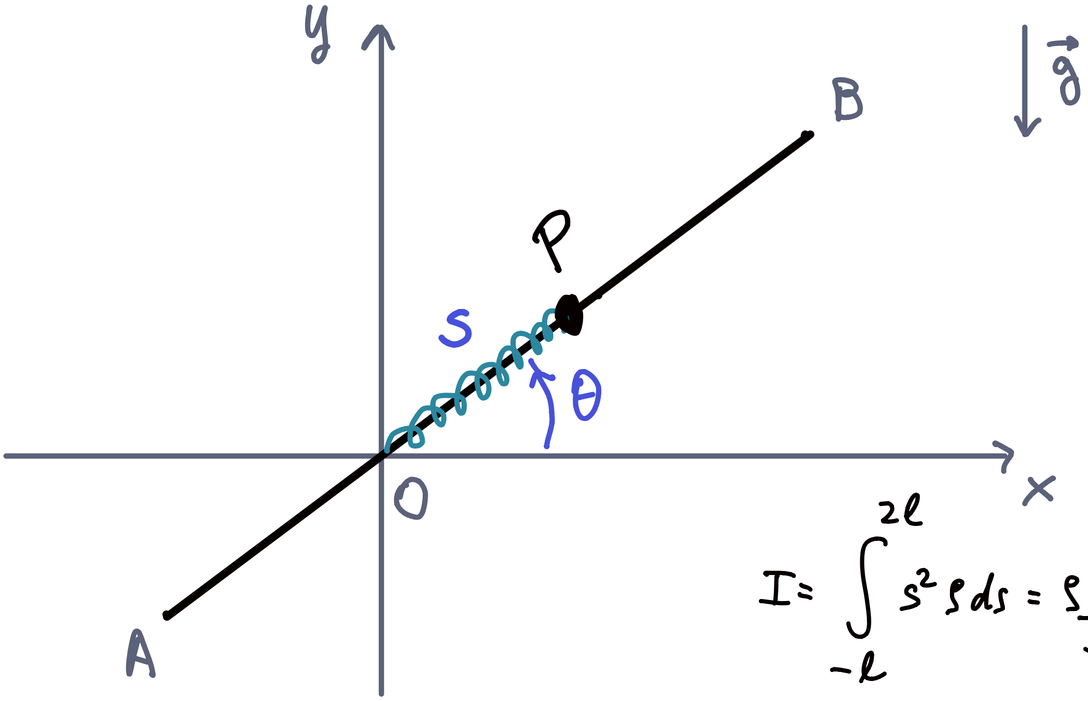

$$I = \int_{-\ell}^{2\ell} s^2 \rho \, ds = \rho \frac{s^3}{3}\Big|_{-\ell}^{2\ell} = \frac{\rho \ell^3}{3}(8+1) = 3\rho\ell \cdot \ell^2 = m\ell^2$$

1) $x_P = s\cos\theta \qquad \dot{x}_P = \dot{s}\cos\theta - s\dot{\theta}\,\text{sen}\,\theta$

$y_P = s\,\text{sen}\,\theta \qquad \dot{y}_P = \dot{s}\,\text{sen}\,\theta + s\dot{\theta}\cos\theta$

$$T = \frac{1}{2}m(\dot{s}^2 + s^2\dot{\theta}^2) + \frac{1}{2}m\ell^2\dot{\theta}^2$$

$$V_{grav} = mg\,y_P + mg\,y_{c.m.} = mgs\,\text{sen}\,\theta + mg\frac{\ell}{2}\text{sen}\,\theta$$

$$V_{elast.} = \frac{1}{2}Ks^2$$

$$L = \frac{1}{2}m\dot{s}^2 + \frac{1}{2}m(s^2+\ell^2)\dot{\theta}^2 - mg\left(s+\frac{\ell}{2}\right)\text{sen}\,\theta - \frac{1}{2}Ks^2$$

$$a = \begin{pmatrix} m & 0 \\ 0 & m(s^2+\ell^2) \end{pmatrix}$$

2) $\frac{d}{dt}\frac{\partial L}{\partial \dot{s}} = m\ddot{s} \qquad \frac{\partial L}{\partial s} = ms\dot{\theta}^2 - mg\,\text{sen}\,\theta - Ks$

$$\ddot{s} = s\dot{\theta}^2 - g\,\text{sen}\,\theta - \frac{K}{m}s$$

$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = m(s^2+\ell^2)\ddot{\theta} + 2ms\dot{s}\dot{\theta} \qquad \frac{\partial L}{\partial \theta} = -mg\left(s+\frac{\ell}{2}\right)\cos\theta$

$$\ddot{\theta} + \frac{2s}{s^2+\ell^2}\dot{s}\dot{\theta} + \frac{g(s+\ell/2)\cos\theta}{s^2+\ell^2} = 0$$

3) ...

4) $V = mg\left(s+\frac{l}{2}\right)\operatorname{sen}\theta + \frac{1}{2}Ks^2$

$$\frac{\partial V}{\partial s} = mg\operatorname{sen}\theta + Ks = 0 \qquad s = -\frac{mg}{K}\operatorname{sen}\theta$$

$$\frac{\partial V}{\partial \theta} = mg\left(s+\frac{l}{2}\right)\cos\theta = 0$$

$\hookrightarrow$ $\theta = \pm\pi/2 \rightarrow s = \mp\frac{mg}{K}$

$s = -\frac{l}{2} \rightarrow \operatorname{sen}\theta^*_{1,2} = \frac{Kl}{2mg}$

$\downarrow$

Esiste solo se $Kl \leq 2mg$

$$b \equiv \partial^2 V = \begin{pmatrix} K & mg\cos\theta \\ mg\cos\theta & -mg\left(s+\frac{l}{2}\right)\operatorname{sen}\theta \end{pmatrix}$$

$$b\left(\frac{mg}{K}, -\frac{\pi}{2}\right) = \begin{pmatrix} K & 0 \\ 0 & mg\left(\frac{mg}{K}+\frac{l}{2}\right) \end{pmatrix}$$

def. pos.
$\Rightarrow$ pto eq. STAB.

$$b\left(-\frac{mg}{K}, \frac{\pi}{2}\right) = \begin{pmatrix} K & 0 \\ 0 & mg\left(\frac{mg}{K}-\frac{l}{2}\right) \end{pmatrix}$$

def. pos. se $Kl \leq 2mg$
(STAB)
altrim. INSTAB.

$$b\left(-\frac{l}{2}, \theta^*_{1,2}\right) = \begin{pmatrix} K & mg\cos\theta^*_{1/2} \\ mg\cos\theta^*_{1/2} & 0 \end{pmatrix}$$

$\det = -\left(mg\cos\theta^*_{1/2}\right)^2 \leq 0$

$\downarrow$

non è def. pos.

$\rightarrow$ sempre INSTAB.
(quando esiste)

5) ...

6) $\left(\frac{mg}{K}, -\frac{\pi}{2}\right)$: $B = \begin{pmatrix} K & 0 \\ 0 & mg\left(\frac{mg}{K} + \frac{\ell}{2}\right) \end{pmatrix}$

$$A = a\left(\frac{mg}{K}, -\frac{\pi}{2}\right) = \begin{pmatrix} m & 0 \\ 0 & m\left(\left(\frac{mg}{K}\right)^2 + \ell^2\right) \end{pmatrix}$$

$$\zeta \equiv s - \frac{mg}{K}$$

$$\varphi \equiv \theta + \frac{\pi}{2}$$

$$\hat{L} = \frac{1}{2} m \dot{\zeta}^2 + \frac{1}{2} m\left(\frac{m^2g^2}{K^2} + \ell^2\right)\dot{\varphi}^2 - \frac{1}{2} K \zeta^2 - \frac{mg}{2}\left(\frac{mg}{K} + \frac{\ell}{2}\right)\varphi^2$$

$$0 = \det(B - \lambda A) \rightarrow \quad \lambda_\zeta = \frac{K}{m}$$

$$\lambda_\varphi = \frac{g\left(\frac{mg}{k} + \frac{\ell}{2}\right)}{\ell^2 + \frac{m^2g^2}{K^2}} = \frac{g}{2\ell}\left(\frac{\frac{2mg}{K\ell} + 1}{\left(\frac{mg}{K\ell}\right)^2 + 1}\right)$$

$$= \frac{g}{2\ell}\left(1 - \frac{\left(\frac{mg}{K\ell}\right)^2}{1 + \left(\frac{mg}{K\ell}\right)^2}\right)$$

7)
$$\zeta(t) = A_\zeta \cos\left(\sqrt{\lambda_\zeta}\, t + c_\zeta\right)$$

$$\varphi(t) = A_\varphi \cos\left(\sqrt{\lambda_\varphi}\, t + c_\varphi\right)$$

$$s(t) = \frac{mg}{K} + A_\zeta \cos\left(\sqrt{\lambda_\zeta}\, t + c_\zeta\right)$$

$$\theta(t) = -\frac{\pi}{2} + A_\varphi \cos\left(\sqrt{\lambda_\varphi}\, t + c_\varphi\right)$$

Quiz.

$p = P^{\alpha} \cos Q \qquad q = b P^{\beta} \operatorname{sen} Q$

$$\{q, p\} = \frac{\partial q}{\partial Q}\frac{\partial p}{\partial P} - \frac{\partial q}{\partial P}\frac{\partial p}{\partial Q} =$$

$$= b P^{\beta} \cos Q \; \alpha P^{\alpha-1} \cos Q - b\beta P^{\beta-1} \operatorname{sen} Q \; P^{\alpha}(-\operatorname{sen} Q)$$

$$= b P^{\beta+\alpha-1} (\alpha \cos^2 Q + \beta \operatorname{sen}^2 Q)$$

$\alpha = \beta$

$\alpha = \beta = \frac{1}{2}$

$\frac{b}{2} = 1 \Rightarrow b = 2$