EQUILIBRIA OF COLLISIONLESS SYSTEMS

INTRO OF BT2

MEAN FREE PATH OF A STAR BETWEEN COLLISIONS WITH ANOTHER STAR

$\lambda = \frac{1}{n\sigma}$ n = number density , σ = cross section

star ~ sun $R_\odot \sim 7 \cdot 10^8 m = 2.3 \cdot 10^{-8} pc$

$\sigma = \pi (2R_\odot)^2$

10^{11} stars in a disk of radius 10 kpc a thickness 0,5 kpc , $n \sim 0.6\ pc^{-3}$

$\lambda \simeq 2 \cdot 10^{14}\ pc$

interval between collisions $\frac{\lambda}{v}$ v = random velocity $\sim 50\ km\ s^{-1}$

$\hookrightarrow \sim 5 \cdot 10^{18}\ yr$

10^8 times > age of galaxy

$\Longrightarrow$ collisions are rare

CAP 4. BT1

THE RELAXATION TIME

gas ≠ stellar system

$\hookrightarrow$ $V \sim$ cost + collisions

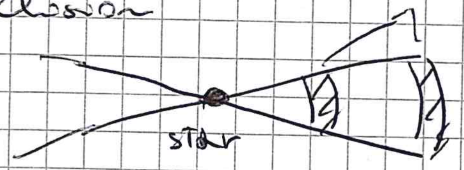
$\rightarrow$ Force acts at distance + NO collision

the same effect!

$\downarrow$

gross structure is important

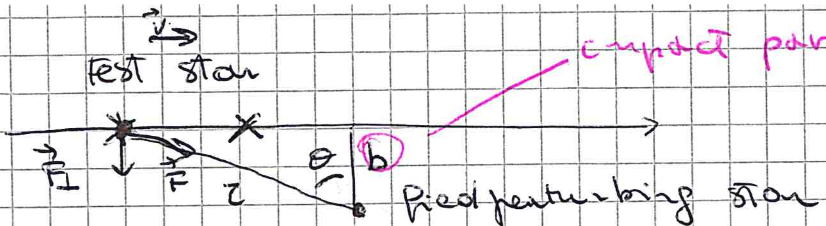
stars accelerate smoothly through the force field that is generated by the galaxy as a whole



$F \propto r^{-2}$

$m \propto r^2$

N stars of mass m and focus on motion of one star , which is $\Delta \bar{v}$ after a crossing of the system?



$$r^2 = b^2 + x^2$$

$$F_\perp = \frac{Gm^2}{b^2+x^2}\cos\theta = \frac{Gm^2}{b^2+x^2}\cdot\frac{b}{\sqrt{b^2+x^2}} = \frac{Gm^2 b}{(b^2+x^2)^{3/2}}$$

$$b^2+x^2 = b^2\left(1+\left(\frac{x}{b}\right)^2\right) \sim b^2\left(1+\left(\frac{vt}{b}\right)^2\right)$$

$x \sim vt$ that is $\frac{\delta v}{v} \ll 1$

impulsive approximation = straight line trajectory

$$F_\perp \sim \frac{Gm^2}{b^2}\left(1+\left(\frac{vt}{b}\right)^2\right)^{-3/2} \qquad (4,1)$$

$$m\bar{a}_\perp = \bar{F}_\perp \qquad m\dot{\bar{v}}_\perp = \bar{F}_\perp \qquad \text{Newton law}$$

$$\left|m\frac{d\bar{v}_\perp}{dt}\right| \sim \frac{Gm^2}{b^2}\left(1+\left(\frac{vt}{b}\right)^2\right)^{-3/2}$$

Notice: $\boxed{\delta v_\parallel = 0}$
before accel.
then decel.

$$\left|\frac{d\bar{v}_\perp}{dt}\right| \sim \frac{Gm}{b^2}\left(\quad\right)^{-3/2}$$

$$\left|\int d\bar{v}_\perp\right| \sim \int_{-\infty}^{+\infty}\frac{Gm}{b^2}\left(\quad\right)^{-3/2}dt =$$

change of variable
$s = \frac{vt}{b} \quad ds = \frac{v}{b}dt$

$$\sim \frac{Gm}{b^2}\cdot\frac{b}{v}\int_{-\infty}^{+\infty}(1+s^2)^{-3/2}ds =$$

$$\left[\int (1+x^2)^{-3/2}dx = \frac{x}{\sqrt{x^2+1}} + \text{const}\right.$$

wolframalpha.com

$$= \frac{Gm}{b^2}\cdot\frac{b}{v}\left|\frac{s}{\sqrt{s^2+1}}\right|_{-\infty}^{+\infty} =$$

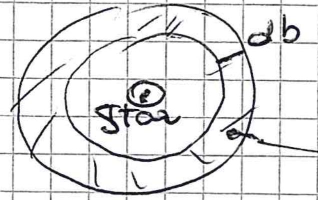
$$= \frac{Gmb}{b^2 v}\left(1-(-1)\right) \sim 2\frac{Gm}{b^2}\cdot\frac{b}{v} \qquad (4,3)$$

accel. when distance $= b$ max accel. !

time of encounter

as expected
v small $\to \delta v$
v large $\to$ no δv = no time to interact

Now: many encounters galaxy + test star
Radius $\equiv R$



δn encounters due to

$$\delta n = \underbrace{\frac{N}{\pi R^2}}_{\text{surface density}} 2\pi\, b\, db$$

$$\delta n = \frac{2N}{R^2}\, b\, db$$

after δn encounters

$$\delta v_\perp^2 \approx \left(\frac{2Gm}{bv}\right)^2 \frac{2N}{R^2}\, b\, db$$

integral over b

$$\Delta v_\perp^2 \approx \int_{b_{min}}^{R} \delta v_\perp^2 \sim 8N\left(\frac{G^2 m^2}{v^2 R^2}\right)\int_{b_{min}}^{R} \frac{\cancel{b}}{b^{\cancel{2}}}\, db$$

$$= 8N\left(\frac{Gm}{Rv}\right)^2 \ln\left(\frac{R}{b_{min}}\right) = \underline{8N\left(\frac{Gm}{Rv}\right)^2 \ln\Lambda}$$

$\Lambda = \frac{R}{b_{min}}$ $\quad \ln\Lambda$ Coulomb logarithm

b_{min}? if $\delta v_\perp \gtrsim v$ ($\rightarrow$ 1 star) NO! impulsive approx. is invalid!

$$\frac{2Gm}{bv} \lesssim v \qquad b \gtrsim b_{min} = \frac{Gm}{v^2}$$

virial theorem $\quad v^2 \sim \frac{GNm}{R} \qquad R \sim \frac{GNm}{v^2}$

typical v of star

% variation

$$\frac{\Delta v_\perp^2}{v^2} \sim \frac{8\cancel{N}\, G^2 m^2}{\frac{G^2 N^2 m^2}{v^2}\, \cancel{v^2}} \ln\Lambda \sim \frac{8\ln\Lambda}{N} \qquad 4.8$$

DEF
n crossing to have $\frac{\Delta v_\perp^2}{v^2} \sim 1$ i.e. strong variation of $\vec{v}$

$\boxed{\equiv n_{relax}}$ $\qquad \boxed{T_{relax}} \equiv n_{relax} \cdot t_{cross}$

$$\frac{n_{relax} \cdot 8 \ln \Lambda}{N} \sim 1$$

$$\underline{n_{relax}} \sim \frac{N}{8 \ln \Lambda} \sim \boxed{\frac{0.1\, N}{\ln N}}$$

$$\boxed{T_{cross} \equiv \frac{R}{V}}$$

$$\Lambda = \frac{R}{b_{min}} = \frac{R}{\frac{Gm}{V^2}} = \frac{\frac{G N m}{V^2}}{\frac{Gm}{V^2}} = N$$

$T_{relax} = n_{relax} \cdot T_{cross}$.

cf. with T_{univ} $\sim 13 \cdot 10^9$ yr $\sim 10^{10}$ yr

if $T_{relax} < T_{univ}$ 2-body relax. is important.

CONCLUSIONS if $T_{relax} > T_{univ}$ NO and system is said COLLISIONLESS!

we can study dynamics studying orbits of stars in a smooth average potential! $\Longrightarrow$ statistical treatment

		n_{relax} $\frac{0.1 N}{\ln N}$	R	V km/s	t_{cross} R/V y	$T_{relax} = n_{rel} \cdot T_{cross}$	
gals	10^{11} *	$\sim 5 \cdot 10^8$	10 Kpc	10^2	10^8	$5 \cdot 10^{16}$	NO coll.
GC globular clusters	10^5 * core $\sim 10^4$	$\sim 10^3$	$\lesssim 10$ pc	10	10^6 y.	10^9	coll.
GC galaxy cluster	10^3 gals	~ 15	1 Mpc	10^3	10^9 y	$1{,}5 \cdot 10^{10}$	NO coll. est. si!
groups	10	0,5	$< 0{,}5$ Mpc	10^2	$2 \cdot 10^9$ y	$2 \cdot 10^9$	coll.

$$1 \text{ Km/s} \sim \frac{1 \text{ pc}}{10^6 \text{ y}}$$