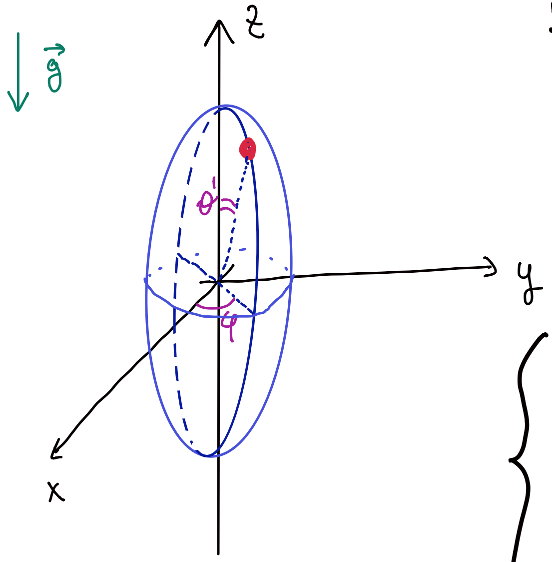


Elissoide

$$\operatorname{tg}\theta = \frac{b}{a}\,\frac{r}{z} = \frac{b}{a}\operatorname{tg}\theta'$$

$$r = a \operatorname{sen}\theta$$

$$z = b\cos\theta$$

$$\begin{cases} x = a \operatorname{sen}\theta \cos\varphi \\ y = a \operatorname{sen}\theta \operatorname{sen}\varphi \\ z = b\cos\theta \end{cases} \qquad \begin{cases} \dot{x} = a\cos\theta\cos\varphi\,\dot{\theta} - a\operatorname{sen}\theta\operatorname{sen}\varphi\,\dot{\varphi} \\ \dot{y} = a\cos\theta\operatorname{sen}\varphi\,\dot{\theta} + a\operatorname{sen}\theta\cos\varphi \\ \dot{z} = -b\operatorname{sen}\theta\,\dot{\theta} \end{cases}$$

1.

$$T = \frac{m}{2}\left[a^2\operatorname{sen}^2\theta\,\dot{\varphi}^2 + \left(a^2\cos^2\theta + b^2\operatorname{sen}^2\theta\right)\dot{\theta}^2\right]$$

$$V = mgb\cos\theta$$

- Si scrive la metrica cinetica

$$L = \frac{m}{2}\left[a^2\operatorname{sen}^2\theta\,\dot{\varphi}^2 + \left(a^2\cos^2\theta + b^2\operatorname{sen}^2\theta\right)\dot{\theta}^2\right] - mgb\cos\theta$$

$$\text{Metrica cinetica} = \begin{pmatrix} m a^2\operatorname{sen}^2\theta & 0 \\ 0 & m\left(a^2\cos^2\theta + b^2\operatorname{sen}^2\theta\right) \end{pmatrix}$$

2. vedi teoria

3. $$\frac{\partial L}{\partial \dot\varphi} = m a^2 \operatorname{sen}^2\theta\, \dot\varphi \qquad \dot\varphi = \frac{\ell}{m a^2 \operatorname{sen}^2\theta}$$

$$L^* = \frac{m}{2}\left(a^2\cos^2\theta + b^2\operatorname{sen}^2\theta\right)\dot\theta^2 - mgb\cos\theta - \frac{\ell^2}{2ma^2\operatorname{sen}^2\theta}$$

4. vedi teoria

5.

Eq. Lagr. : $$\frac{d}{dt}\frac{\partial L^*}{\partial\dot\theta} = \frac{d}{dt}\left[m\left(a^2\cos^2\theta + b^2\operatorname{sen}^2\theta\right)\dot\theta\right] = m\left(a^2\cos^2\theta + b^2\operatorname{sen}^2\theta\right)\ddot\theta + 2m\left(b^2 - a^2\right)\cos\theta\operatorname{sen}\theta\,\dot\theta^2$$

$$\frac{\partial L^*}{\partial\theta} = m\left(b^2 - a^2\right)\cos\theta\operatorname{sen}\theta\,\dot\theta^2 + mgb\operatorname{sen}\theta + \frac{\ell^2\cos\theta}{ma^2\operatorname{sen}^3\theta}$$

$$m\left(a^2\cos^2\theta + b^2\operatorname{sen}^2\theta\right)\ddot\theta = -m\left(b^2 - a^2\right)\cos\theta\operatorname{sen}\theta\,\dot\theta^2 + mbg\operatorname{sen}\theta + \frac{\ell^2\cos\theta}{ma^2\operatorname{sen}^3\theta}$$

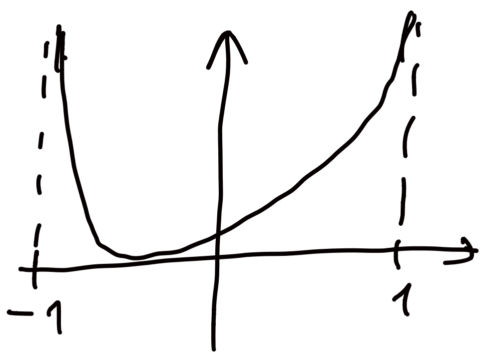
6.

$$V_{eff} = mgb\cos\theta + \frac{l^2}{2ma^2\sin^2\theta}$$

$x = \cos\theta \qquad V_{eff}(\theta) = f(x)$

$$mgbx + \frac{l^2}{2ma^2(1-x^2)}$$

$$f(x) = \alpha x + \frac{\beta}{2(1-x^2)}$$



$(\alpha, \beta > 0)$

$$f'(x) = \alpha + \frac{x\beta}{(1-x^2)^2}$$

← ha sempre uno zero per $x^* < 0 \rightarrow \frac{\pi}{2} < \theta^* < \pi$

$$\boxed{V'_{eff}(\theta) = -\left(\alpha + \frac{\cos\theta\,\beta}{\sin^4\theta}\right)\sin\theta}$$

7.

$$L^{*}_{l=0} = \frac{m}{2}\left(a^2\cos^2\theta + b^2\operatorname{sen}^2\theta\right)\dot{\theta}^2 - mgb\cos\theta$$

$V = mgb\cos\theta$

min $\rightarrow$ $\theta = \pi$ def. $\theta = \pi + \delta\theta$ $\dot{\theta} = \delta\dot{\theta}$

$\cos(\pi + \delta\theta) = -\cos\delta\theta$

$$\hat{L}^{*}_{l=0} = \frac{m}{2}a^2\delta\dot{\theta}^2 + mgb\left(1 - \frac{\delta\theta^2}{2}\right)$$

$$\rightarrow \frac{m}{2}a^2\delta\dot{\theta}^2 - \frac{mgb}{2}\delta\theta^2 \longleftrightarrow \frac{1}{2}A\delta\dot{\theta}^2 - \frac{1}{2}B\delta\theta^2$$

$$\lambda = \frac{B}{A} \Rightarrow \lambda = \omega^2 = g\frac{b}{a^2} \rightarrow \omega = \frac{1}{a}\sqrt{gb}$$