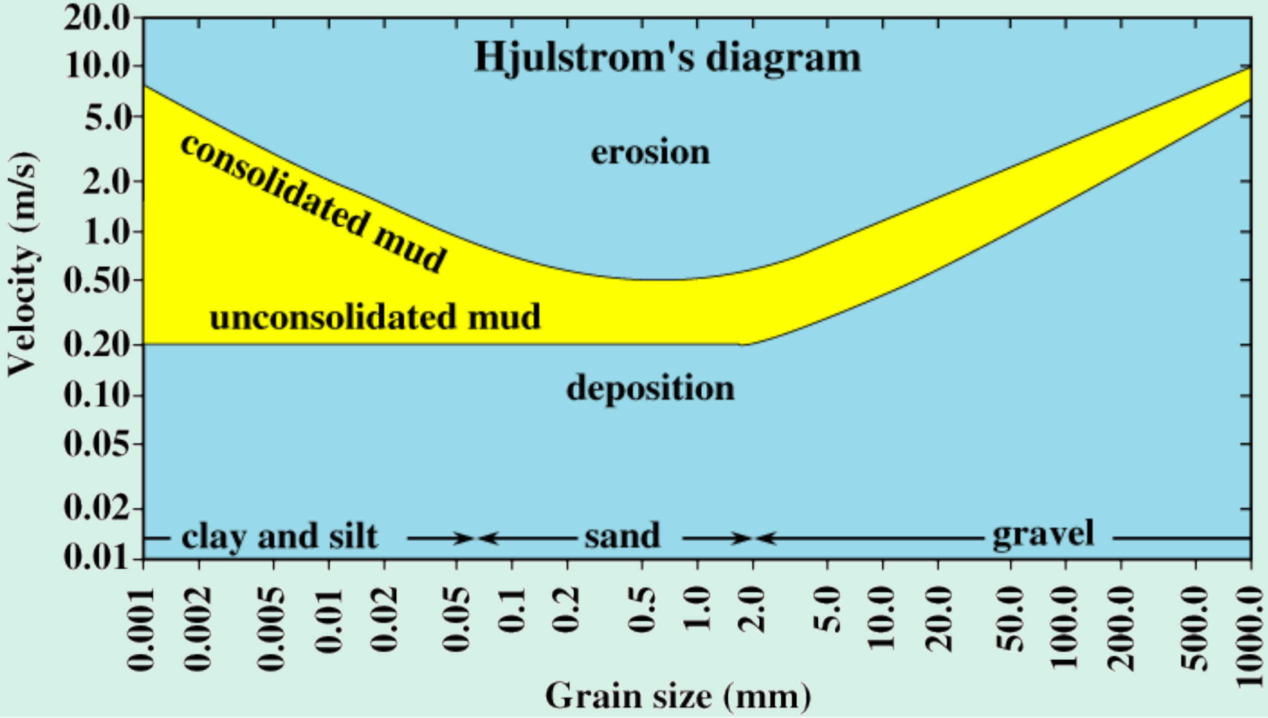
For finer grain sizes (with cohesive clay minerals) the finer the grain size the greater the critical velocity for transport.

This is because the more mud is present the greater the cohesion and the greater the resistance to erosion, despite the finer grain size.

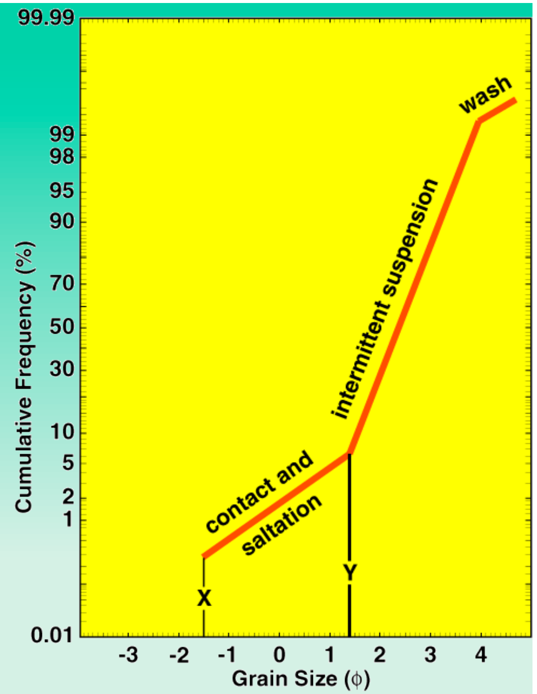


In our example, the coarsest grain size was 2.8 mm.



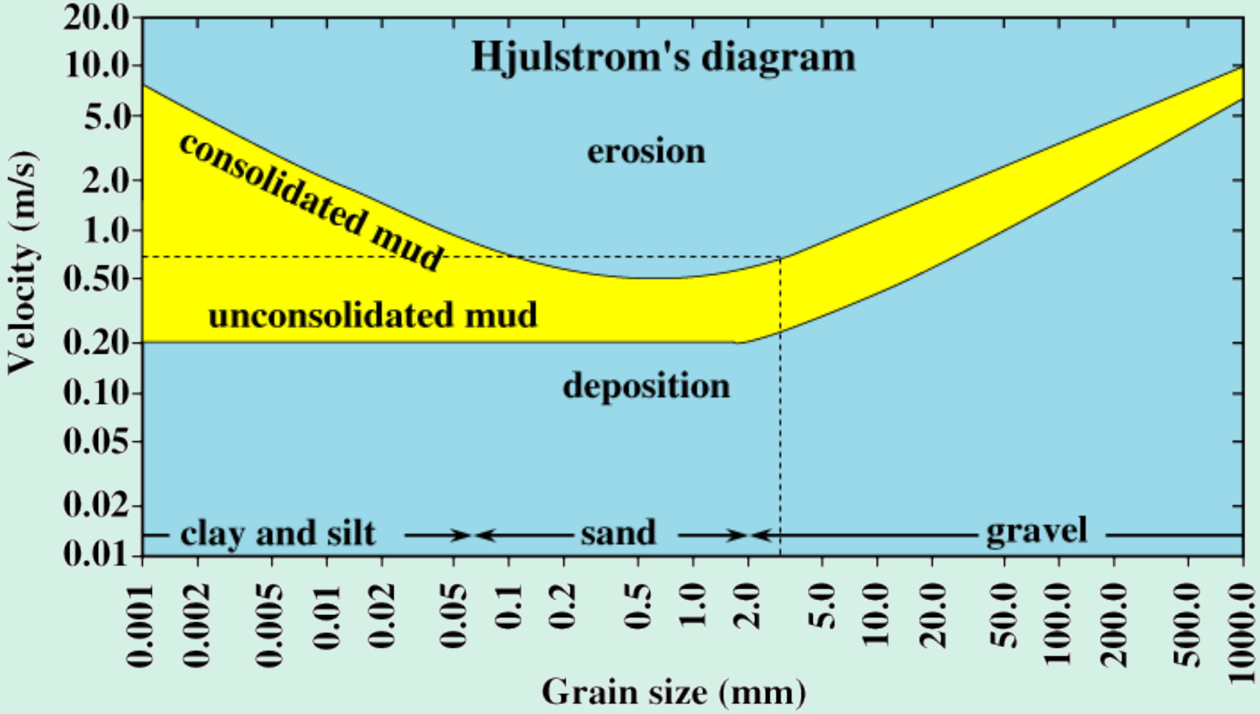


In our example, the coarsest grain size was 2.8 mm.



According to Hjulstron’s diagram, that grain size would require a flow with a velocity of approximately 0.65m/s.

Therefore, the sediment shown in the cumulative frequency curve was transported by currents at 0.65 m/s.



The problem is that the forces that are required to move sediment are not only related to flow velocity.

Boundary shear stress is a particularly important force and it varies with flow depth.

$$\tau_o = \rho g D \sin\theta$$

Therefore, Hjulstrom's diagram is reasonably accurate only for sediment that has been deposited under flow depths of 1 m.

i) Shield’s criterion for the initiation of motion

Based on a large number of experiments Shield’s criterion considers the problem in terms of the forces that act to move a particle.

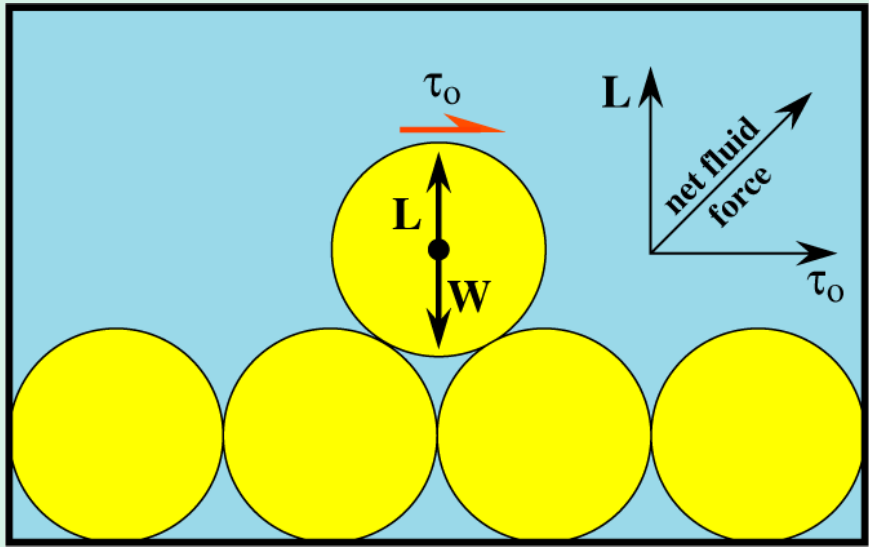
The criterion applies to beds of spherical particles of uniform grain size.

Forces that are important to initial motion:

1. The submerged weight of the particle ($\frac{\pi}{6}(\rho_s - \rho)gd^3$) which resists motion.

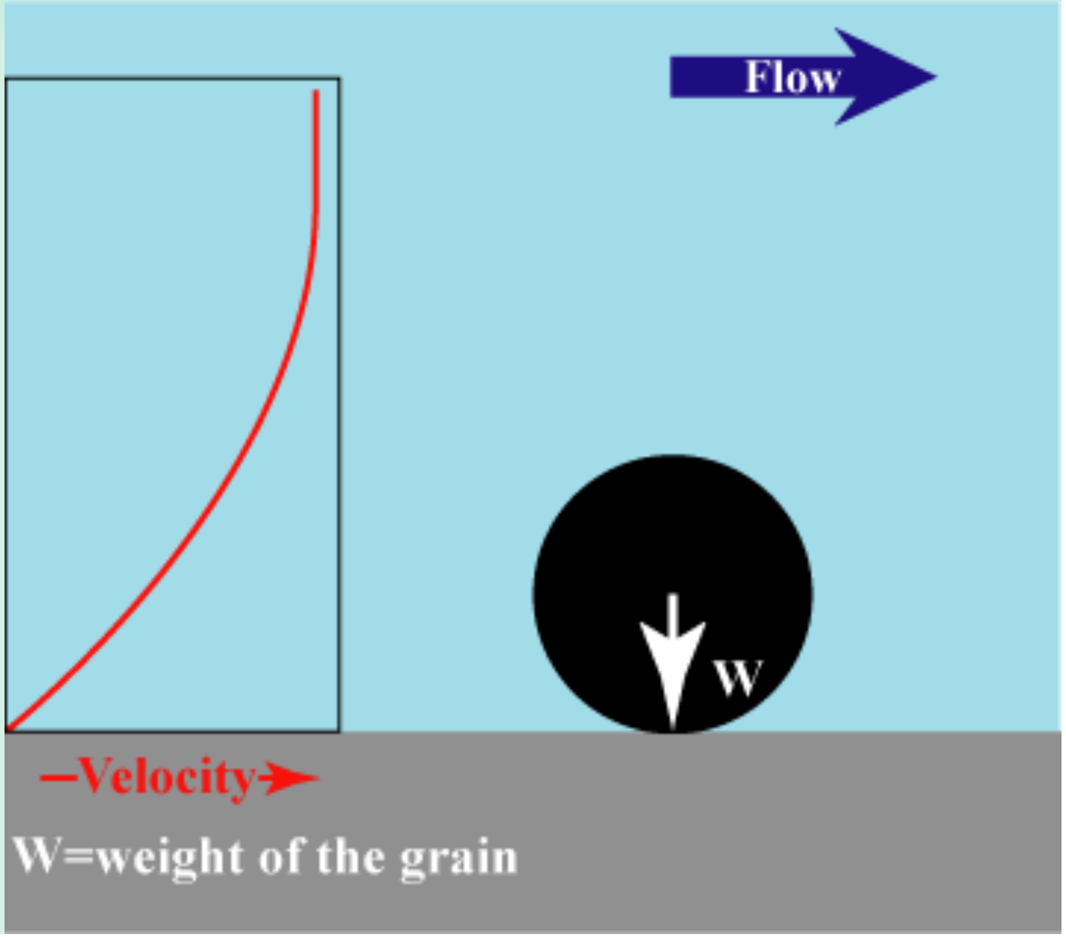
2. τ_o which causes a drag force that acts to move the particle down current.

3. Lift force (L) that reduces the effective submerged weight.



What's a Lift Force?

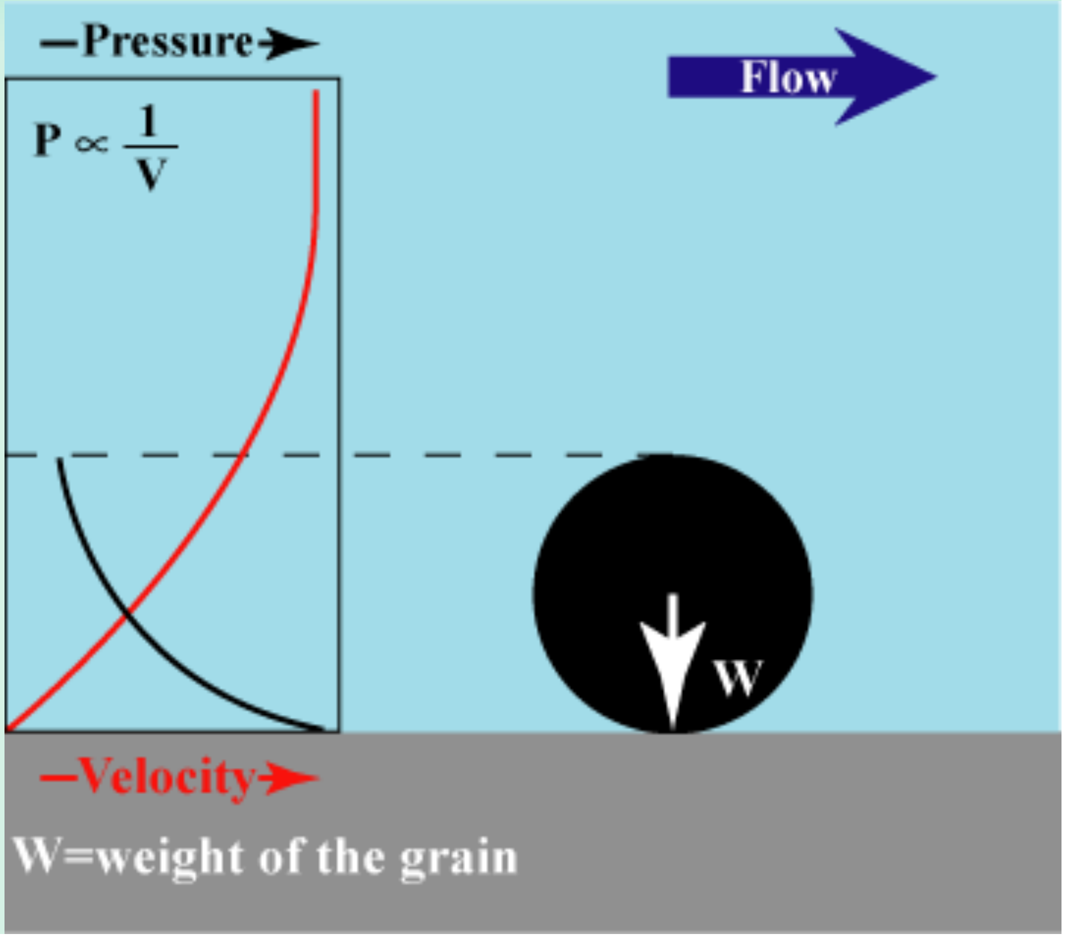
The flow velocity that is "felt" by the particle varies from approximately zero at its base to some higher velocity at its highest point.



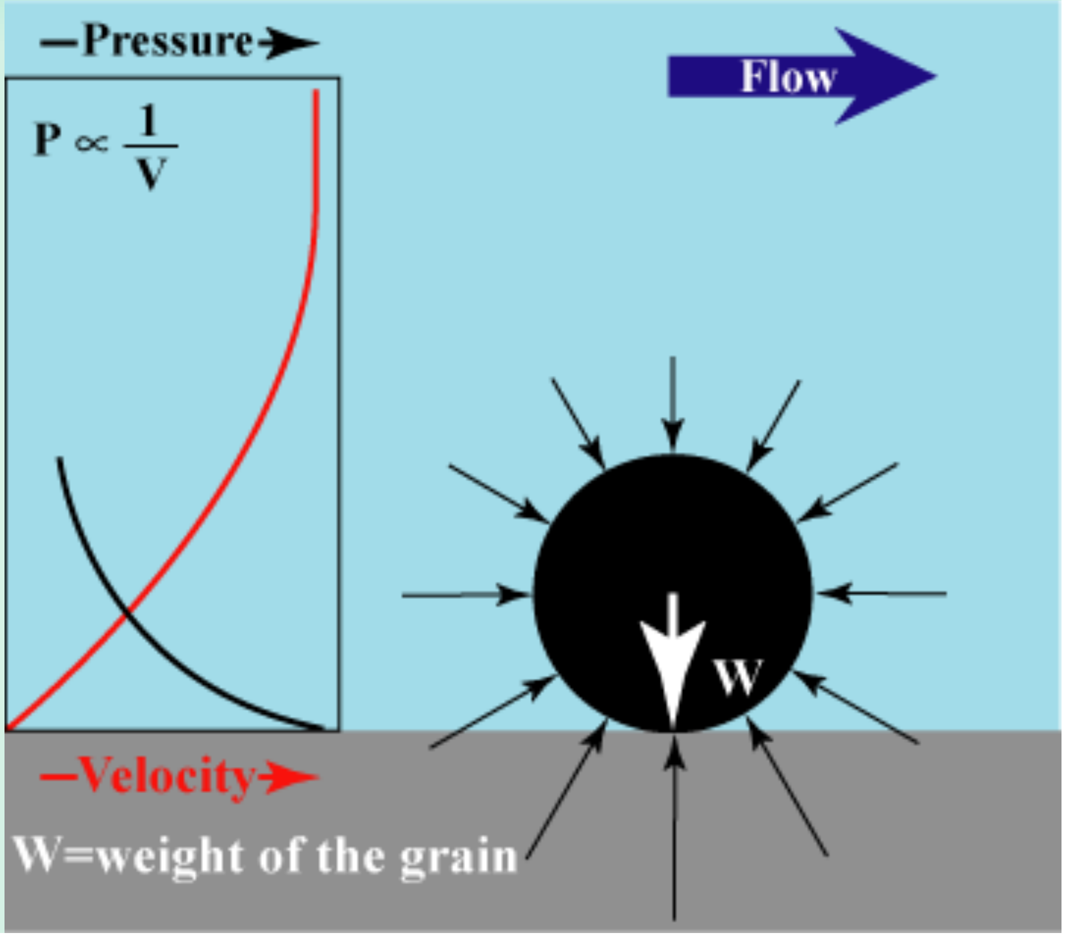
Pressure (specifically "dynamic pressure" in contrast to static pressure) is also imposed on the particle and the magnitude of the dynamic pressure varies inversely with the velocity:

Higher velocity, lower dynamic pressure.

Maximum dynamic pressure is exerted at the base of the particle and minimum pressure at its highest point.



The dynamic pressure on the particle varies symmetrically from a minimum at the top to a maximum at the base of the particle.

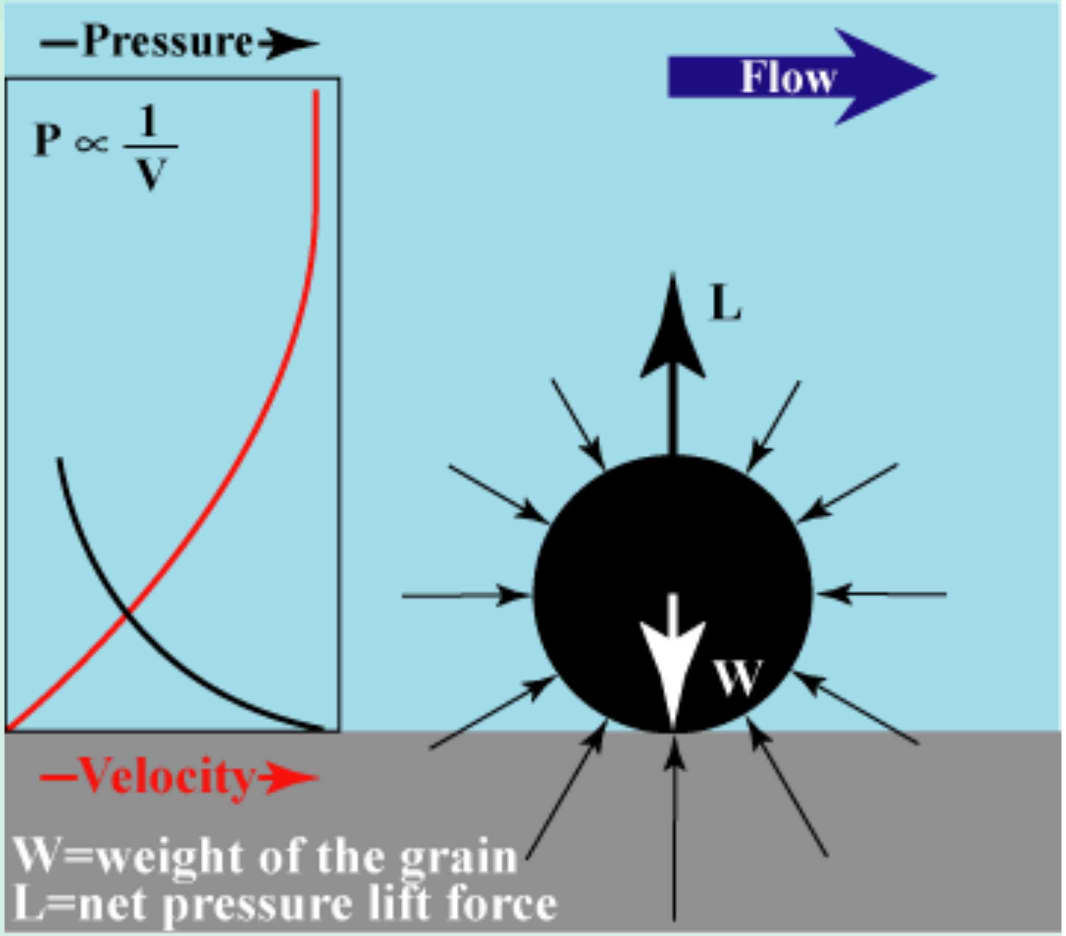


This distribution of dynamic pressure results in a net pressure force that acts upwards.

Thus, the pressure force (known as the Lift Force) acts oppose the weight of the particle (reducing its effective weight).

This makes it easier for the flow to roll the particle along the bed.

The lift force reduces that drag force that is required to move the particle.

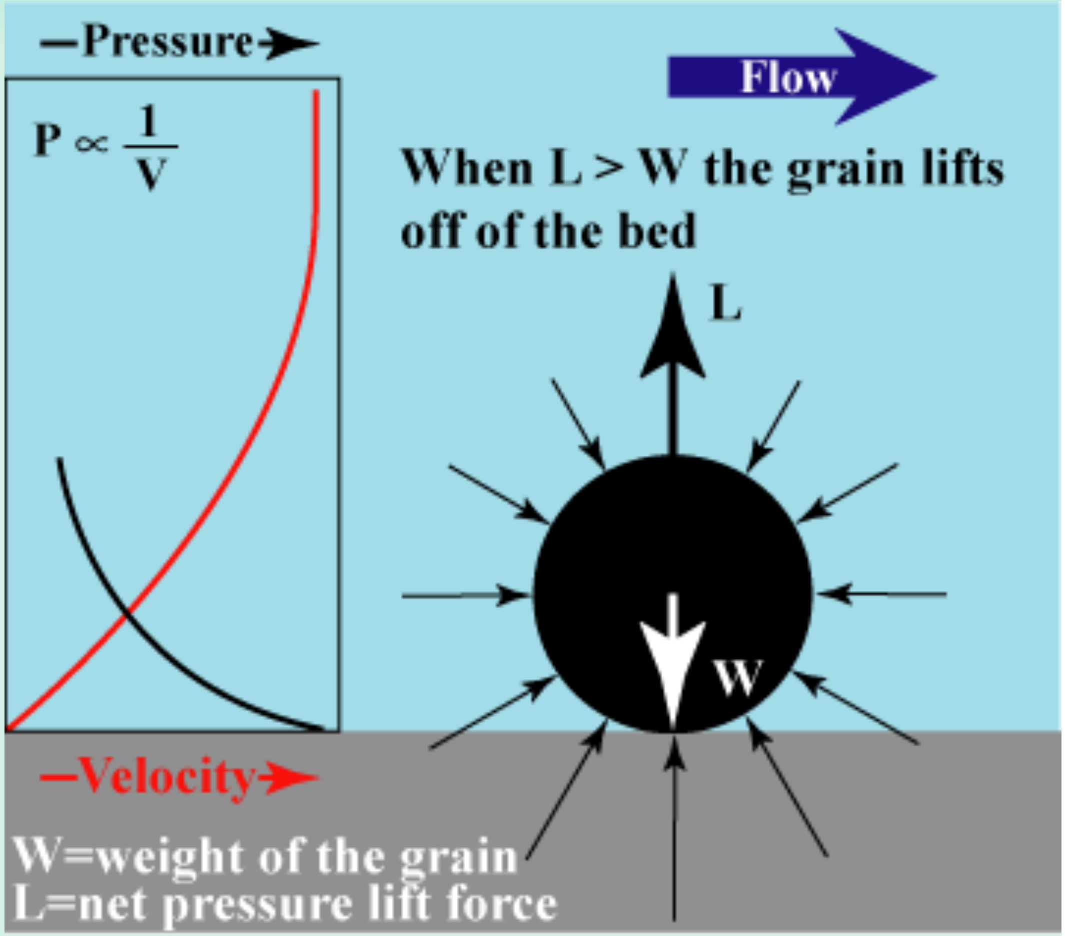


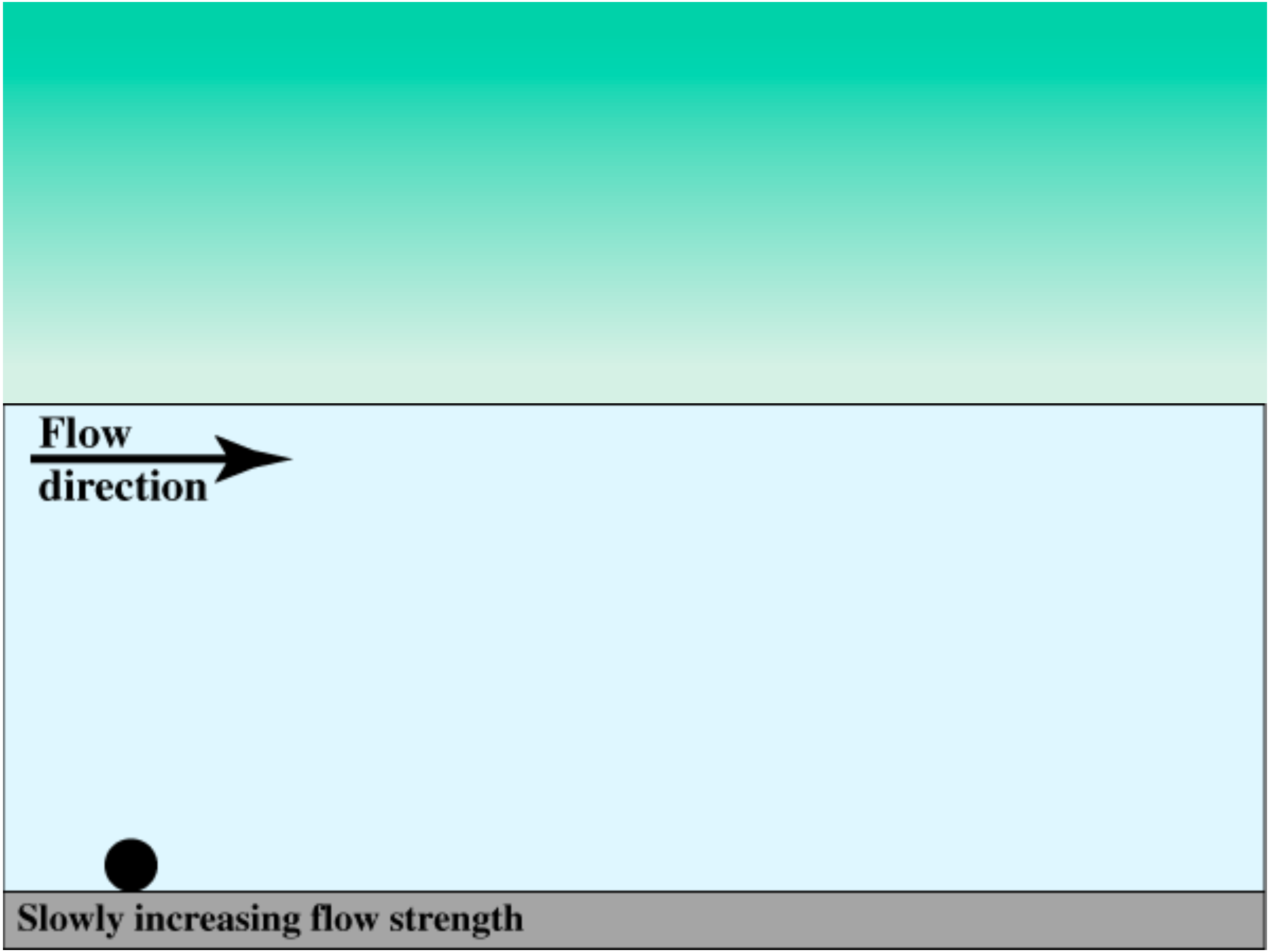
A quick note on saltation……

If the particle remains immobile to the flow and the velocity gradient is large enough so that the Lift force exceeds the particle's weight….it will jump straight upwards away from the bed.

Once off the bed, the pressure difference from top to bottom of the particle is lost and it is carried down current as it falls back to the bed….

following the ballistic trajectory of saltation.





Flow
direction
Slowly increasing flow strength

Shield's experiments involved determining the critical boundary shear stress required to move spherical particles of various size and density over a bed of grains with the same properties.

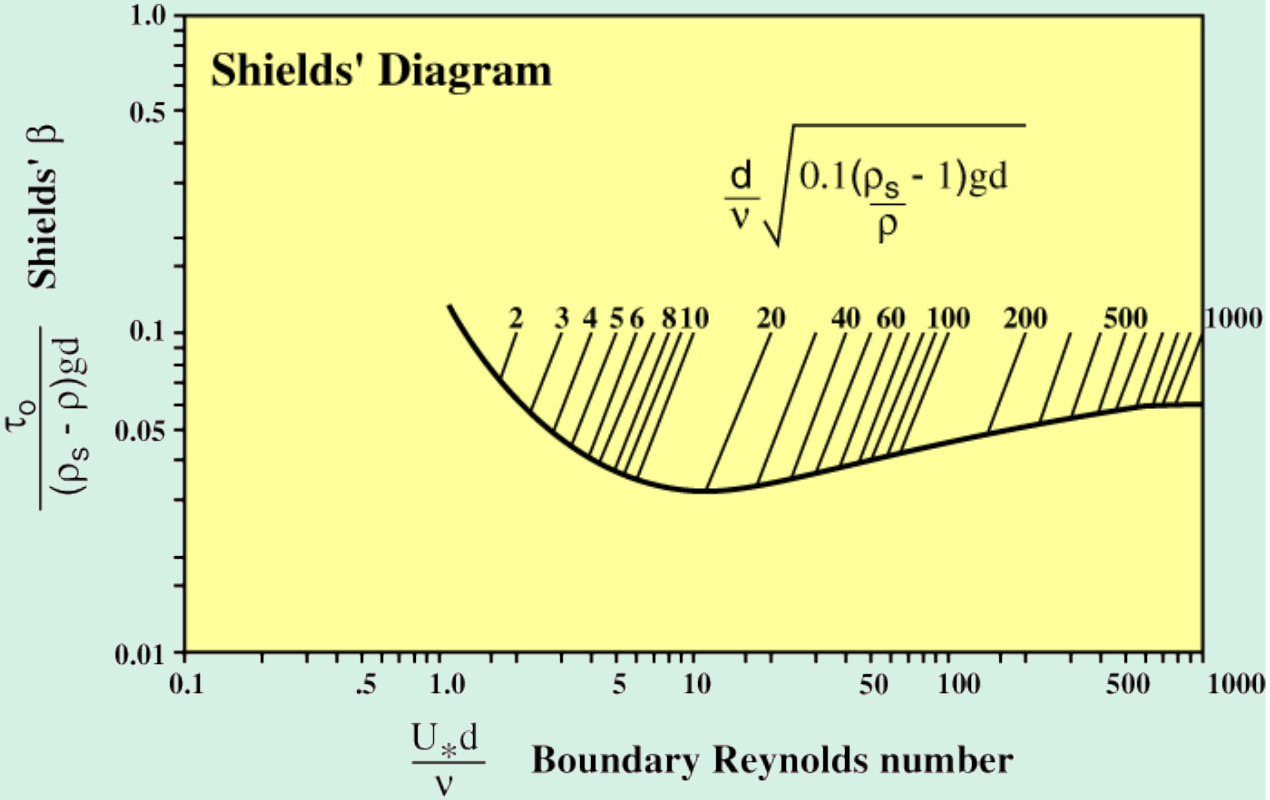
He produced a diagram that allows the determination of the critical shear stress required for the initiation of motion.

A bivariate plot of "Shield's Beta" versus Boundary Reynolds' Number:

$$\beta = \frac{\tau_o}{(\rho_s - \rho)gd}$$

vs

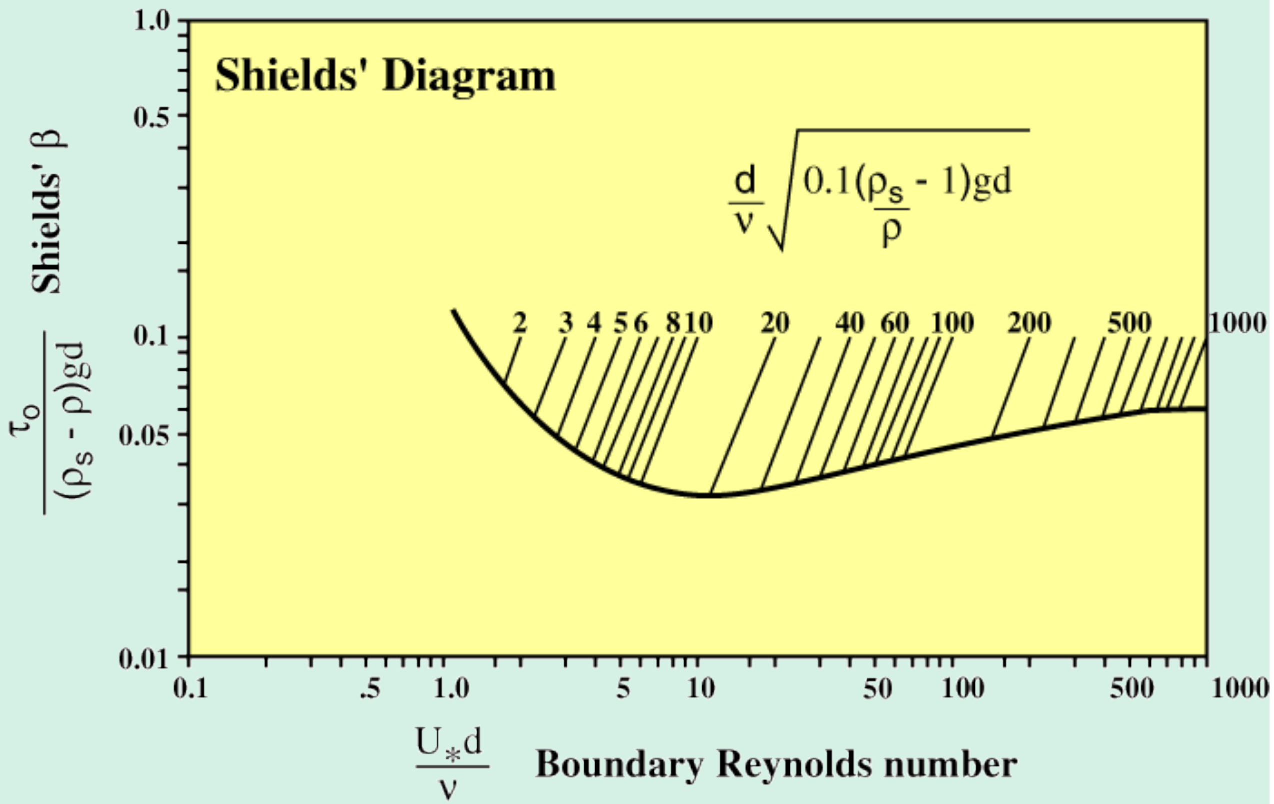
$$R_* = \frac{U_* d}{\upsilon}$$



$$\beta = \frac{\tau_o}{(\rho_s - \rho)gd}$$

Critical shear stress for motion.

Submerged weight of grains per unit area on the bed.

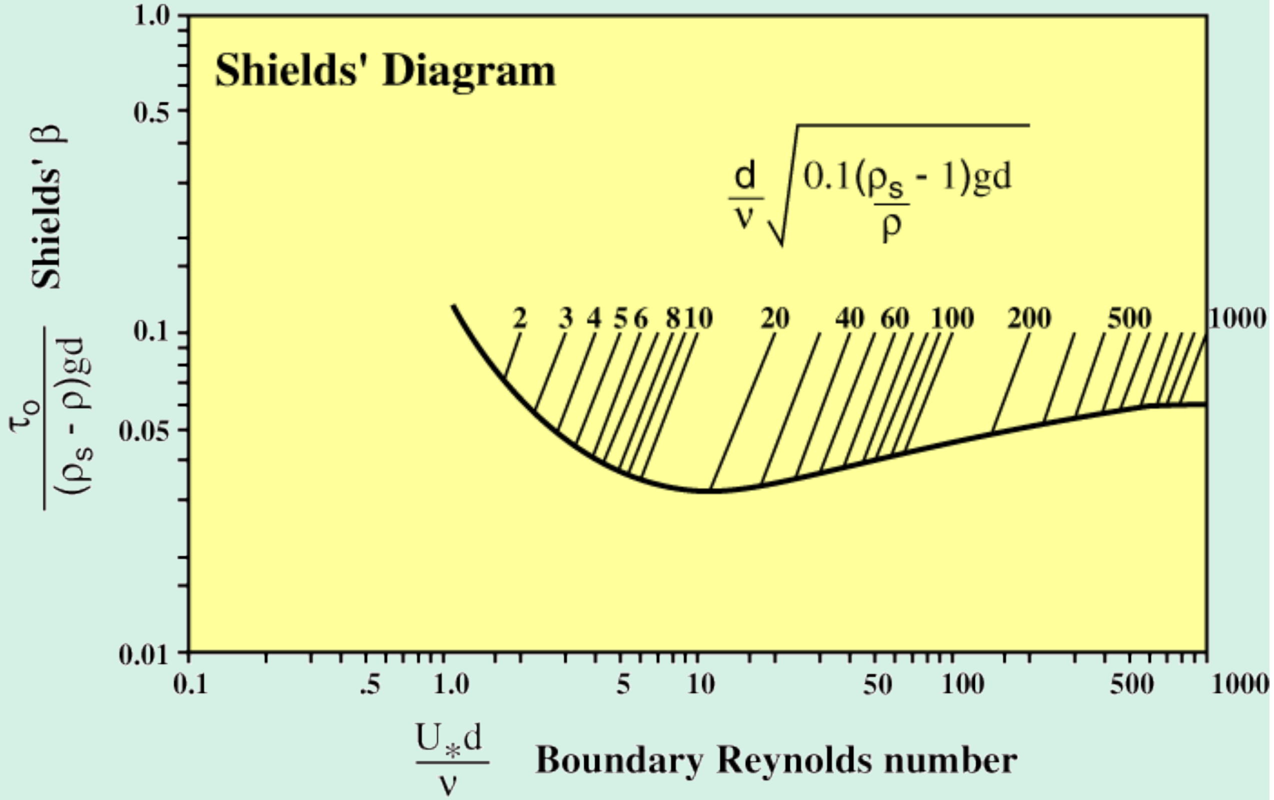


$$\beta = \frac{\tau_o}{(\rho_s - \rho)gd} = \frac{\text{Force acting to move the particle (excluding Lift)}}{\text{Force resisting movement}}$$

As the Lift Force increases β will decrease (lower τ_o required for movement).

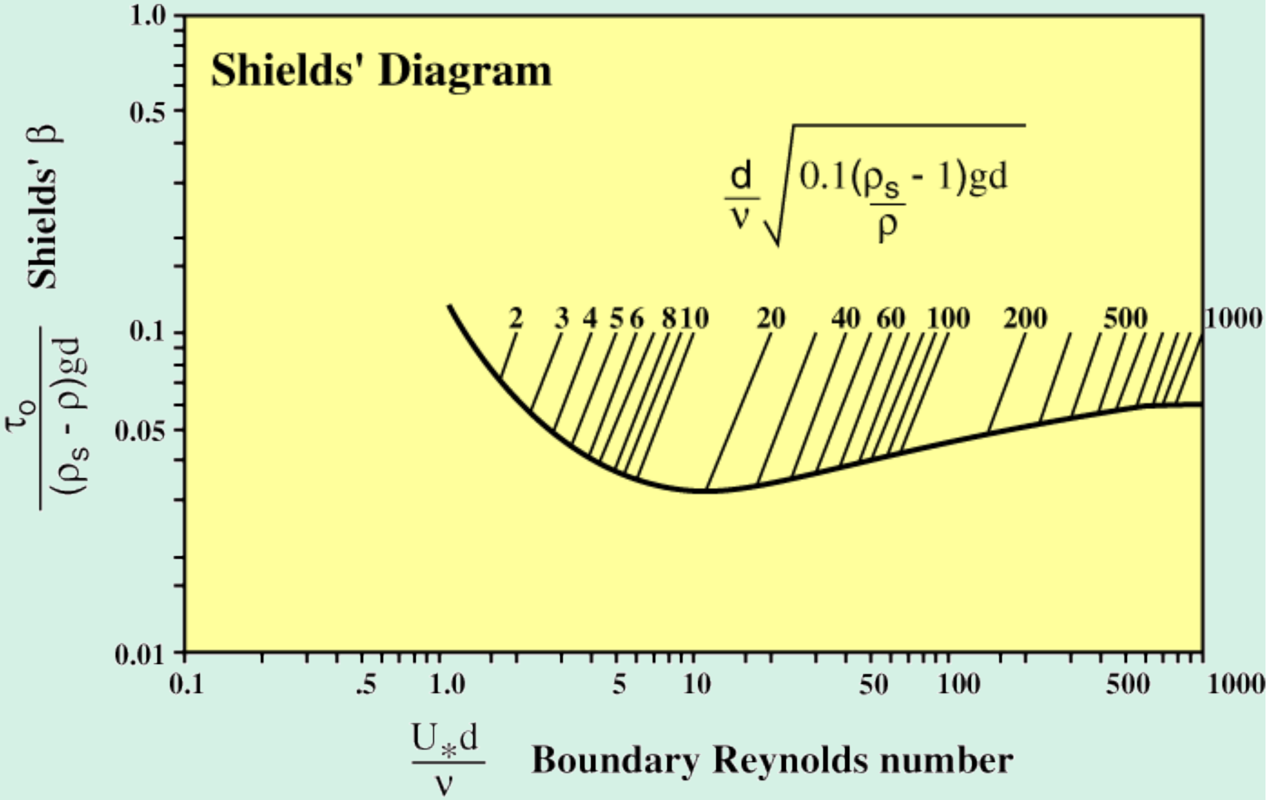
$$R_* = \frac{U_* d}{\upsilon}$$

Reflects something of the lift force (related to the velocity gradient across the particle.



For low boundary Reynold's numbers Shield's β decreases with increasing R_*.

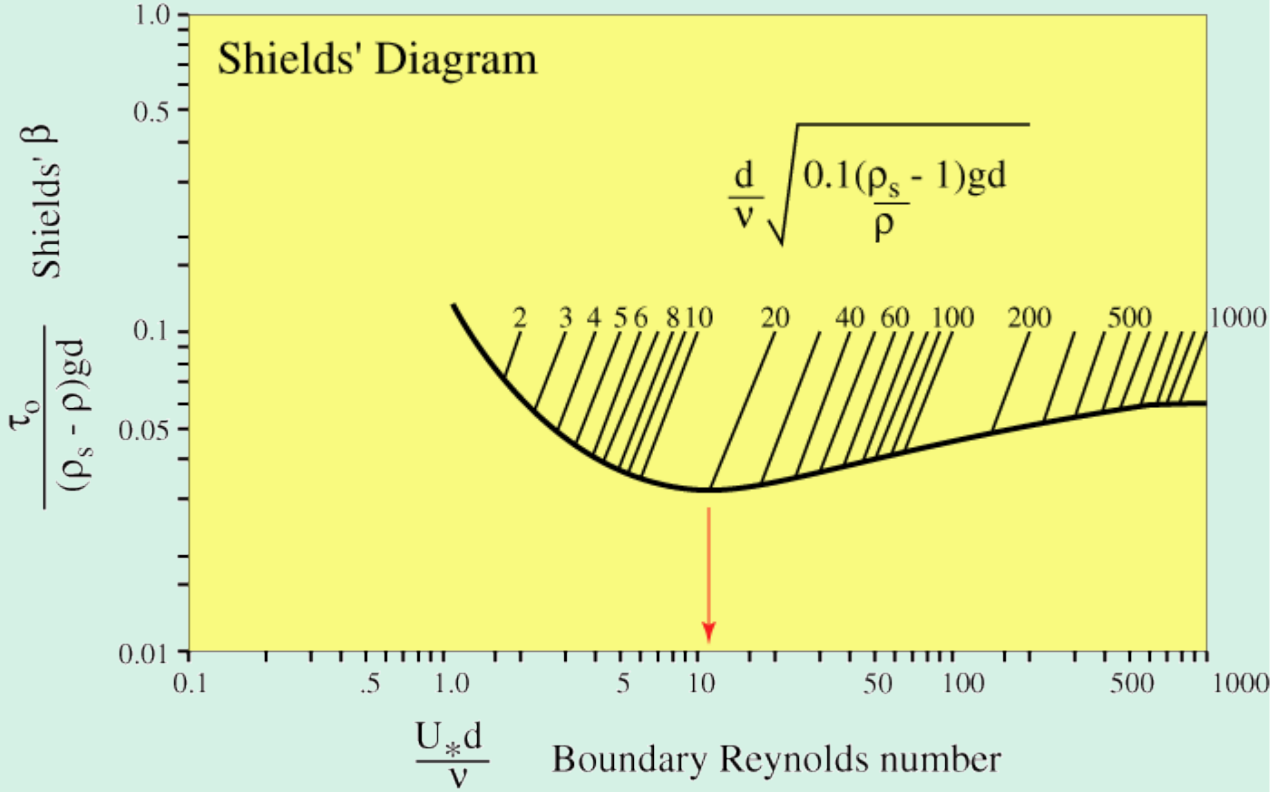
For high boundary Reynold's numbers Shield's β increases with increasing R_*.



For low boundary Reynold's numbers Shield's β decreases with increasing R_*.

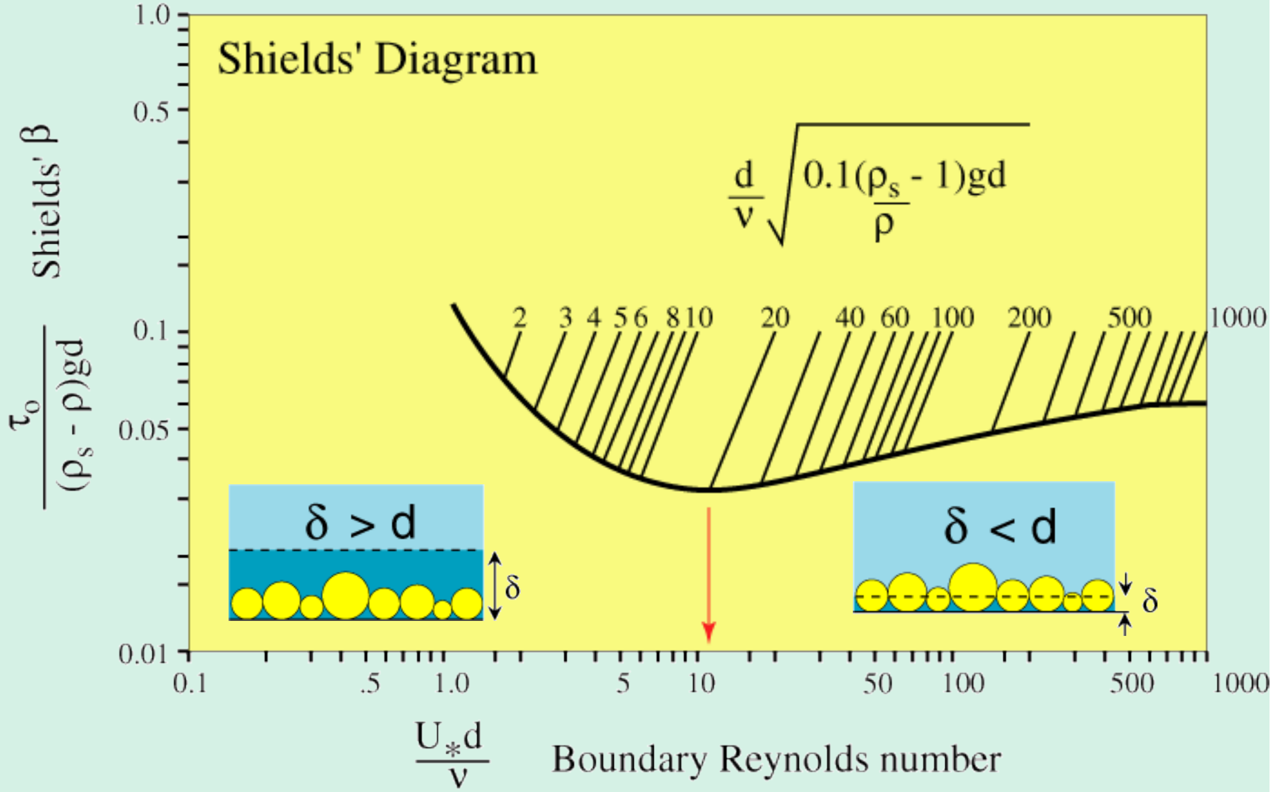
For high boundary Reynold's numbers Shield's β increases with increasing R_*.

The change takes place at $R^* \approx 12$.

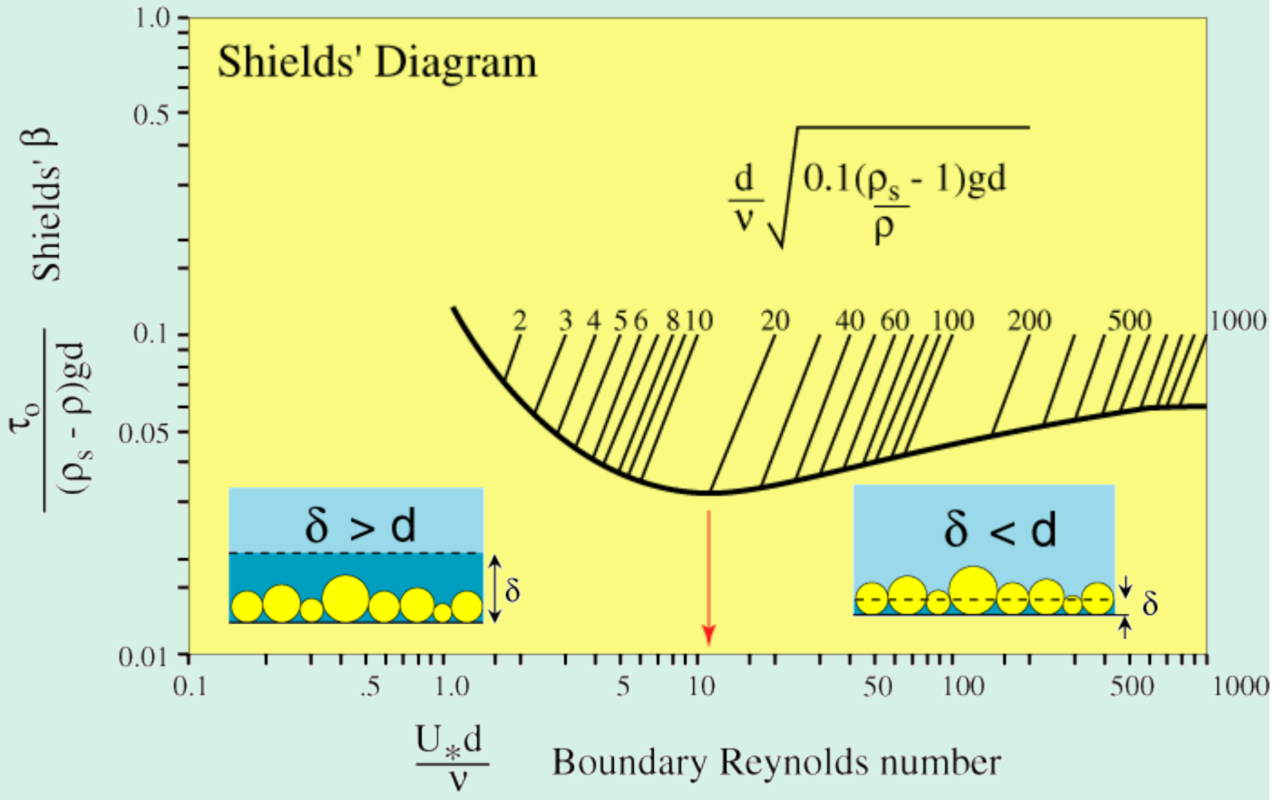


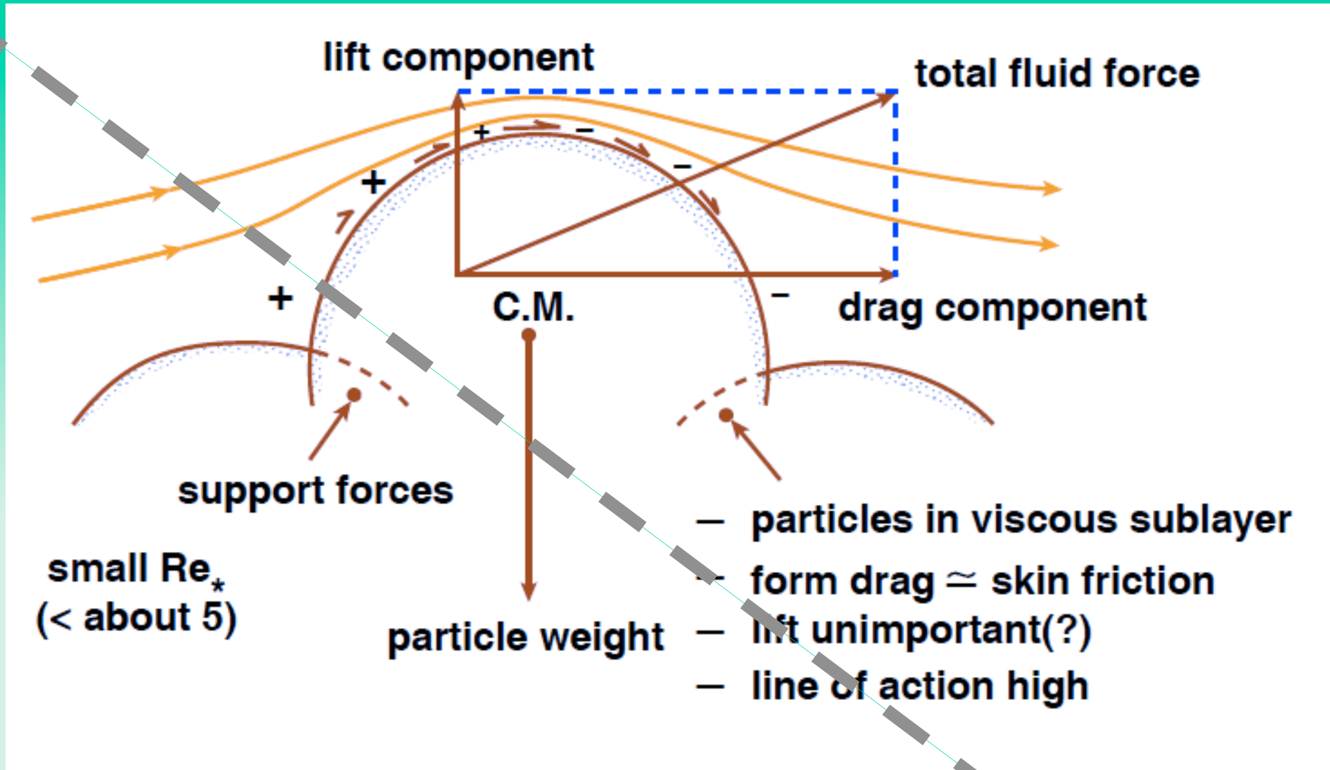
At boundary Reynold’s numbers less than 12 the grains on the bed are entirely within the viscous sublayer.

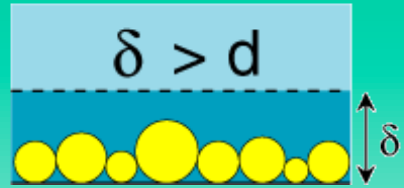
At boundary Reynold’s numbers greater than 12 the grains on the bed extend above the viscous sublayer.



As Shield's β decreases ($R_* < 12$) the critical shear stress required for motion decreases for a given grain size.



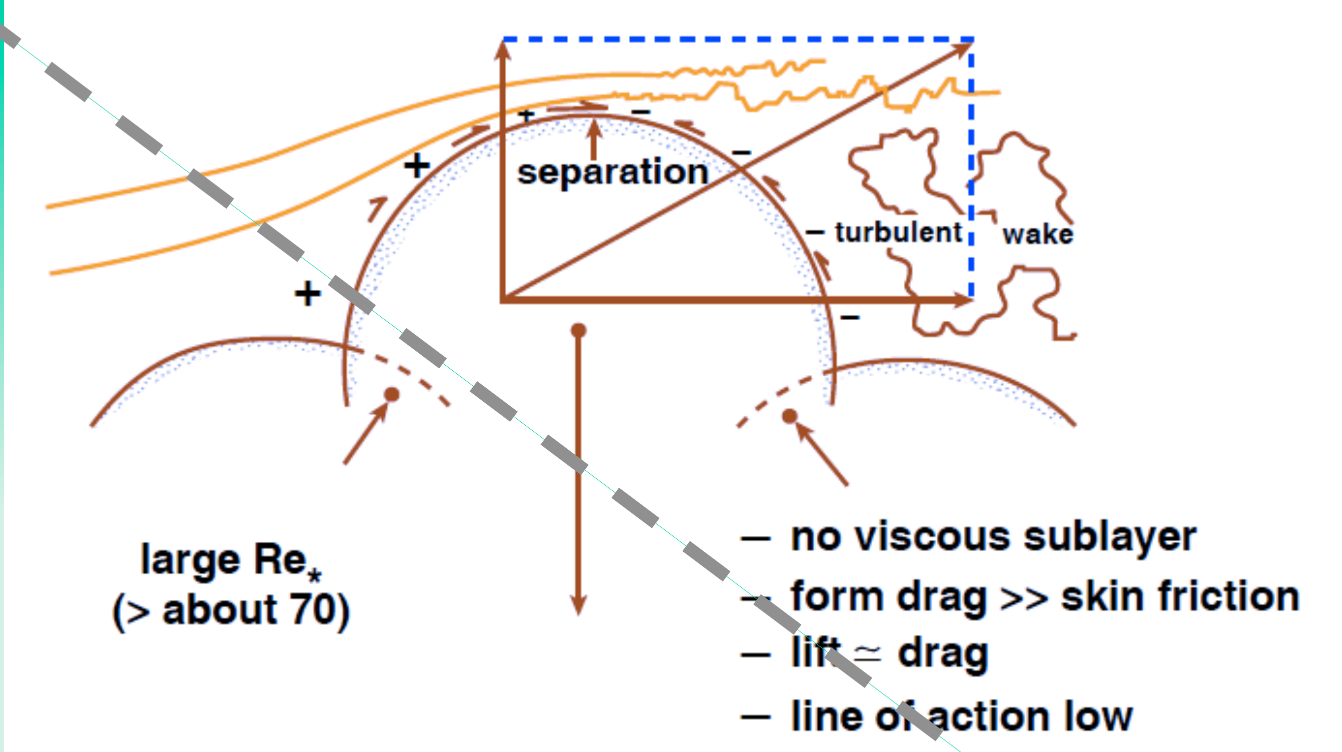


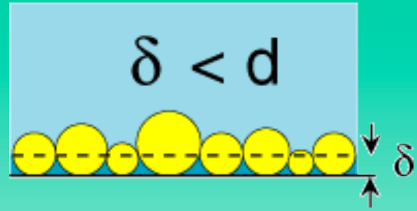


For **small R$_*$** (which corresponds to relatively small values of a Reynolds number based on local flow velocity around the particle) there is no well defined boundary layer along the top surface of the particle, and there is no flow separation behind the particle.

Both viscous forces and pressure forces are important. The line of action of the resultant force lies well above the center of mass of the particle, because the viscous forces are strongest on the uppermost surface of the particle.

Figure by MIT OpenCourse Ware





For **large R***** (which corresponds to relatively large values of a Reynolds number based on local flow velocity) there is a well defined local boundary layer on the surface of the particle, and pronounced flow separation, with a turbulent wake behind the particle.

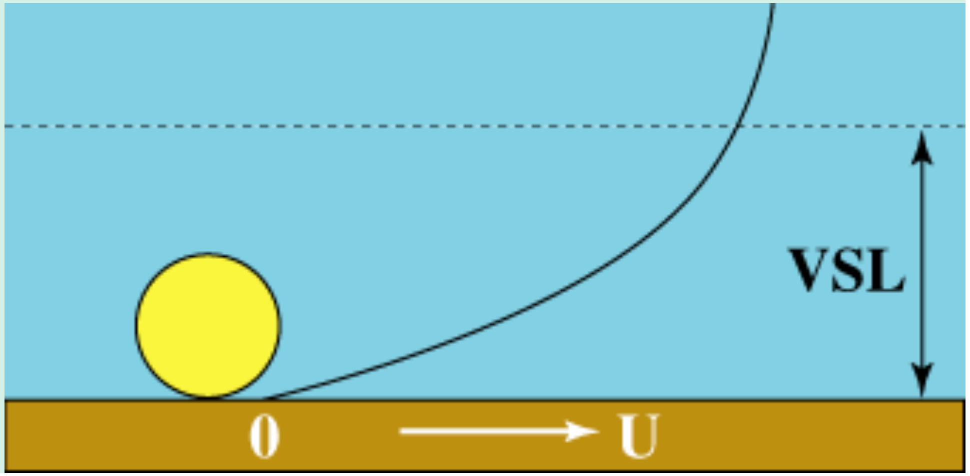
Pressure forces far outweigh viscous forces, and because the net pressure force comes about mainly by the difference in pressure from front to back the line of action of the resultant force is closer to the center of mass of the particle.

Figure by MIT OpenCourse Ware

At low boundary Reynolds numbers (< 12) the grains experience a strong velocity gradient within the VSL.

As R_* increases towards a value of 12 the VSL thins and the velocity gradient becomes steeper, increasing the lift force acting on the grains.

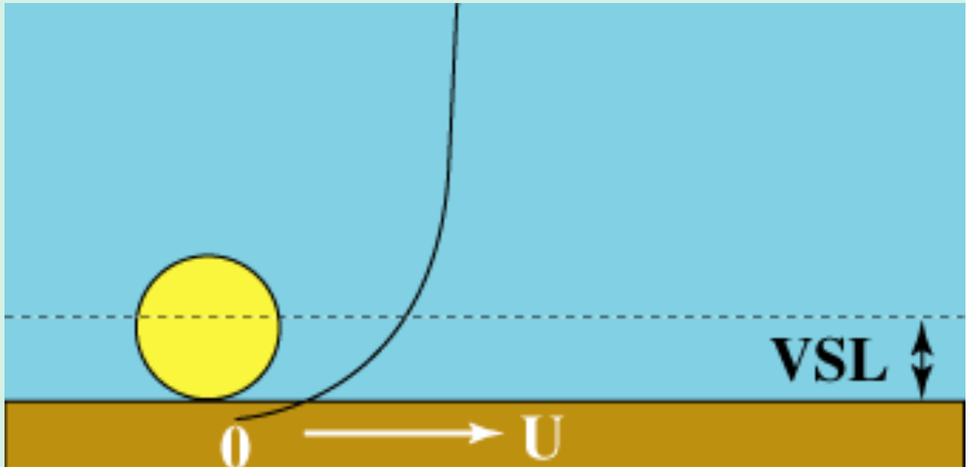
The greater lift force reduces the effective weight of the grains and reduces the boundary shear stress that is necessary to move the grain.



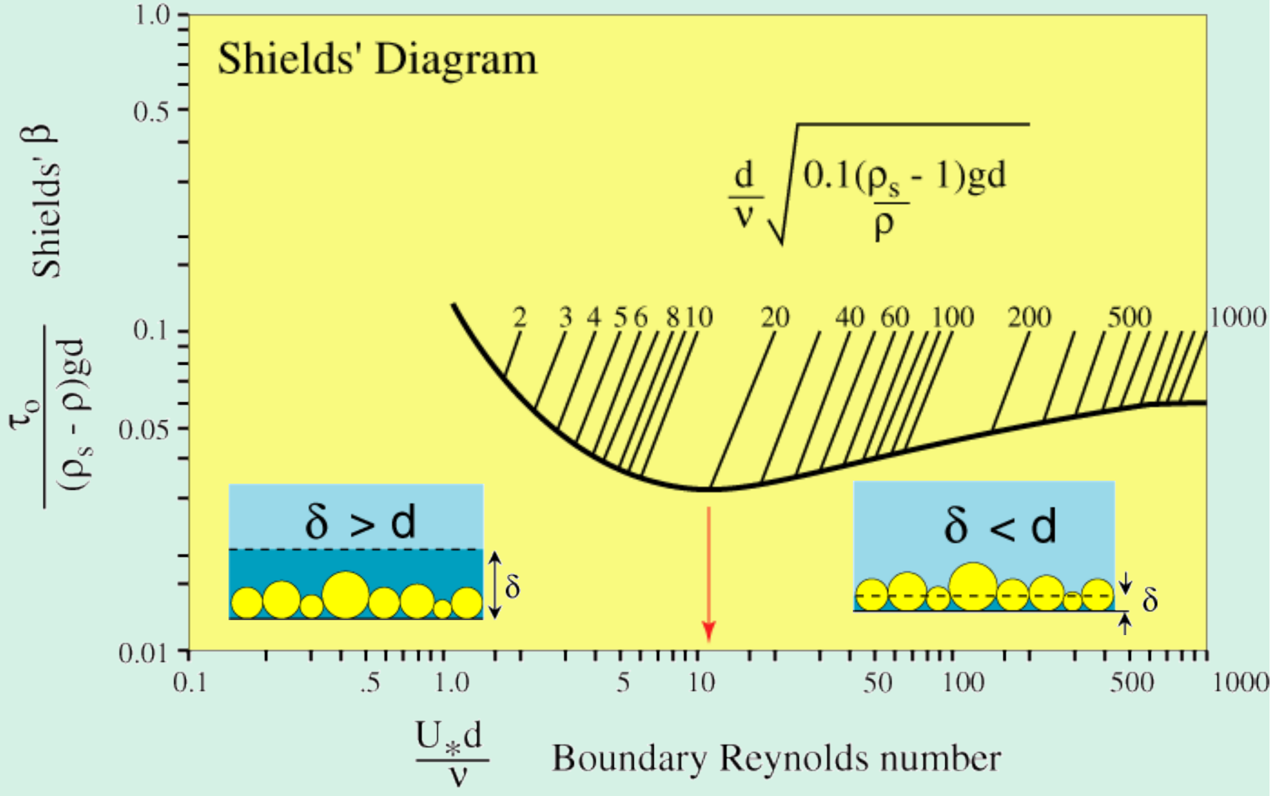
At high boundary Reynolds numbers (> 12) the grains protrude through the VSL so that the region of strong velocity gradient is below the grains, leading to lower lift forces.

As R_* increases the velocity gradient acting on the grains and resulting lift forces are reduced.

The lower lift force leads to an increase in the effective weight of the grains and increases the boundary shear stress that is necessary to move the grains.



The boundary Reynold's number accounts for the variation in lift force on the grains which influences the critical shear stress required for motion.



How to use Shield's Diagram

What is the boundary shear stress required to move 2.8 mm sand?

$d = -1.5\phi = 2.8$ mm $= 0.0028$ m

$\nu = 1.1 \times 10^{-6}$ m^2/s (water at 20°C)

$\rho = 998.2$ kg/m^3 (density of water at 20°C)

$\rho_s = 2650$ kg/m^3 (density of quartz)

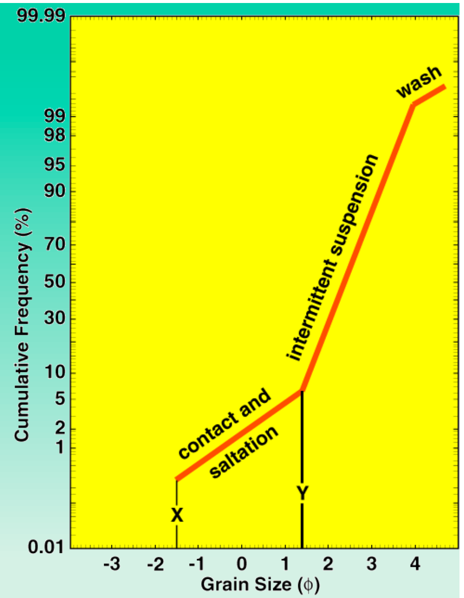
$g = 9.806$ m/s^2

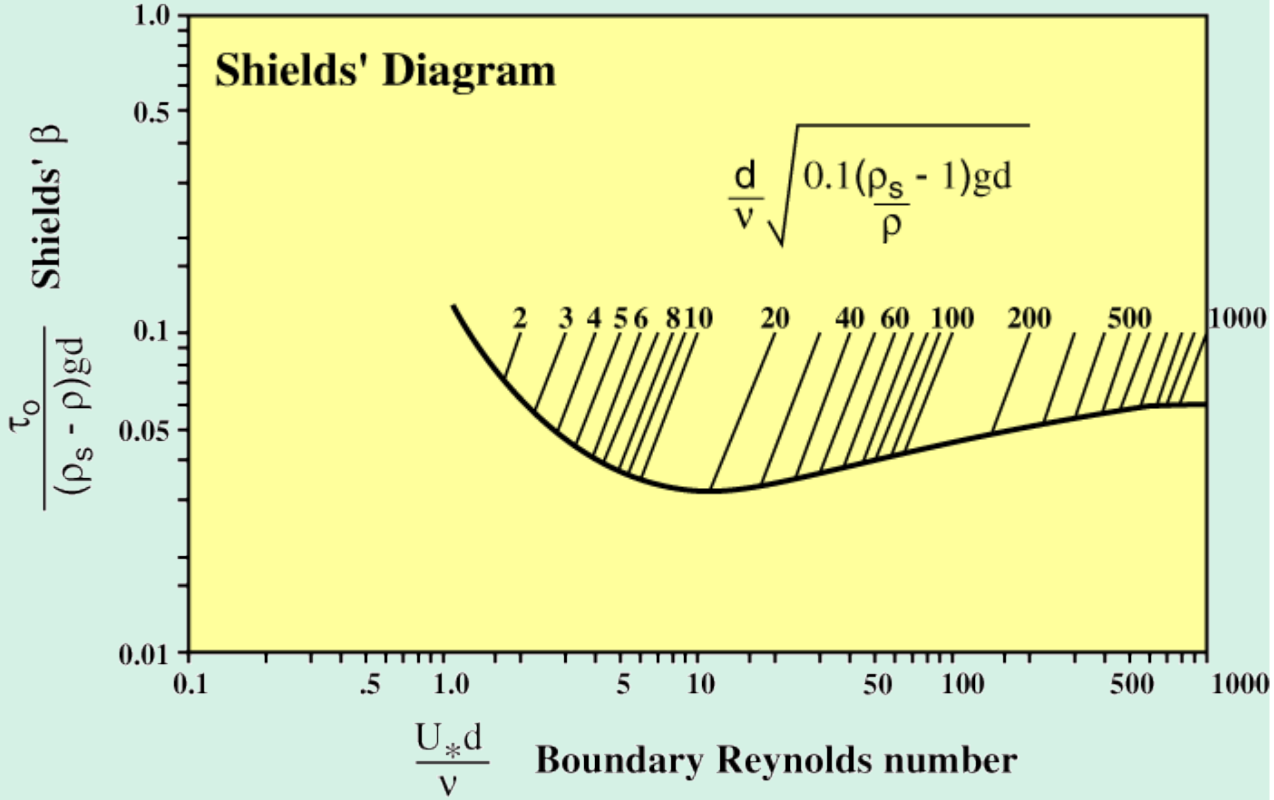
Note the assumptions regarding the water

Calculate:

$$\frac{d}{\upsilon}\sqrt{0.1\left(\frac{\rho_s}{\rho}-1\right)gd}$$

=172





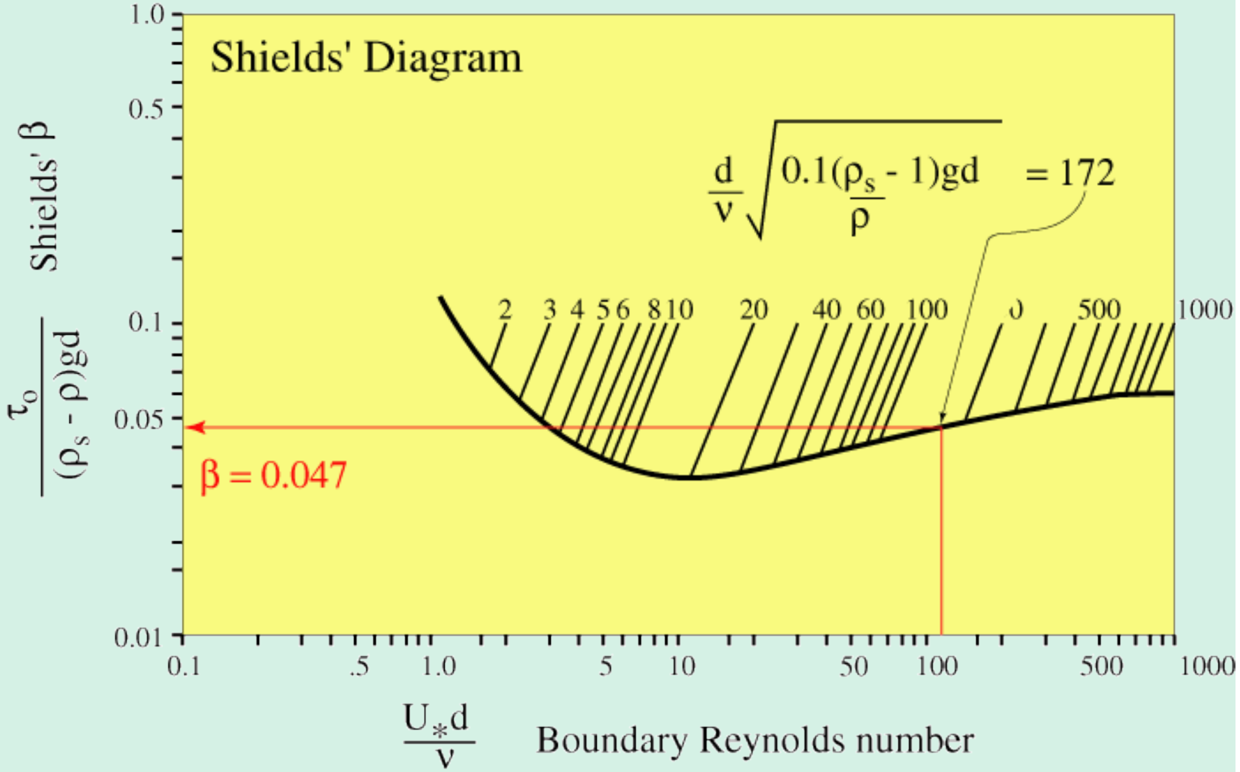
$$\beta = \frac{\tau_o}{(\rho_s - \rho)gd} = 0.047$$

Rearranging: $\tau_o = \beta(\rho_s - \rho)gd$

$$= 0.047(\rho_s - \rho)gd$$

$$= 2.13 \text{ N/m}^2$$

Remember:
using the Hjulstrom diagram we obtained a critical velocity $V_c = 0.65$ m/s for the same grain size



Limitations of Shield's Criterion:

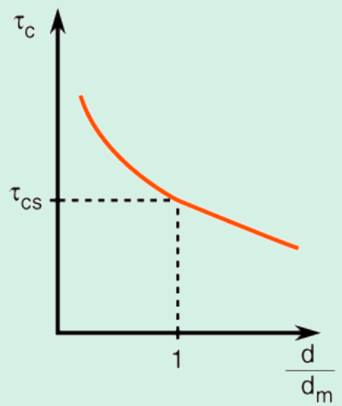
1. It applies only to spherical particles; it doesn't include the influence of particle shape.

It will underestimate the critical shear stress required for motion for angular grains.

2. It assumes that the material on the bed is of uniform size.

It underestimates the critical shear stress for small grains on a bed of larger grains

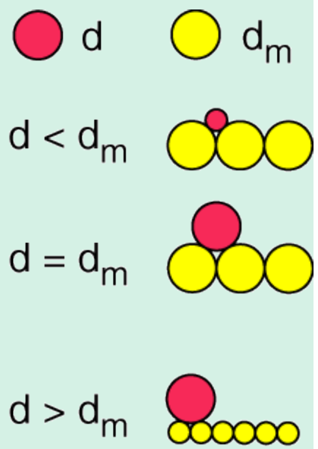
It overestimates the critical shear stress for large grains on a bed of finer grains



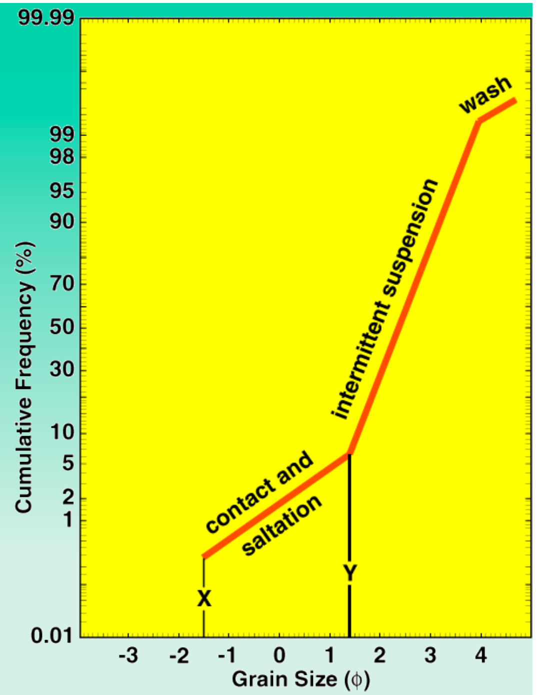
$\tau_c > \tau_{cs}$: $\frac{d}{d_m} < 1$; $d < d_m$

$\tau_c = \tau_{cs}$: $\frac{d}{d_m} = 1$; $d = d_m$

$\tau_c < \tau_{cs}$: $\frac{d}{d_m} > 1$; $d > d_m$



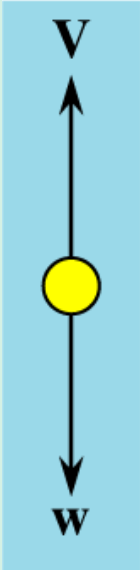
b) The threshold for suspension



The coarsest grain size in the intermittent suspension load is the coarsest sand that the current will suspend.

Sediment is suspended by the upward component of turbulence (velocity V).

Middleton's criterion for suspension:



The largest particle to be suspended by a current will be that particle with a settling velocity (ω) that is equal to V.

Suspension when $V \geq \omega$

where V is the upward component of velocity due to turbulence and ω is the settling velocity of the particle.

Experiments have shown that $V \approx U_*$ for a given current.

Therefore, Middleton's criterion is:

A particle will be taken into suspension by a current when the shear velocity of the current equals or exceeds the settling velocity of the particle.

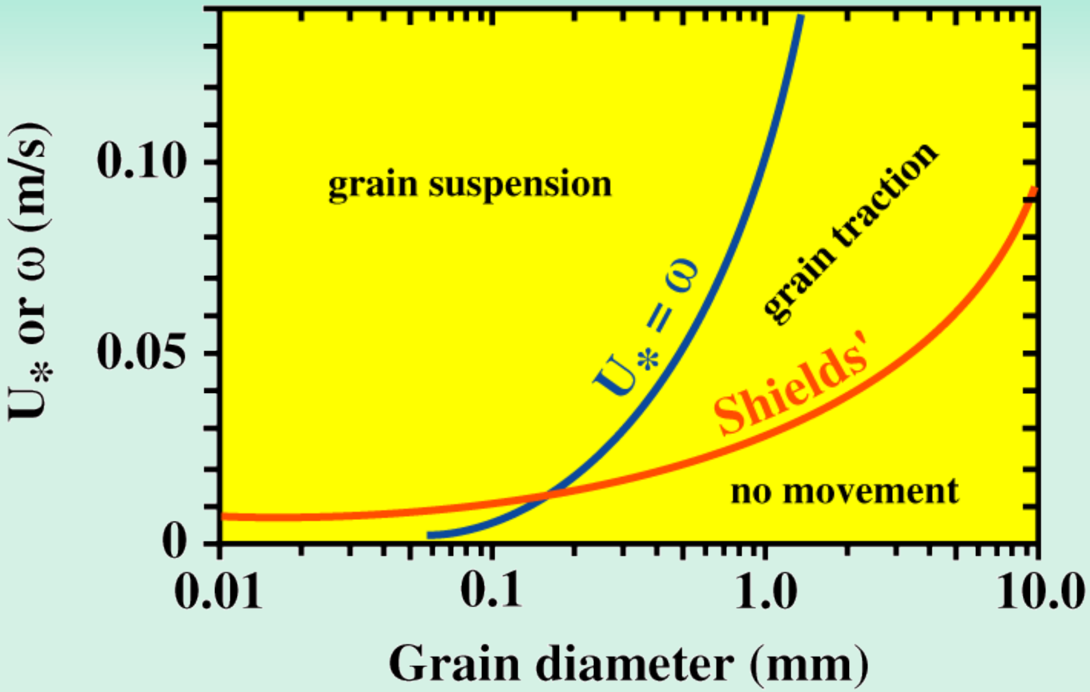
$$\boxed{U^* \geq \omega}$$

Comparisons of the settling velocity of the largest grain size in the intermittent suspension load found in the bed material of major rivers show that they compare very favorably to the measured shear velocity during peak flow in those rivers.

River	**U*** **(m/s)**	**ω** **(m/s)**
Middle Loup	7 – 9	7 - 9
Middle Loup	≈ 20	≈20
Niobrara	7 - 10	7 - 9
Elkhorn	7 - 9	2.5 – 5.0
Mississippi (Omaha)	6.5 – 6.8	2.5 – 5.0
Mississippi (St. Louis)	9 - 11	3 - 12
Rio Grande	8 - 12	≈10

This diagram shows the shear velocity required to suspend particles as a function of their size (the curve labeled $U_* = \omega$).

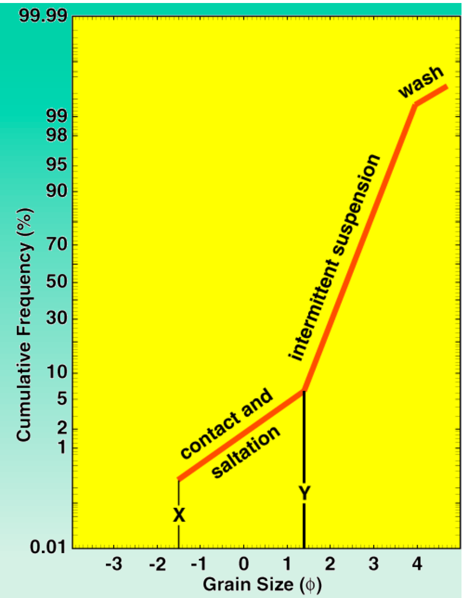
For comparison it also shows the critical shear velocity required to move a particle on the bed based on Shield's criterion.

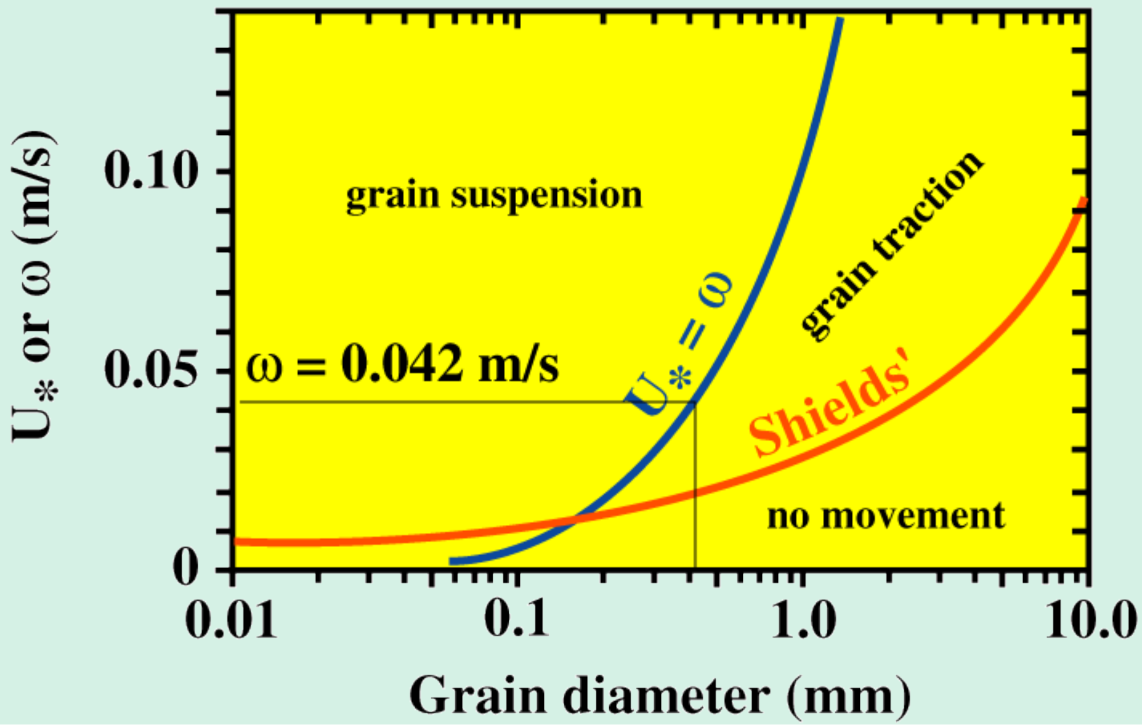


The $U_* = \omega$ curve can also be used to estimate settling velocity for grains coarser than 0.1 mm (the upper limit for Stoke's Law).

Note that for grain sizes finer than approximately 0.015 mm the grains will go into suspension as soon as the flow strength is great enough to move them (i.e., they will not move as contact load).

What is the critical shear velocity required to suspend 0.41 mm sand?





The critical shear velocity for suspension is 0.042 m/s.

How do our estimates based on the coarsest grains size in transport on the bed and the coarsest grain size in suspension compare?

Middleton's criterion: $U_* = 0.042$ m/s

Shield's criterion: $\tau_o = 2.13$ N/m^2

$$U_* = \sqrt{\frac{\tau_o}{\rho}}$$

$\rho = 998.2$ kg/m^3 (density of water at 20°C)

$U_* = 0.046$ m/s

Very close!