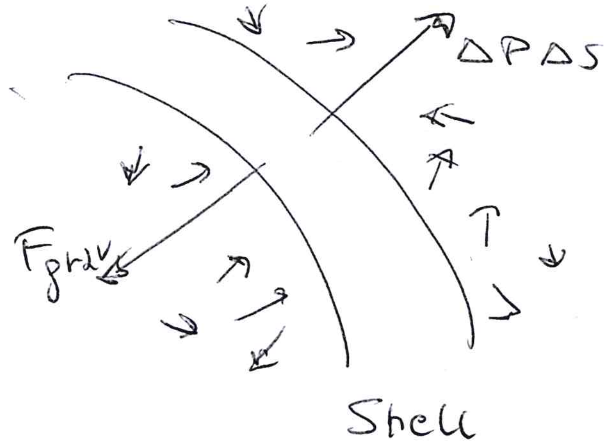
JEANS's EQUATION

FOR SPHERICAL BODY (SEE MAMON COURSE)



LOCAL EQUILIB.

$$\frac{\partial(\nu \bar{v}_r)}{\partial t} + \frac{\partial(\nu \overline{v_r^2})}{\partial r} + \frac{\nu}{r}\left[2\overline{v_r^2} - \left(\overline{v_\theta^2} + \overline{v_\phi^2}\right)\right] = -\nu \frac{\partial \Phi}{\partial r}$$

DEM. IN MAMON PAGES

EQ. OF JEANS (STATIONARY) $\frac{\partial}{\partial t} = 0$ $\quad \frac{\partial}{\partial r} \to \frac{d}{dr}$

$$\frac{d(\nu \overline{v_r^2})}{dr} + \frac{\nu}{r}\left[2\overline{v_r^2} + \left(\overline{v_\theta^2} + \overline{v_\phi^2}\right)\right] = -\nu \frac{d\Phi}{dr}$$

velocity dispersion $\overline{v_i^2} = \sigma_i^2 - \overline{v_i}^2$ $\quad i = r, \theta, \phi$

spherical system with $\bar{v}_\theta = \bar{v}_\phi = 0$ (no rotations)

still $\bar{v}_r \neq 0$ (i.e. radial motions are possible)

$$\frac{d(\nu \sigma_r^2)}{dr} + \frac{d(\nu \overline{v_r}^2)}{dr} + 2\frac{\beta \sigma_r^2}{r} + 2\frac{\nu \overline{v_r}^2}{r} = -\nu \frac{d\Phi}{dr}$$

with, due to symmetry, $\sigma_\theta^2 = \sigma_\phi^2 = \frac{\sigma_t^2}{2}$ tangential

$$\beta = 1 - \frac{\sigma_t^2}{\sigma_r^2}$$

velocity anisotropy parameter

$$\beta = \begin{cases} 1 & \text{with radial anisotropy} \\ 0 & \text{isotropic system} \\ -\infty & \text{tangential anisotropy} \end{cases}$$

{ es. of models $\rightarrow$ $\beta(r) = \dfrac{r^2}{r^2 + r_a^2}$ (Mamon e Lokas) 2005

Osipkov-Merritt 1985 $\quad \beta(r) = \dfrac{1}{2}\dfrac{r}{r+r_a}$ }

No radial motions $\bar{v}_r = 0$

$$\frac{d(\nu\sigma_r^2)}{dr} + 2\beta\frac{\nu\sigma_r^2}{r} = -\nu\frac{d\phi}{dr}$$

(the standard version)

if $\beta = 0$ (isotropic velocities)

$$\frac{d(\nu\sigma_r^2)}{dr} = -\nu\frac{d\phi}{dr}$$

cfr with hydrostatic equilibrium for gas of ICM in clusters

$\nu \rightarrow \nu_{gas}$

$$\frac{\sigma_r^2}{\frac{kT}{\mu m_p}} = 1 = \beta_{spec}$$