Lavoriamo in coordinate cartesiane ortogonali

$$\to \boxed{\overline{\nabla} f^\alpha = \sum_i \bar{e}_i \frac{\partial f^\alpha}{\partial x^i} = \sum_i \bar{e}_i \alpha f^{\alpha-1} \frac{\partial f}{\partial x^i} = \alpha \cdot f^{\alpha-1} \overline{\nabla} f}$$

$$\bullet\ \overline{\nabla}_x \left(\frac{1}{|\bar{x}' - \bar{x}|} \right) = \overline{\nabla}_x \left\{ \left[(\bar{x}' - \bar{x})^2 \right]^{-1/2} \right\} = -\frac{1}{2} \frac{1}{\left[(\bar{x}'-\bar{x})^2\right]^{3/2}} \cdot \overline{\nabla}_x (\bar{x}' - \bar{x})^2$$

ma

$$\overline{\nabla}_x (\bar{x}' - \bar{x})^2 = \sum_i \bar{e}_i \frac{\partial}{\partial x^i} (\bar{x}' - \bar{x})^2 = \sum_i \bar{e}_i \frac{\partial}{\partial x^i} (x^{i\prime} - x^i)^2 =$$

$$= \sum_i \bar{e}_i \cdot 2 (x^{i\prime} - x^i) \cdot -1 = -2 \sum_i \bar{e}_i (x^{i\prime} - x^i) = -2 (\bar{x}' - \bar{x})$$

da cui

$$\boxed{\overline{\nabla}_x \left(\frac{1}{|\bar{x}' - \bar{x}|} \right) = -\frac{1}{2} \cdot \frac{1}{|\bar{x}' - \bar{x}|^3} \cdot -2 (\bar{x}' - \bar{x}) = \frac{\bar{x}' - \bar{x}}{|\bar{x}' - \bar{x}|^3}} \text{ cvd}$$

$$\to \boxed{\overline{\nabla} \cdot (\lambda \bar{u}) = \sum_i \frac{\partial}{\partial x^i} (\lambda u^i) = \sum_i \lambda \frac{\partial u^i}{\partial x^i} + \sum_i u^i \frac{\partial \lambda}{\partial x^i} = \lambda \overline{\nabla} \cdot \bar{u} + \bar{u} \cdot \overline{\nabla} \lambda}$$

$$\bullet\ \overline{\nabla}_x \cdot \frac{\bar{x}' - \bar{x}}{|\bar{x}' - \bar{x}|^3} = \frac{1}{|\bar{x}' - \bar{x}|^3} \cdot \overline{\nabla}_x \cdot (\bar{x}' - \bar{x}) + (\bar{x}' - \bar{x}) \cdot \overline{\nabla}_x \left(\frac{1}{|\bar{x}' - \bar{x}|^3} \right)$$

Ma $\overline{\nabla}_x \cdot \bar{x} = \sum_i \frac{\partial}{\partial x^i} (x^i) = 1+1+1 = 3 \to \boxed{\overline{\nabla}_x \cdot (\bar{x}' - \bar{x}) = -3}$

Inoltre

$$\overline{\nabla}_x \left(\frac{1}{|\bar{x}' - \bar{x}|^3} \right) = \overline{\nabla}_x \left\{ \left[(\bar{x}' - \bar{x})^2 \right]^{-3/2} \right\} = -\frac{3}{2} \frac{1}{\left[(\bar{x}'-\bar{x})^2\right]^{5/2}} \cdot \overline{\nabla} (\bar{x}' - \bar{x})^2 =$$

$$= -\frac{3}{2} \frac{1}{|\bar{x}' - \bar{x}|^5} \cdot -2 (\bar{x}' - \bar{x}) = +3 \frac{\bar{x}' - \bar{x}}{|\bar{x}' - \bar{x}|^5}$$

Allora

$$\boxed{\overline{\nabla}_x \cdot \left(\frac{\bar{x}' - \bar{x}}{|\bar{x}' - \bar{x}|^3} \right) = -3 \cdot \frac{1}{|\bar{x}' - \bar{x}|^3} + 3 \frac{(\bar{x}' - \bar{x}) \cdot (\bar{x}' - \bar{x})}{|\bar{x}' - \bar{x}|^5}}$$

Se $\bar{x}' - \bar{x} \neq 0$ posso semplificare un fattore $|\bar{x}' - \bar{x}|^2$ sopra e sotto nel 2° termine del 2° membro, per cui ho

$$\boxed{\overline{\nabla}_x \cdot \left(\frac{\bar{x}' - \bar{x}}{|\bar{x}' - \bar{x}|^3} \right) = \emptyset \quad \text{se } \bar{x}' \neq \bar{x}}$$