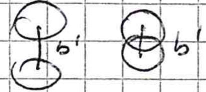
7.2 HIGH SPEED ENCOUNTERS

study the internal of systems, their structure

high speed ⟹ small effect ~ small perturbation of steady state systems

colliding systems (e.g. galaxies) M_1, M_2 r_1, r_2 median radii

at the closest approach V, b'
rel. vel. distance → possono essere "compenetrati"

duration of the encounter $t_{enc} \sim \frac{\max(r_1, r_2, b')}{V}$ 

σ_1, σ_2 v. dispersions

$t_i \equiv \frac{r_i}{\sigma_i}$ characteristic times of stars $i = 1, 2$

impulsive approximation $t_i \gg t_{enc}$

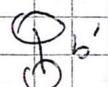
$V \gg \sigma_i \frac{\max(r_1, r_2, b')}{r_i}$ [7.34]

7.34 → small effect | Effect of impulse approx.
we can assume that stars do not change their position with respect to the center of the system.

↓

(numerical simulation is ok even when 7.34 is not strictly satisfied)

→ density distribution is the same → two rigid bodies

IF 7.34, and assume $r_i \lesssim b'$ 
⟺ the centers travel at nearly uniform velocity

reduced particle $\mu \to \frac{M_1 M_2}{M_1 + M_2}$ travel in the Keplerian potential of $M_1 + M_2$

The reduced particle travels at constant velocity so long as its pot. energy $-\frac{G(M_1+M_2)}{r}$ is smaller than kinetic energy $\frac{1}{2}V_\infty^2$

$$\frac{G(M_1+M_2)}{r} \ll \frac{1}{2}V^2$$

virial Th.

$$\frac{G(M_1+M_2)}{r} \lesssim \frac{G(M_1+M_2)}{b'} \simeq \frac{r_1\sigma_1^2 + r_2\sigma_2^2}{b'} \ll \frac{V^2(r_1^3+r_2^3)}{b'^3}$$

$r_i \lesssim b'$ 7.34 7.35e

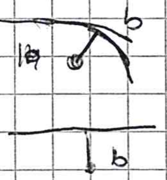
$$\sigma_i^2 \ll \frac{V^2 r_i^2}{b'^2}$$

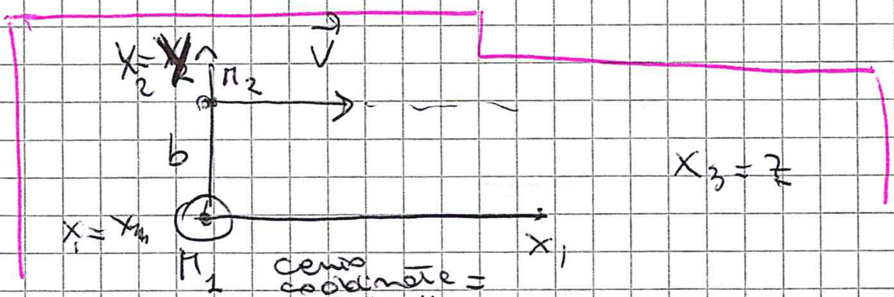
So

$$\frac{G(M_1+M_2)}{r} \ll V^2\,\frac{r_1^3+r_2^3}{b'^3} < 2V^2 \qquad 7.35b$$

$r_i < b_i$

So uniform V $b' = b$ impact parameter of Fig 7.2

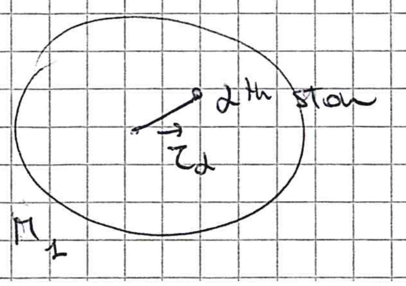




M_2 motion $X(t) \sim (0, b, Vt)$ 7.36 — circa poco a mente

M_1 = perturbed system

How change its structure due to M_2? perturbing system



$\Delta\vec{V}_\alpha$? → idea of how much is modified M_1

$$\Delta\bar{V}'_\alpha = \Delta\bar{V}_\alpha + \Delta\bar{V}$$

$\Delta\bar{V}_\alpha$: with respect to mass centre; $\Delta\bar{V}$ → of the mass centre

$$\Delta \bar{V} = \frac{\sum_\beta m_\beta \Delta \bar{V}'_\beta}{\sum_\beta m_\beta} \qquad 7.37e$$

↙ of mass centre.

↳ is the average on all stars

$$\Delta \bar{V}_\alpha \equiv \Delta \bar{V}'_\alpha - \Delta \bar{V} \qquad 7.37b$$

$\phi(\bar{r}, t)$ is the potential generated by the perturber at each point $\bar{r}$ of the perturbed

rate of change of $\bar{V}'_\alpha$

$$\dot{\bar{V}}'_\alpha = - \bar{\nabla} \phi(\bar{r}_\alpha, t) \qquad 7.38$$

impulse approximation → $\bar{r}_\alpha$ is constant

$$\Delta \bar{V}'_\alpha = - \int_{-\infty}^{+\infty} \bar{\nabla} \phi(\bar{r}_\alpha, t) \qquad 7.39$$

$$\Delta \bar{V}_\alpha = - \int_{-\infty}^{+\infty} \left[\bar{\nabla} \phi(\bar{r}_\alpha, t) - \frac{1}{M_1} \sum_\beta m_\beta \bar{\nabla} \phi(\bar{r}_\beta, t) \right] dt \qquad \boxed{7.40}$$

Velocity impulse

→ onde le altre stelle di M_1 eq. 7.37e

impulse approx → the potential energy of system does not change during the encounter but change the internal kinetic energy

$$\Delta E = E_f - E_i = \frac{1}{2} \sum_\alpha m_\alpha \left[(\bar{V}_\alpha + \Delta \bar{V}_\alpha)^2 - \bar{V}_\alpha \right] =$$

$$= \frac{1}{2} \sum m_\alpha \left[|\Delta \bar{V}_\alpha|^2 + 2 \bar{V}_\alpha \Delta \bar{V}_\alpha \right]$$

In any static axis symmetric system $\sum m_\alpha \bar{V}_\alpha \Delta \bar{V}_\alpha = 0$

↖ if there is a particle with $\Delta \bar{V}_\alpha$ +, there is another with $-\bar{V}_\alpha$

$$\Delta E = \frac{1}{2} \sum m_\alpha |\Delta \bar{V}_\alpha|^2 \qquad \boxed{7.42}$$

↳ is positive!

How the system reacts to the injection of energy?

(a) relaxation to return to equilibrium

M_1 is no longer in virial equilibrium $2T + U = 0$

↳ changes → virial eq.

is bigger than the change due to encounter

at the beginning $E_0 = T_0 + U_0 \overset{\text{virial}}{=} T_0 - 2T_0$

$T_0 = -E_0$ (internal energies)

encounter $T_0 \to T_0 + \delta T$ ($4\delta T$)

$E_1 = E_0 + \delta T$ end of encounter

($-2\delta T$) $T_1 = -E_1 = -(E_0 + \delta T) = T_0 - \delta T$

ends of violent relaxation

$\to T_0 - \delta T$

(b) Mass loss

stars with high $\bar{v}_\lambda$ escapes from the system so the total energy of the system is lower

(es. cosmo')

Sims, obs. in clusters, plumes, groups destroyed in fields tidal,

collisional tidal stripping

Richstone x clusters

$$\frac{1}{m_g}\frac{dm_g}{dt} = -\frac{1}{10^{10}\,yr}\left(\frac{n}{10^{-3} M_\odot pc^{-3}}\right)\left(\frac{\sigma_{cl}}{10^3\,km/s}\right)\left(\frac{r_g}{10\,kpc}\right)^2\left(\frac{\sigma_{cl}}{4\sigma_g}\right)^{-\gamma}$$

$2 < \gamma < 3$

check with sim.

the rate of encounter $(n\,\sigma_{cl}\,\pi\,r_p^2)$

if $\sigma_{cl} \gg \sigma_g$ no important

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c) Adiabatic invariance

7.34 valid for most stars

at the center, orbital time is very short
7.34 no valid
~~orbits deform~~
orbits deform but the change will be reversed as the perturber departs

d) Tidal approximation

if $r_1 \ll b'$ approx for velocity impulse
$r_2 \ll b'$ $\Delta \bar{v}_\alpha$ [7.40]

$\Delta \bar{v}_\alpha =$

$$\Delta \bar{v}(x) = -\int_{-\infty}^{\infty} \left[\bar{\nabla}\phi(\bar{x},t) - \frac{1}{M_1}\int \rho(x')\,\bar{\nabla}\phi(\bar{x}',t)\,d^3\bar{x} \right. \quad 7.46$$

invece di sommatoria

centro delle coordinate in centro M_1

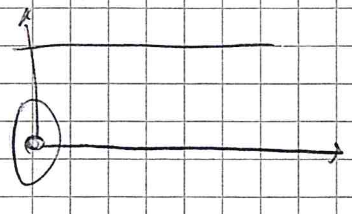
Taylor series

k component

$$\nabla_k \phi(\bar{x},t) = \phi_k^{(1)} + \sum_j \phi_{kj}^{(2)} x_j + O(|\bar{x}|^2) \quad 7.47_e$$

$$\phi_k^{(1)} = \left.\frac{\partial \phi}{\partial x_k}\right|_{x=0} \qquad \phi_{kj}^{(2)} \equiv \left.\frac{\partial^2 \phi}{\partial x_k \partial x_j}\right|_{x=0}$$

$$\Delta \bar{v}(\bar{x}) = \frac{2GM_2}{b^2 V}(-x, y, 0) + O(\bar{r}^2) \quad [7.54]$$



7.42 $\Delta E = ..$
+ 7.54 ..

ΔE in the tidal approx.
(Spitzer 58)

$$\Delta E = \frac{1}{2}\sum m_\alpha |\Delta v_\alpha|^2 \quad 7.42$$

$$\Delta E = \frac{1}{2}\frac{4G^2M_2^2}{b^4 V^2}\int \rho(\bar{x})(x^2+y^2)\,d^3\bar{x} = \frac{4G^2M_2^2M_1}{3b^4V^2}\,\bar{r}^2 \quad [7.55]$$

$\bar{r}^2$ medio del sistema 1

medio

$$\bar{r}^2 = \frac{\int \rho(\bar{x})(x^2+y^2+z^2)\,d^3x}{\int \rho(\bar{x})\,d^3\bar{x}}$$

$$\int \rho(\bar{x})(x^2+y^2+z^2)\,d^3x = \bar{r}^2 \int \rho(\bar{x})\,d^3x = \bar{r}^2 M_1$$

~~$= \frac{4\pi \int \rho(x) d^3x_1}{4\pi}$~~

$\int \rho(\bar{x})(x^2+y^2) = \overline{r^2} \cdot \frac{2}{3} M_1$

$$\Delta E = \frac{4 G^2 M_2^2 M_1}{3 b^4 V^2} \overline{r^2} \qquad 7.55$$

$\overline{r^2}$ → del sistema (1)

$\Delta E \propto M_2^2$!

$\Delta E \propto V^{-2}$

$\Delta E \propto b^{-4}$

$\Delta E \propto \overline{r^2}$

ok with simulations

effect is larger if $M_2 \uparrow$ $V \downarrow$ $b \downarrow$

$\overline{r^2} \uparrow$ system 1 is less compact!

(e)

Penetrating encounters

we need to know the potential of the perturber ↓ more computation

7.55 close encounters are important!

we need approx. for the case $\frac{b}{r_i}$ small

e.g. more term in the Taylor approx.

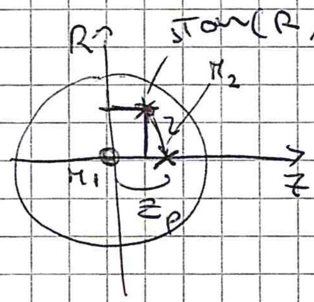
here: To compute ΔE at $b = 0$ — head on

i) we ignore Δv of stars that lie along the M_2 trajectory

$\Delta v_z \rightarrow$

$-\Delta v_z \leftarrow$ opposite impulse

2) by symmetry $\Delta \bar{v}_z$ will point toward the line of motion of the perturber



cylindrical coordinates (R, z) → perturber's trajectory

$(R = 0, z = z_p(t) = Vt)$ → z del perturber

$(R = 0, z = 0)$ center of perturbed

$r(R, z, t)$ distance from center of M_1

M_2 spherical with $\phi(r)$

R-component of the perturber grav. field

$$-\frac{d\phi}{dr}\frac{R}{r}$$

be a star at (R, z)

$$\Delta v_R = -\int_{-\infty}^{+\infty} \frac{d\phi}{dr}\frac{R}{r}\, dt =$$

$dz_p = V\,dt$
$dt = \frac{dz_p}{V}$

$$= -\frac{2R}{V}\int_0^{\infty} \frac{d\phi}{dr}\frac{dz_p}{r}$$

$r = \sqrt{(z_p - z)^2 + R^2} = r(z_p)$

→ symmetric

Δv_R depends only on the distance between star and the perturber

$$\Delta E = \int_0^{\infty} \frac{1}{2}\left[\Delta v_R(R)\right]^2 2\pi R\, dR\, \Sigma(R) =$$

$\int$ for all stars

↳ surface density of the perturber

$$= \pi \int_0^{\infty} \left[\Delta v_R(R)\right]^2 \Sigma(R) R\, dR \qquad 7.57$$

we need a function for $\phi(r)$ of the perturber → ranges 7 bis, 7 ter

$$\Delta v_R = -\frac{R G M_2}{V}\frac{2}{(R^2 + a^2)} \qquad 7.58$$

$$\Delta E = \frac{G^2 M_2^2 M_1}{3 V^2 a^2} \qquad 7.59$$

read on formula

$a \uparrow$ $\rho \downarrow$ $\Delta E \downarrow$

a parità di M_2 e V

sistema + concentrato

7

Cfr. 7.55 and 7.58

$\frac{b^4}{r^2} \to 4a^4$

one can interpolate between 7.55 and 7.58

APPLICATIONS

(a) Disruption of open clusters

open clusters $10^2 M_\odot \lesssim M_{clust} \lesssim 10^3 M_\odot$ $r_{half} \sim 1\,pc$ $\sigma_* \sim 1\,km/s$

& giant molecular clouds $M_{cl} \gtrsim 10^5 M_\odot$ $r_{cloud} \sim 10\,pc$

random velocity within the disk $\sim 7\,km/s$ > $\sigma_* \sim 1\,km/s$ → use of impulse approx.

7.55 $\Delta E \propto M_2^2$

clouds ~ # open clusters
→ but more massive

$t_d = \left(\frac{dE_{pw}}{dt}\frac{1}{E_{bind}}\right)^{-1} \sim 5.7\cdot 10^8 \left(\frac{M_{cl}}{250 M_\odot}\right)\left(\frac{1\,pc}{r_h}\right)^3\left(\frac{r_h}{\overline{r^2}}\right) yr$

→ to dissolve open cluster; r_h → median radius of open cluster

we do not expect to see open clusters older than 10^9 yr
→ YES from obs.

(b) Disruption of wide binaries f(semi-major axes a)

(c) Disk shocking of globular clusters $\sigma_* \sim 5\,km/s$ $r_h \sim 10\,pc$ $r_t \sim 50\,pc$

GCs pass through the disk $V_\perp \sim 100\,km/s$

$t_{shock} \sim 6\cdot 10^9 yr$ 10 times T_d

old GCs can exist

microlensing constraint: $m \lesssim 10^{-7} M_\odot$ or $m \gtrsim 30 M_\odot$

from NO heat the disk $m \lesssim 5\cdot 10^6 M_\odot$

BT2 (e) ~~Machos~~ MACHOs: BHs or ~~non luminous stars~~
→ on binary stars

If a population of binaries with $a \gtrsim 2\cdot 10^4$ AU exist

8

$$= \frac{3M_1}{2\pi a^2}\int_{\frac{R}{a}}^{\infty}\frac{(1+x^2)^{-5/2}\cdot x\,dx}{\sqrt{x^2-\frac{R^2}{a^2}}} = \frac{3M_1}{2\pi a^2}\,\frac{2}{3\left[\left(\frac{R}{a}\right)^2+1\right]^2} =$$

$$= \frac{M_1 a^2}{\pi(R^2+a^2)}$$

wolfram

ΔE 7.57 $\Delta E = \pi\int_0^{\infty}[\Delta V_R(R)]^2\,\Sigma(R)\,R\,dR$

$$\Delta E = \pi\,\frac{4G^2M_2^2}{V^2}\int_0^{\infty}\frac{R^2}{(R^2+a^2)^2}\,\frac{M_1a^2}{(R^2+a^2)^2\pi}\,R\,dR =$$

$$= \frac{4G^2M_1M_2^2a^2}{V^2}\int_0^{\infty}\frac{R^3\,dR}{(R^2+a^2)^4} = \frac{G^2M_2^2M_1}{3V^2a^2} \qquad (7.58)$$

wolfram $\frac{1}{2a^4}$

NOTA $a\uparrow$ $\rho\uparrow$ $\Delta E\downarrow$

perturbato meno "perturbato"

a parità di M_2, V

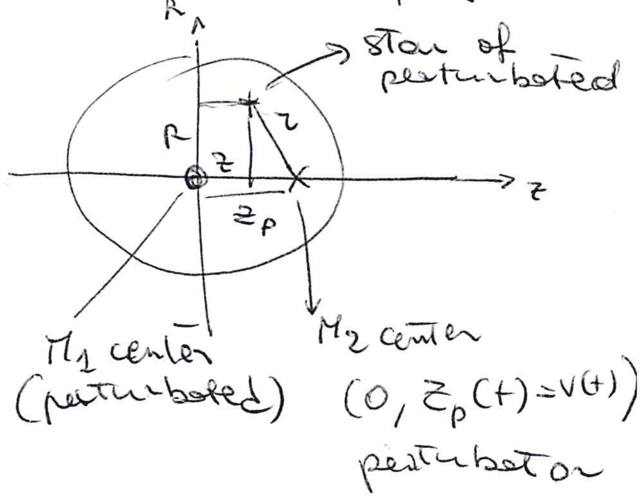
cfr. (7.58) $b=0$ with (7.54) (tidal approximation) $b>r$

interpolation

~~Bins~~

7this

FROM. EQ. 7.56 → 7.58 → 7.59



$$\Delta v_R = -\frac{R}{V}\int_{-\infty}^{+\infty} \frac{d\phi}{dr}\frac{dz_p}{r}$$

$$\phi_{Plummer} = -\frac{GM_2}{\sqrt{r^2+a^2}} = \phi$$

→ per perturbatore

$$\frac{d\phi}{dr} = -GM_2\left(-\frac{1}{2}\right)(r^2+a^2)^{-3/2}\,2r = GM_2\frac{r}{(r^2+a^2)^{3/2}}$$

$$\Delta v_R = -\frac{R}{V}GM_2\int_{-\infty}^{\infty}\frac{r}{(r^2+a^2)^{3/2}}\frac{dz_p}{r} \qquad r=\sqrt{(z_p-z)^2+R^2}$$

$$\Delta v_R = -\frac{RGM_2}{V}\int_{-\infty}^{\infty}\frac{dX}{(X^2+R^2+a^2)^{3/2}} \qquad X = z_p - z$$

$$\Delta v_R = -\frac{RGM_2}{V}\frac{2}{(R^2+a^2)} \qquad \text{eq. 7.58}$$

$$\rho(r)_{Plummer} = \frac{3M_1}{4\pi a^3}\left(1+\frac{r^2}{a^2}\right)^{-5/2} \quad \text{density}$$

X Perturbato NOTA $a\uparrow\ \rho\uparrow$

$$\Sigma(R) = 2\int_R^{\infty}\frac{\rho(r)\,r\,dr}{\sqrt{r^2-R^2}} = \frac{6M_1}{4\pi a^3}\int_R^{\infty}\frac{\left(1+\frac{r^2}{a^2}\right)^{-5/2} r\,dr}{\sqrt{r^2-R^2}} =$$

↓ projected density

Abel integral

$$= \frac{3}{2\pi}\frac{M\,a^3}{a^3\cdot a}\int_R^{\infty}\frac{\left(1+\frac{r^2}{a^2}\right)^{-5/2}\frac{r}{a}\,d\left(\frac{r}{a}\right)}{\sqrt{\frac{r^2}{a^2}-\frac{R^2}{a^2}}} =$$

$$x = \frac{r}{a}$$

7 bis

BT 2

8

(8) High speeds interactions in clusters of galaxies

$\sigma_{gal} \sim 2\cdot10^3$ Km/s $\rightarrow$ $\sigma_{1D} \sim 800$ Km/s
(3D)

$\sigma_* \sim 2-400$ Km/s

→ impulsive approximation

cluster ~ singular isothermal sphere $\rho(r) = \frac{\sigma^2}{2\pi G r^2}$

gas ~ DM distributions $\Rightarrow M^* = \frac{\rho}{n} \sim$ const.
n → numerical density of gal.

$L^*_R = 3\cdot10^{10} L_{\odot,R}$

$\Upsilon_R = \left(\frac{M_\odot}{L_R}\right)_{cluster} \sim 200 \left(\frac{M_\odot}{L_{\odot,R}}\right)$

$M^* = \Upsilon_R L^*_R \sim 6\cdot10^{12} M_\odot$ $\qquad n(r) = \frac{\rho(r)}{M^*} = \frac{\sigma^2}{2\pi G M^* r^2}$

$t_{coll} \sim (n\, \sigma_{gal} r_h^2)^{-1}$

$t_{coll} \sim 6$ Gyr $\left(\frac{800 \text{ Km/s}}{\sigma}\right)^3 \left(\frac{20 \text{ Kpc}}{r_h}\right)^2 \left(\frac{r}{0{,}5 \text{ Mpc}}\right)^2 \frac{M^*}{5\cdot10^{12} M_\odot}$

r → internal cluster region

collisional stripping of Richstone

so ~ 1 encounter
spirals → lenticulars?
(better ram pressure to loose gas and interaction to change morphology)

size of dark
but dark halos several 10^2 Kpc

most of dark-halo mass has therefore been stripped

Groups $\sigma \sim 300$ Km/s
encounters have stronger effect, more galaxy mergers
more dynamical friction → PRE-PROCESSING of galaxies
es. cD also in poor clusters