Correzione prova esame 24/06/2021

Problema (D)

a) $$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} = 72,6$$

x_i	$\bar{x}$	$x_i - \bar{x}$
50	72,6	−22,6
84	"	11,4
72	"	−0,6
81	"	8,4

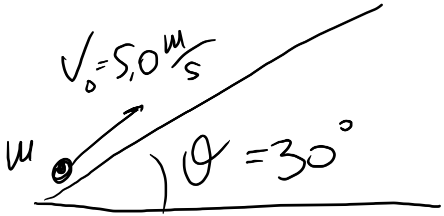
b) $$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$$

c) $$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}$$

incertezza della media

$$\bar{x} \pm \sigma_{\bar{x}}$$

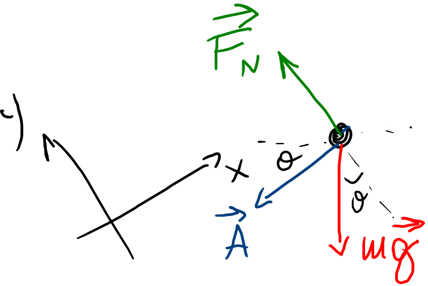
Problema A



$\mu_d = 0,30$

$h_{max} = ?$

$V_f = 0 \frac{m}{s}$



Seconda legge di Newton

$$\sum \vec{F} = m\vec{a} \qquad \vec{F}_N + m\vec{g} + \vec{A} = m\vec{a}$$

In componenti lungo

// piano ← $x: \begin{cases} -mg\sin\vartheta - A = ma \Rightarrow \\ F_N - mg\cos\vartheta = 0 \Rightarrow F_N = mg\cos\vartheta \end{cases}$

⊥ piano ← $y:$

$$A = \mu_d \cdot F_N = \mu_d \, mg\cos\vartheta$$

Quindi

$$-\cancel{m}g\sin\theta - \mu_d \cancel{m}g\cos\theta = \cancel{m}a$$

$$a = -g\left(\sin\theta + \mu_d\cos\theta\right) = -7{,}45 \ \frac{m}{s^2}$$

Applico

dove $V = 0 \ \frac{m}{s}$ $\quad x - x_0 = d = ?$

$$d = \frac{V^2 - V_0^2}{2a} =$$

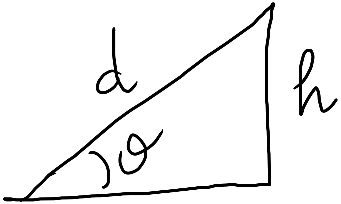
$$= \frac{0 - \left(5 \frac{m}{s}\right)^2}{2 \cdot \left(-7{,}45 \frac{m}{s^2}\right)} = 1{,}68 \ m$$

FOMULE MOTO UNIF. ACCEL.

$$S = S_0 + V_0 t + \frac{1}{2} a t^2$$

$$V = V_0 + at$$

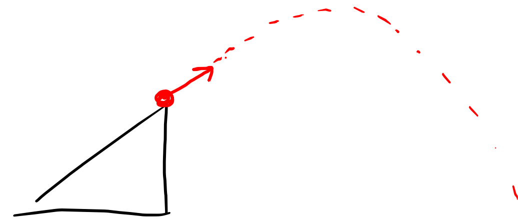
$$\boxed{V^2 = V_0^2 + 2a(x - x_0)}$$



$$h = d \sin\theta$$

$$d = \frac{h}{\sin\theta}$$

$$h = d \sin\theta = 0{,}84 \text{ m}$$



Problema (B)

$V_{20} = 0\ \frac{m}{s}$

$V_{2f} = ?$

In TUTTI gli urti la quantità di moto si conserva

Negli urti elastici si conserva l'energia cinetica $K_i = K_f$

Negli urti completamente anelastici le due masse restano attaccate $\rightarrow$ $V_{1f} = V_{2f}$

a) ELASTICO

$$\begin{cases} P_i = P_f \\ K_i = K_f \end{cases} \Longrightarrow \begin{cases} m_1 V_{1o} = m_1 V_{1f} + m_2 V_{2f} & (1) \\ \frac{1}{2} m_1 V_{1o}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2 & (2) \end{cases}$$

2 eq. in 2 incognite

Isolo V_{1f} in (1)

$$V_{1f} = \frac{m_1 V_{1o} - m_2 V_{2f}}{m_1} \quad (3)$$

Sost. (3) in (2)

$$m_1 V_{1o}^2 = \cancel{m_1} \frac{(m_1 V_{1o} - m_2 V_{2f})^2}{m_1^{\cancel{2}}} + m_2 V_{2f}^2 \quad (4)$$

Isolo V_{2f} in (4)

$$V_{2f} = \cancel{\frac{m_2 - m_1}{m_2 + m_1} V_{2i}} + \frac{2m_1}{m_2 + m_1} V_{1i}$$

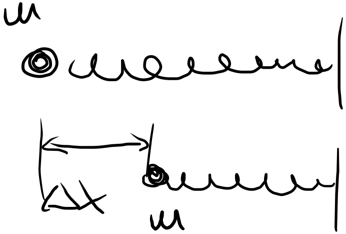
$$= \frac{2m_1}{m_2 + m_1} V_{1i} = 0{,}067 \, \frac{m}{s}$$ → ELASTICO

b) COMPLETAMENTE ANELASTICO

$$\begin{cases} p_i = p_f \\ V_{1f} = V_{2f} = V_f \end{cases} \qquad \begin{cases} m_1 V_{10} = m_1 V_{1f} + m_2 V_{2f} \\ V_{1f} = V_{2f} = V_f \end{cases}$$

COMPLET. ANELASTICO

$$m_1 V_{10} = (m_1 + m_2) V_f \qquad V_f = \frac{m_1}{m_1 + m_2} V_{10} = 0{,}033 \, \frac{m}{s}$$



Conservazione dell'energia

$$K_i + E_i = K_f + E_f$$

energia elastica

$$E_i = 0$$

$$K_f = 0$$

$$K_i = \frac{1}{2} m v_{2f}^2$$

$$E_f = \frac{1}{2} k (\Delta x)^2$$

$$\longrightarrow \cancel{\frac{1}{2}} m v_{2f}^2 = \cancel{\frac{1}{2}} k (\Delta x)^2$$

$$\Delta x = \sqrt{\frac{m}{k}}\, v_{2f}$$

$$\Delta x = \sqrt{\frac{m}{K}}\, V$$

a) CASO ELASTICO

$$\Delta x = \sqrt{\frac{m_2}{K}}\, V_{2f} = 3{,}0\ cm$$

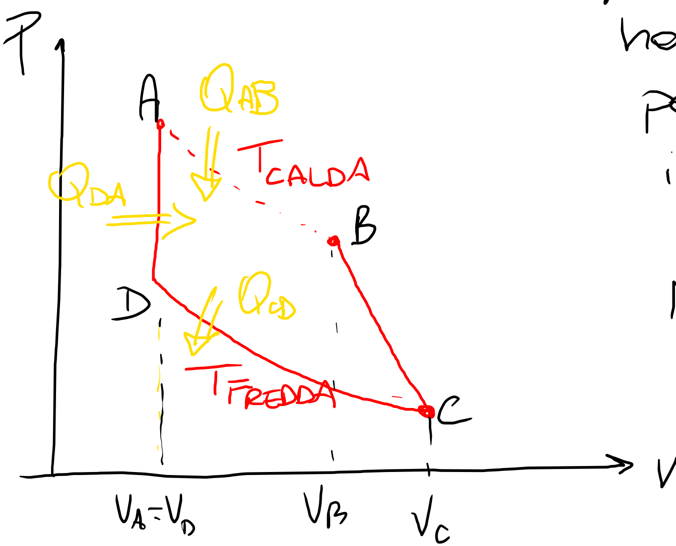
b) CASO COMPL. ANELASTICO

$$\Delta x = \sqrt{\frac{m_1+m_2}{K}}\, V_f =$$

$$= \sqrt{\frac{m_1+m_2}{K}} \cdot \frac{m_1}{m_1+m_2} V_{10} =$$

$$= \frac{1}{\sqrt{K(m_1+m_2)}}\, m_1 V_{10} = 1{,}8\ cm$$

Problema C



Nelle t. irreversibili non è definito il percorso tra lo stato iniziale e quello finale

↓

Non è possibile determinare le variabili termodinamiche tra A e B.

$$\cancel{W = \int_i^f p\,dV}$$

NO PER LE IRREVERSIBILI

$$\cancel{W = nRT} \ln\left(\frac{V_f}{V_i}\right)$$

NO PER ISOTERMA IRREVERSIBILE

L'energia si conserva sempre

$\downarrow$

Il primo principio resta valido

$$\Delta U = Q - W$$ OK

Gas ideale biatomico

$C_V = \frac{5}{2} R$

$C_P = \frac{7}{2} R$

$\gamma = \frac{C_P}{C_V} = \frac{7}{5}$

$W_{DA} = 0$

$W_{AB} \Rightarrow \Delta U_{AB} = 0 = Q_{AB} - W_{AB}$

$W_{AB} = Q_{AB} = 2100\,J$

Nei gas ideali U (en. interna) dipende <u>solo</u> da T

$\Delta U = n C_V \Delta T$

W_{BC} è un adiabatica $(Q=0)$ $\quad \Delta U_{BC} = -W_{BC}$

$$W_{BC} = -nC_V(T_C - T_B) = nC_V T_B \left(1 - \left(\frac{V_B}{V_C}\right)^{\gamma-1}\right)$$

$$TV^{\gamma-1} = \text{costante}$$

$$T_C = T_B \left(\frac{V_B}{V_C}\right)^{\gamma-1} = 850\,K$$

$$\frac{V_B}{V_C} = \frac{20\,\ell}{30\,\ell} = \frac{2}{3}$$

$$\frac{V_D}{V_C} = \frac{V_A}{V_C} = \frac{10\,\ell}{30\,\ell} = \frac{1}{3}$$

$$W_{CD} = \int_i^f p\,dV = \int_{V_C}^{V_D} nRT_C \frac{dV}{V} = nRT_C \ln\left(\frac{V_D}{V_C}\right)$$

$$W_{TOT} = W_{AB} + W_{BC} + W_{CD} = Q_{AB} + nC_V \overset{=T_A}{T_B}\left(1 - \left(\frac{V_B}{V_C}\right)^{\gamma-1}\right) +$$

$$+ nRT_C \ln\left(\frac{V_D}{V_C}\right) = Q_{AB} + nC_V T_A \left(1 - \left(\frac{2}{3}\right)^{2/5}\right) + nRT_A \left(\frac{2}{3}\right)^{2/5} \cdot \ln\left(\frac{1}{3}\right) =$$

$$W_{TOT} = 2100\,J + 1244\,J - 3105\,J = 239\,J$$

$$\eta = \frac{W_{TOT}}{Q_{ass}} = \frac{239\,J}{3344\,J} = 0{,}071 \Longrightarrow 7{,}1\%$$

$$Q_{ass} = Q_{DA} + Q_{AB} = 1244\,J + 2100\,J = 3344\,J$$

$$Q_{DA} = nC_v\,(T_A - T_D) = 1244\,J$$