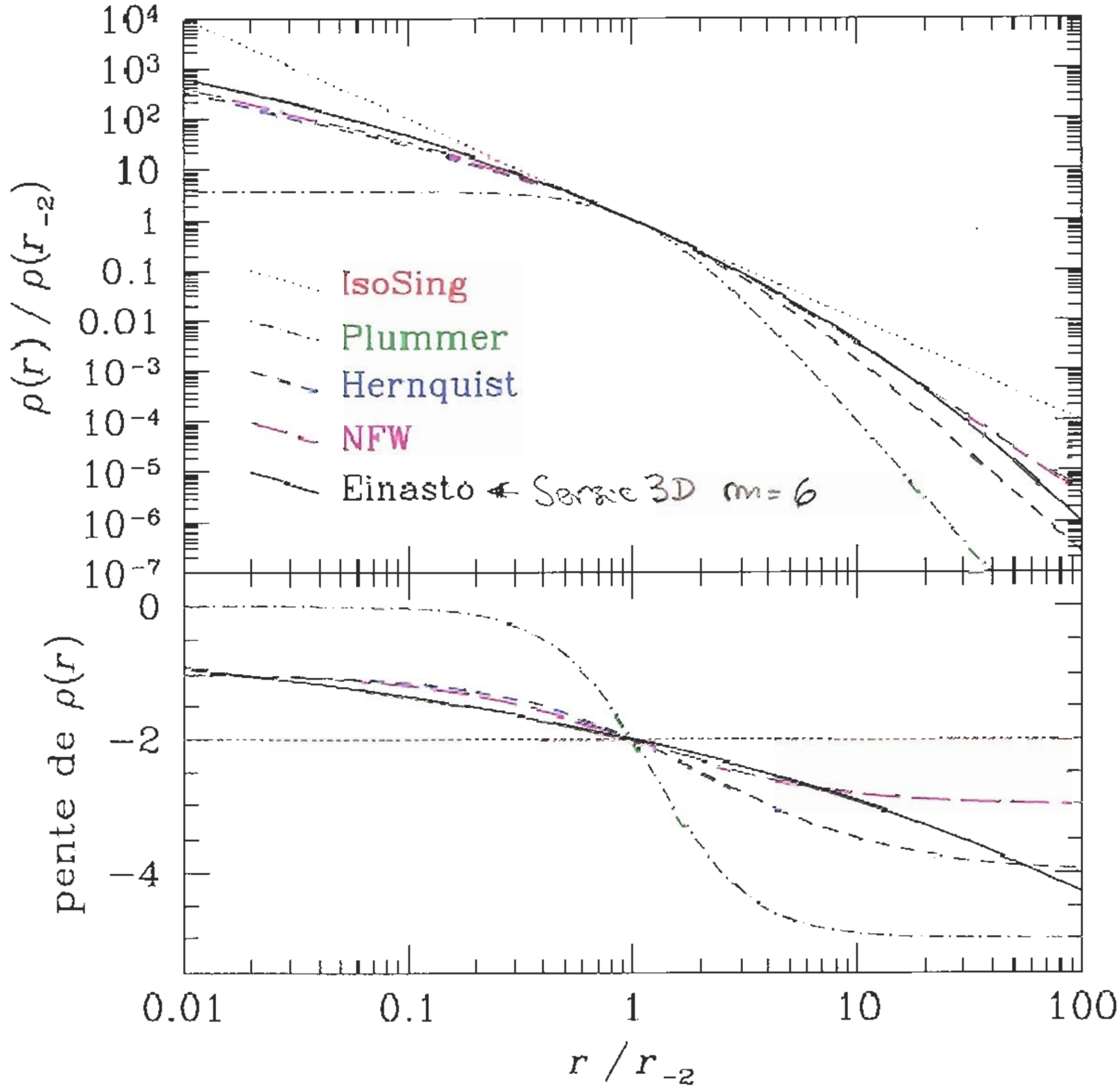


Modello di Sersic 3D : (detto anche di Einasto)

$$\rho_m(r) = \rho_0\, e^{-(r/a)^{1/m}}$$

Hanno il vantaggio che se $r \to 0$ $\rho(r) \to \rho_0$ finito ed hanno anche una massa che converge:

$$M = \int_0^\infty 4\pi \frac{r^2}{a^2} \rho_0\, e^{-(r/a)^{1/m}}\, d\frac{r}{a} \cdot a^3 = 4\pi a^3 \rho_0 \int_0^\infty x^2 e^{-x^{1/m}}\, dx$$

funzione Gamma

$$\int_0^\infty x^{m-1} e^{-x}\, dx = \Gamma(m)$$

$$x^{1/m} \equiv u \;\to\; x = u^m,\quad x^2 = u^{2m},\quad dx = m\, u^{m-1}\, du$$

$$\boxed{M = 4\pi a^3 \rho_0 \int_0^\infty u^{3m-1} e^{-u}\, du = 4\pi a^3 \rho_0 \cdot m\, \Gamma(3m)}$$

$$\ln\rho = \ln\rho_0 - (r/a)^{1/m} \to \frac{d\ln\rho}{dr} = -\frac{1}{m}\left(\frac{r}{a}\right)^{1/m-1} / r \to \frac{d\ln\rho}{d\ln r} = -\frac{1}{m}\, r \cdot \left(\frac{r}{a}\right)^{1/m-1} \cdot \frac{a}{a}$$

$$\boxed{\frac{d\ln\rho}{d\ln r} = -\frac{a}{m}\left(r/a\right)^{1/m}}$$

Inter

Modelli N-body di Dark Matter

Navarro et al 2004 :

Mon. Not. R. Astron. Soc. 349, 1039–1051 (2004)

After some experimentation, we have found that a density profile where $\beta(r)$ is a power law of radius is a reasonable compromise that satisfies these constraints whilst retaining simplicity, i.e.

$$\beta_\alpha(r) = -\mathrm{d}\ln\rho/\mathrm{d}\ln r = 2(r/r_{-2})^\alpha, \qquad (4)$$

$\alpha \equiv 1/m$

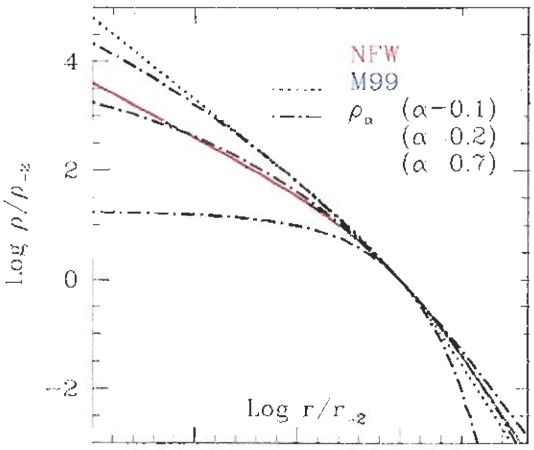
which corresponds to a density profile of the form

$$\ln(\rho_\alpha/\rho_{-2}) = (-2/\alpha)[(r/r_{-2})^\alpha - 1]. \qquad (5)$$

ρ_{-2} : densità a $r = r_{-2}$, r a cui la pendenza $= -2$

This profile has finite total mass (the density cuts off exponentially at large radius) and has a logarithmic slope that decreases inward more gradually than the NFW or M99 profiles. The thick dot-dashed curves in Figs 3 and 4 show that equation (5) (with $\alpha \sim 0.17$) $= \frac{1}{6}$ does indeed reproduce fairly well the radial dependence of $\beta(r)$ and $\beta_{\max}(r)$ in simulated haloes.

È modello di Sersic 3D con $m=6$



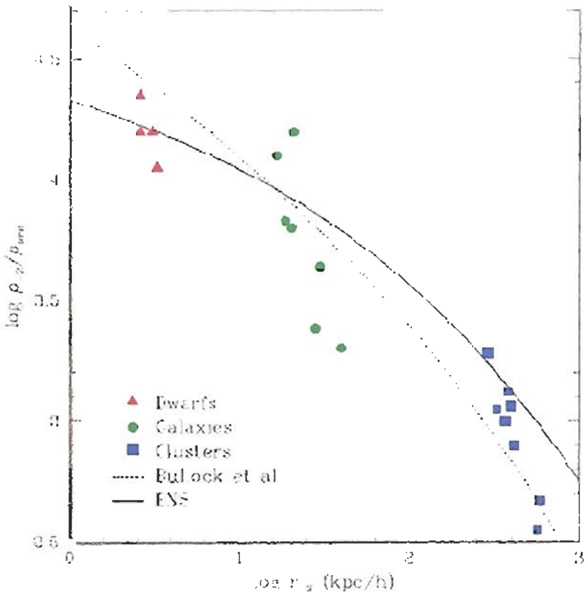


Figure 8. The radius, r_{-2}, where the logarithmic slope of the density profile takes the 'isothermal' value, $\beta(r_{-2}) = 2$, plotted versus the local density at that radius, $\rho_{-2} = \rho(r_{-2})$, for all simulated haloes in our series. This figure illustrates the mass dependence of the central concentration of dark matter haloes: low-mass haloes are systematically denser than their more massive counterparts. Solid and dotted lines indicate the scale radius-characteristic density correlation predicted by the formalisms presented by Eke et al. (2001) and Bullock et al. (2001). These parameters may be used, in conjunction with equation (5), to predict the mass profile of ΛCDM haloes.

$$\frac{d\ln\rho}{d\ln r} = -2\left(\frac{r}{r_{-2}}\right)^\alpha$$

$$\int d\ln\rho = -2\int\left(\frac{r}{r_{-2}}\right)^\alpha \frac{dr/r_{-2}}{r/r_{-2}} = -2\int x^{\alpha-1}dx$$

$$\ln\rho + \text{cost} = -\frac{2}{\alpha}\left(\frac{r}{r_{-2}}\right)^\alpha$$

Se $r = r_{-2}$ $\rho = \rho_{-2}$ $\rightarrow$ $\ln\rho_{-2} + \text{cost} = -\frac{2}{\alpha}$

$$\boxed{\ln(\rho/\rho_{-2}) = \left(-\frac{2}{\alpha}\right)\left[\left(\frac{r}{r_{-2}}\right)^\alpha - 1\right] \quad \text{cvd}}$$

$$\rho_{cr} = \frac{3H^2}{8\pi G} \sim 2\times10^{-29}h^2\ g/cm^3$$

densità critica dello Universo (vedi Cosmologia)