Appello 22/07/2016

(A) $V_i = 100\ m/s$ $\quad a = -5\ m/s^2$

$V_f = 0\ m/s$ $\quad \Delta t = ?$

$V^2 - V_0^2 = 2a(s - s_0)$

$V = V_0 + a\Delta t$

$S = S_0 + V_0 \Delta t + \frac{1}{2} a (\Delta t)^2$

È un moto unif. accelerato

$$\Delta t = \frac{V_f - V_i}{a} = \frac{0\ m/s - 100\ m/s}{-5\ m/s^2} = 20\ s$$

È sufficiente $\Delta s = 0{,}800\ Km$?

No, non sono sufficienti

$$S = V_i \cdot \Delta t + \frac{1}{2} a (\Delta t)^2 = 100\ m/s \cdot 20\ s + \frac{1}{2}\left(-5\ m/s^2\right)\cdot(20\ s)^2 =$$

$$= 2000\ m - 1000\ m = 1000\ m = 1\ Km$$

Ⓒ I° principio termodinamica

$$\underbrace{\Delta U}_{\text{en. interna}} = \underbrace{Q}_{\text{calore}} - \underbrace{W}_{\text{lavoro}}$$

In questo caso $\Delta U = 0$

$$Q = W = \int_i^f p\,dV = p(V_f - V_i)$$

$$= 101325\ Pa \cdot (15 - 5) \cdot 10^{-3}\ m^3$$

$$= 1013{,}25\ J$$

$$\frac{1013{,}25\ J}{4{,}184\ J/cal} = 242{,}2\ cal$$

$V_i = 5L$

$V_f = 15L$

$P_i = P_f = 1\ atm$

$= 101325\ Pa$

$Q = ?$ $\Delta U = 0$

$1L = 1\ dm^3$

$1\ cal = 4{,}184\ J$

Appello del 9/11/16

Problema A:

$$x = t + 2{,}0\,t^3$$

$m = 4{,}00\ Kg$

$v = ?$

$a = ?$

$$v = \frac{dx}{dt} = 1 + 6{,}0\,t^2$$

$$K = \frac{1}{2} m v^2 = \frac{1}{2}(4{,}00)\cdot(1+6{,}0\,t^2)^2 = 2\cdot(1+6{,}0\,t^2)^2$$

$$a = \frac{dv}{dt} = 12{,}0\,t$$

$$F = ma = 4{,}00 \cdot 12{,}0\,t = 48{,}0\,t$$

$$P = F\cdot v = (48{,}0\,t)\cdot(1+6{,}0\,t^2)$$

$t_i = 0\ s$

$t_f = 2{,}00\ s$

$W = ?$

$$W = \Delta K$$

$$\Delta K = K_f - K_i$$

$$\Delta K = \underbrace{\left[2(1+6{,}0t^2)^2\right]}_{K_f}\Big|_{t=2} - \underbrace{\left[2(1+6{,}0t^2)^2\right]}_{K_i}\Big|_{t=0} =$$

$$1250\ J \quad - \quad 2\ J = 1248\ J$$

B $\vec{F}_p = (2, -1, 2)\,N$

$P = (1, 2, 1)$ $Q = (-2, 2, -3)$

$\vec{\tau}_Q = ?$

$\vec{\tau}_Q = \vec{r} \times \vec{F}$

$$\vec{a} \times \vec{b} = \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$\hat{i} \times \hat{i} = 0$ $\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{j} = 0$ $j \times k = \hat{i}$

$\hat{k} \times \hat{k} = 0$ $k \times i = \hat{j}$

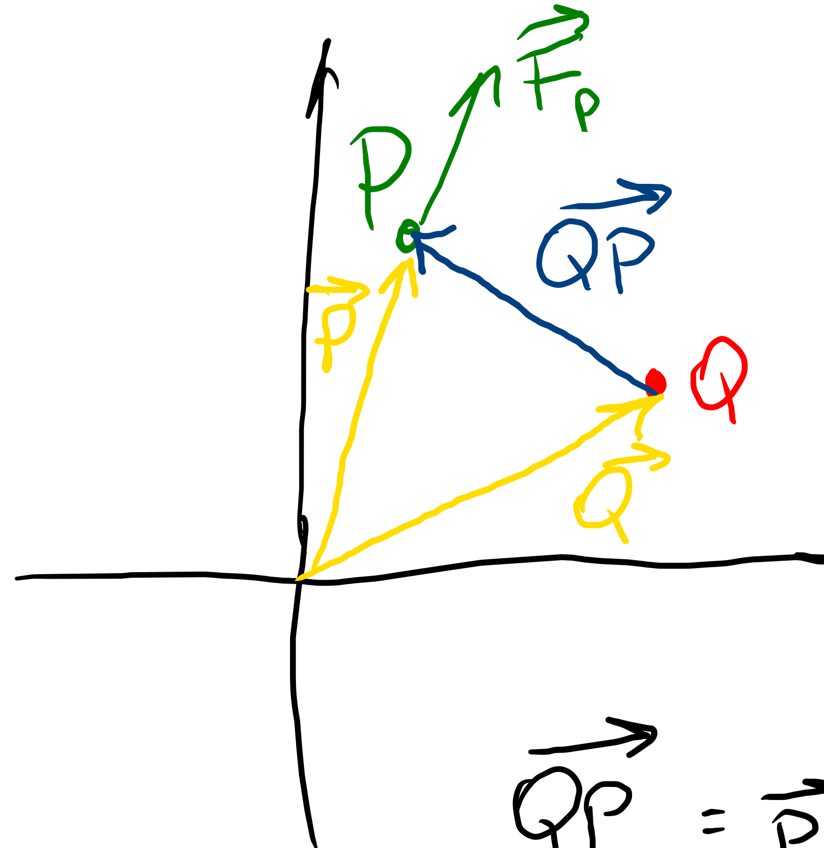
$$(a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}) \times (b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k})$$

$= \ldots\ldots\ldots$

$$\vec{\tau}_Q = \vec{QP} \times \vec{F}_P$$

$$= (3,0,4)^{m} \times (2,-1,2)\,N$$

$$= (4,2,-3)\,N\cdot m$$

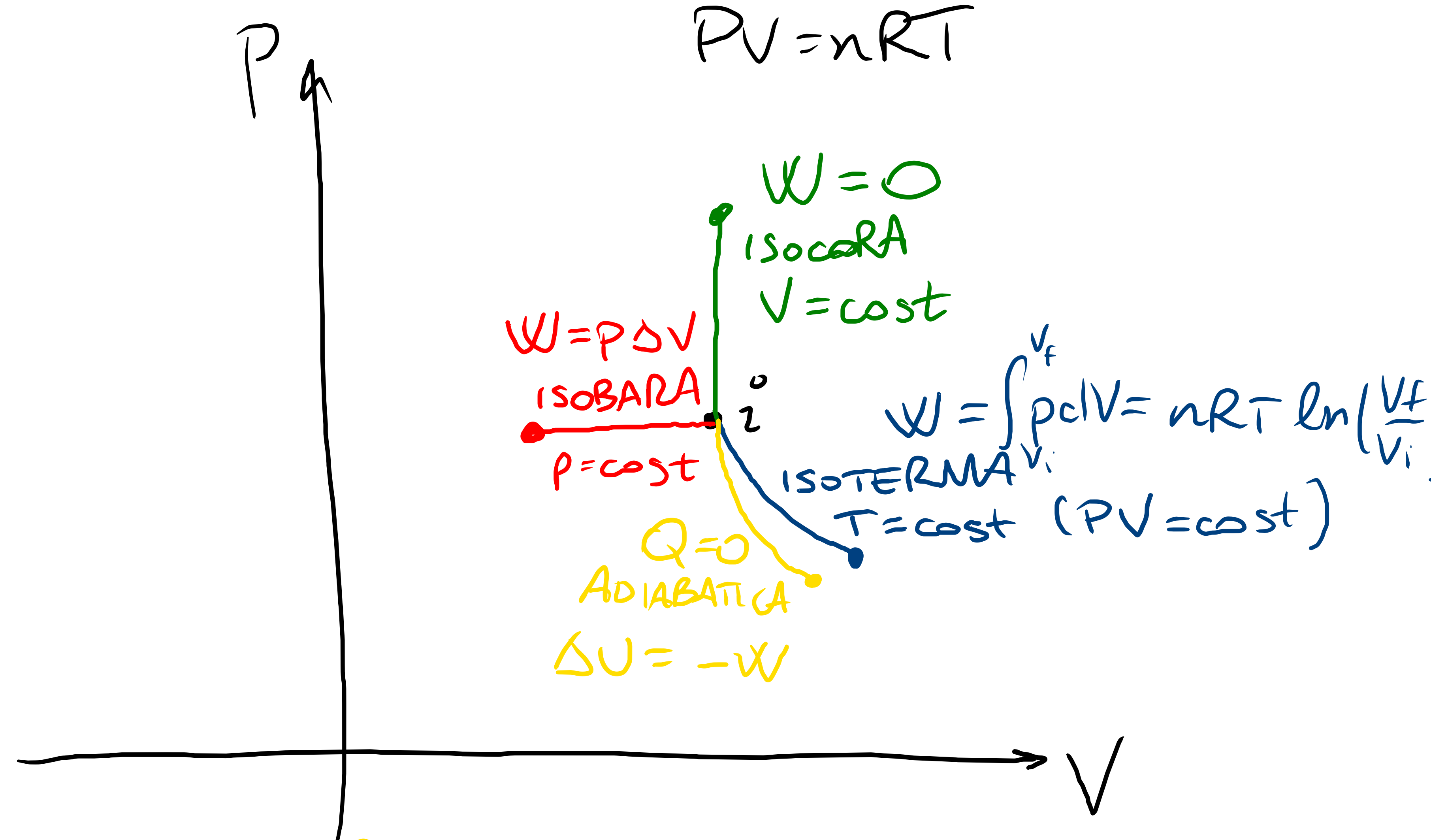


$$\vec{QP} = \vec{P} - \vec{Q}$$

$$= (1,2,1) - (-2,2,-3)$$

$$= (3,0,4)$$

$$\left|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{matrix}\right| \begin{matrix} \hat{i} & \hat{j} \\ a_1 & a_2 \\ b_1 & b_2 \end{matrix} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - a_1b_3)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$



ADIABATICA

$P = \frac{nRT}{V}$ $PV^{\gamma} = cost$

$TV^{\gamma - 1} = cost$

$\gamma = \frac{C_p}{C_v}$

Appello 22/6/2016

Gas ideale monoatomico

$$\gamma = \frac{C_p}{C_V} = \frac{\frac{5}{2}\cancel{R}}{\frac{3}{2}\cancel{R}} = \frac{5}{3}$$

ⓒ $T_i = 400\ K$ $\quad P_i = 1\ atm$

$V_f = \frac{1}{3} V_i$ $\quad Q = 0 \longrightarrow$ trasf. adiabatica

$P_f = ?$ $\quad T_f = ?$

$$P_i V_i^{\gamma} = P_f V_f^{\gamma} \qquad P_f = P_i \left(\frac{V_i}{V_f}\right)^{\gamma} = P_i\, 3^{\gamma} =$$

$$= 1\,atm \cdot 3^{5/3} = 6{,}24\ atm$$

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} = T_i \cdot 3^{\gamma-1} = 400K \cdot 3^{2/3} = 832\ K$$

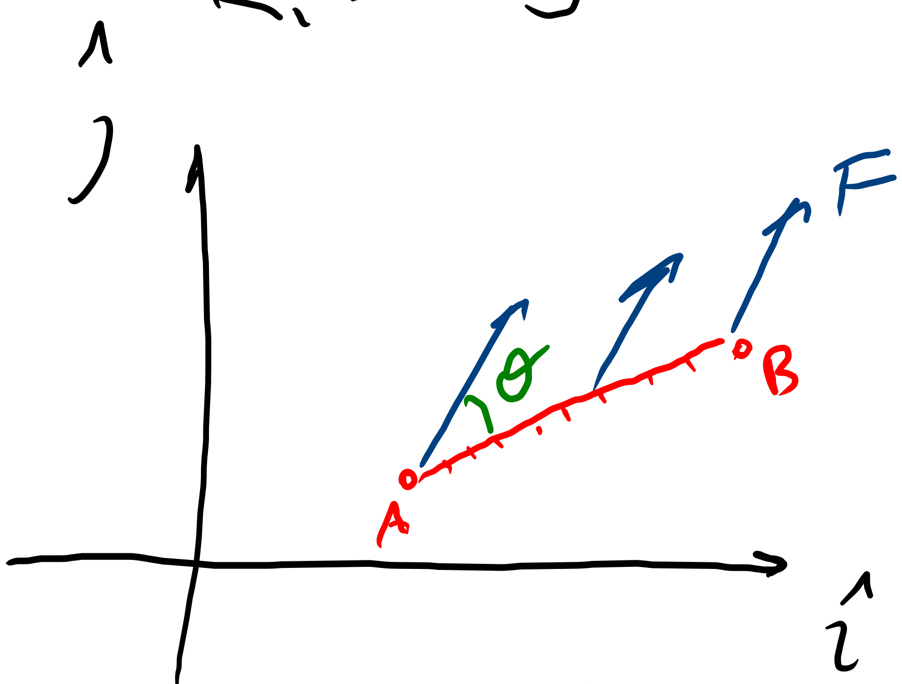
Appello 09/02/2017

(B) $m = 100\,kg$ $A = (20, 10)\,m$ $B = (40, 30)\,m$

$|v_0| = 0\ m/s$ $F = (100, 300)\,N$

$K_i = 0\,J$ $K_f = ?$

$\boxed{\theta_{\vec{F}-\vec{AB}}}$



$\vec{AB} = \vec{B} - \vec{A} = (40, 30) - (20, 10) =$

$= (20, 20)\,m$

$\Delta K = W \Longrightarrow K_f = W = \vec{F} \cdot \vec{AB}$

$= (100, 300)\,N \cdot (20, 20)\,m = (100 \cdot 20 + 300 \cdot 20)\,J = 8000\,J$

Ricordo Prodotto Scalare $\vec{a} = (a_1, a_2, a_3)$ $\vec{b} = (b_1, b_2, b_3)$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\vec{F} \cdot \vec{AB} = |\vec{F}|\,|\vec{AB}| \cos\theta$$

$$\cos\theta = \frac{\vec{F} \cdot \vec{AB}}{|\vec{F}|\,|\vec{AB}|} = \frac{8000}{100\sqrt{10} \cdot 20\sqrt{2}} = \frac{2}{\sqrt{5}}$$

$$|\vec{F}| = \sqrt{100^2 + 300^2} = \sqrt{10^5} = 100 \cdot \sqrt{10}\ \text{N}$$

$$|\vec{AB}| = \sqrt{20^2 + 20^2} = \sqrt{800} = 20\sqrt{2}\ \text{m}$$

$$\theta = \arccos\left(\frac{2}{\sqrt{5}}\right) = 26{,}6°$$