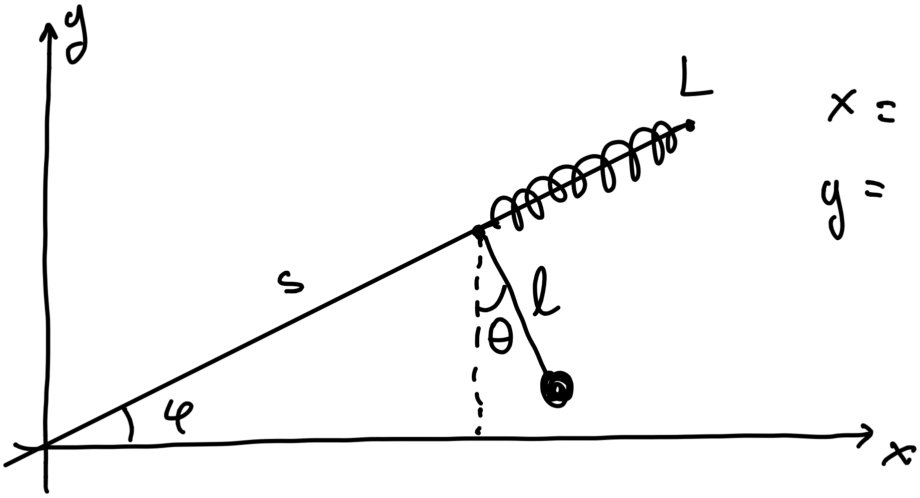

$$x = s\cos\varphi + l\,\mathrm{sen}\,\theta$$
$$y = s\,\mathrm{sen}\,\varphi - l\cos\theta$$

$$\dot{x} = \dot{s}\cos\varphi + l\dot{\theta}\cos\theta$$
$$\dot{y} = \dot{s}\,\mathrm{sen}\,\varphi + l\dot{\theta}\,\mathrm{sen}\,\theta$$

1)
$$T = m\left(\frac{\dot{x}^2}{2} + \frac{\dot{y}^2}{2}\right) = \frac{m}{2}\left(\dot{s}^2 + l^2\dot{\theta}^2 + 2l\dot{s}\dot{\theta}\left(\overbrace{\cos\varphi\cos\theta + \mathrm{sen}\,\varphi\,\mathrm{sen}\,\theta}^{\cos(\theta-\varphi)}\right)\right)$$

$$V = mgy = mgs\,\mathrm{sen}\,\varphi - mgl\cos\theta + \frac{1}{2}K(L-s)^2$$

$$L = \frac{m}{2}\left[\dot{s}^2 + l^2\dot{\theta}^2 + 2l\dot{s}\dot{\theta}\cos(\theta-\varphi)\right] - mgs\,\mathrm{sen}\,\varphi - \frac{1}{2}K(L-s)^2 + mgl\cos\theta$$

Matrice cinetica

$$a = m\begin{pmatrix} 1 & l\cos(\theta-\varphi) \\ l\cos(\theta-\varphi) & l^2 \end{pmatrix}$$

2)

Eq. di Lagrange

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{s}} = \frac{d}{dt}\left[m\dot{s} + ml\dot{\theta}\cos(\theta-\varphi)\right] = m\ddot{s} + ml\ddot{\theta}\cos(\theta-\varphi) - ml\dot{\theta}^2\,\mathrm{sen}(\theta-\varphi)$$

$$\frac{\partial L}{\partial s} = -mg\,\mathrm{sen}\,\varphi - K(s-L)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = \frac{d}{dt}\left[ml^2\dot{\theta} + ml\dot{s}\cos(\theta-\varphi)\right] = ml^2\ddot{\theta} + ml\ddot{s}\cos(\theta-\varphi) - ml\cancel{\dot{s}\dot{\theta}}\,\mathrm{sen}(\theta-\varphi)$$

$$\frac{\partial L}{\partial \theta} = \cancel{-ml\dot{s}\dot{\theta}}\,\mathrm{sen}(\theta-\varphi) - mgl\,\mathrm{sen}\,\theta$$

$$\begin{cases} \ddot{s} + l\ddot{\theta}\cos(\theta-\varphi) - l\dot{\theta}^2\,\mathrm{sen}(\theta-\varphi) + g\,\mathrm{sen}\,\varphi + \frac{k}{m}(s-L) = 0 \\ l\ddot{\theta} + \ddot{s}\cos(\theta-\varphi) + g\,\mathrm{sen}\,\theta = 0 \end{cases}$$

3)

Trovare i pti di eq. e loro stabilità

$$V = mgy = mgs\,\mathrm{sen}\,\varphi - mgl\cos\theta + \frac{1}{2}k(s-L)^2$$

$$\frac{\partial V}{\partial s} = mg\,\mathrm{sen}\,\varphi + k(s-L) \qquad s = L - \frac{mg}{k}\mathrm{sen}\,\varphi = s_0$$

$$\frac{\partial V}{\partial \theta} = mgl\,\mathrm{sen}\,\theta \qquad \theta = 0, \pi$$

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$$\partial^2 V = \begin{pmatrix} k & 0 \\ 0 & mgl\cos\theta \end{pmatrix}$$

min

min per $\theta = 0$

max per $\theta = \pi$

5)

Linearizzare attorno pto eq. stabile

$$L = \frac{m}{2}\left[\dot{s}^2 + l^2\dot{\theta}^2 + 2l\dot{s}\dot{\theta}\cos(\theta-\varphi)\right] - mgs\,\mathrm{sen}\,\varphi - \frac{1}{2}k(L-s)^2 + mgl\cos\theta$$

$$\hat{L} = \frac{m}{2}\left(\dot{s}^2 + l^2\dot{\theta}^2 + 2l\dot{s}\dot{\theta}\cos\varphi\right) + mgl\left(\cancel{1}^{\text{cost}} - \frac{\theta^2}{2}\right) - \frac{1}{2}k(s-s_0)^2$$

$$\sigma = s - s_0$$

$$\hat{L} = \frac{m}{2}\dot{\sigma}^2 + \frac{m}{2}l^2\dot{\theta}^2 + ml\cos\varphi\,\dot{\sigma}\dot{\theta} - \frac{k}{2}\sigma^2 - \frac{mgl}{2}\theta^2$$

Freq. delle piccole oscillazioni

$$A = \begin{pmatrix} m & ml\cos\varphi \\ ml\cos\varphi & ml^2 \end{pmatrix} \qquad B = \begin{pmatrix} K & 0 \\ 0 & mgl \end{pmatrix}$$

$$\det(B - \lambda A) = \det\begin{pmatrix} K - \lambda m & -\lambda ml\cos\varphi \\ -\lambda ml\cos\varphi & mgl - ml^2\lambda \end{pmatrix} =$$

$$= m^2 l^2 \left(\frac{K}{m} - \lambda\right)\left(\frac{g}{l} - \lambda\right) - m^2 l^2 \cos^2\varphi\, \lambda^2 = 0$$

$$\underbrace{(1 - \cos^2\varphi)}_{=\sin^2\varphi} \lambda^2 - \left(\frac{k}{m} + \frac{g}{l}\right)\lambda + \frac{k}{m}\frac{g}{l} = 0$$

$$\lambda_{1,2} = \frac{1}{2\sin^2\varphi}\left\{\left(\frac{k}{m} + \frac{g}{l}\right) \pm \sqrt{\left(\frac{k}{m} + \frac{g}{l}\right)^2 - 4\frac{k}{m}\frac{g}{l}(1 - \cos^2\varphi)}\right\}$$

Scrivere risultato con $\varphi = \pi/2$ e spiegarlo

7)

$$\hookrightarrow \begin{cases} \lambda_1 = k/m \\ \lambda_2 = g/l \end{cases}$$

problema disaccoppia in due oscillatori armonici con freq. date da molla o gravità.

6)

$$\hat{L} = \frac{m}{2}\dot{\sigma}^2 + \frac{m}{2}l^2\dot{\theta}^2 + ml\cos\varphi\,\dot{\sigma}\dot{\theta} - \frac{k}{2}\sigma^2 - \frac{mgl}{2}\theta^2$$

$$B-\lambda A = m\begin{pmatrix} k/m - \lambda & -\lambda\, l\cos\varphi \\ -\lambda\, l\cos\varphi & l^2\left(\frac{g}{l}-\lambda\right)\end{pmatrix} \qquad \lambda_{1,2} = \frac{1}{2\sin^2\varphi}\left\{\left(\frac{k}{m}+\frac{g}{l}\right) \pm \sqrt{\left(\frac{k}{m}+\frac{g}{l}\right)^2 - 4\frac{k}{m}\frac{g}{l}(1-\cos^2\varphi)}\right\}$$

$$\varphi = \frac{\pi}{3} \qquad \lambda_{1,2} = \frac{2}{3}\left\{\left(\frac{k}{m}+\frac{g}{l}\right) \pm \sqrt{\left(\frac{k}{m}+\frac{g}{l}\right)^2 - 3\frac{k}{m}\frac{g}{l}}\right\}$$

$$\frac{K}{m} = \frac{g}{l}$$

$$\lambda_{1,2} = \frac{2}{3}\left\{2\left(\frac{g}{l}\right) \pm \left(\frac{g}{l}\right)\right\} = \begin{cases} 2\,(g/l) = 2\lambda_0 \\ \frac{2}{3}(g/l) = \frac{2}{3}\lambda_0\end{cases} \qquad \boxed{\lambda_0 \equiv g/l}$$

$$(B-\lambda A) = \begin{pmatrix} \lambda_0 - \lambda & -\lambda\frac{l}{2} \\ -\lambda\frac{l}{2} & l^2(\lambda_0-\lambda)\end{pmatrix}$$

$$(B-\lambda_1) = \begin{pmatrix} -\lambda_0 & -\lambda_0 l \\ -\lambda_0 l & -\lambda_0 l^2\end{pmatrix} \qquad V_1 = \begin{pmatrix} l \\ -1\end{pmatrix}$$

(sol 1) $$\begin{pmatrix}\sigma(t)\\ \theta(t)\end{pmatrix} = A_1\begin{pmatrix} l \\ -1\end{pmatrix}\cos\left((2\lambda_0)^{1/2}t + \phi_1\right)$$

$$(B-\lambda_2) = \begin{pmatrix} \lambda_0/3 & -\lambda_0/3\, l \\ -\lambda_0/3\, l & \lambda_0/3\, l^2\end{pmatrix} \qquad V_2 = \begin{pmatrix} l \\ 1\end{pmatrix}$$

(sol 2) $$\begin{pmatrix}\sigma(t)\\ \theta(t)\end{pmatrix} = A_2\begin{pmatrix} l \\ 1\end{pmatrix}\cos\left(\left(\frac{2\lambda_0}{3}\right)^{1/2}t + \phi_2\right)$$

La soluzione generale è una combinazione lineare di (sol 1) e (sol 2)