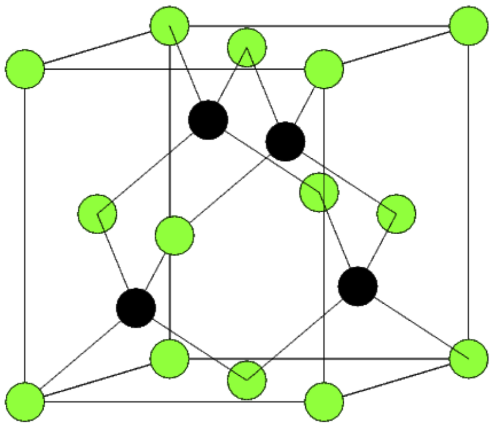
Condensed Matter Physics I
final written test
academic year 2016/2017
September 20, 2017

(*Time: 3 hours*)

Exercise 1: *Crystalline structures*

1. Which kind of crystalline structure is described in the figure below?
2. Describe its primitive unit cell: translation vectors, basis and corresponding vectors.
3. Describe the reciprocal cell and write the general expression of a reciprocal lattice vector.
4. Calculate its structure factor $S(\mathbf{k})$ and in particular its expression on the reciprocal lattice.
5. Calculate which discrete values the structure factor can assume on the reciprocal lattice vectors, and specify in which cases (for which indexes) it assumes such values.
6. Discuss the result when the atomic form factor of the two atomic species are (*i*) equal or (*ii*) opposite.



Exercise 2: *Density of electronic states in different dimensions*

1. Consider a 1D system with lattice parameter a whose energy band is described by:

$$E(k) = -\gamma cos(ak).$$

   Write explicitly the expression for the density of states $g(E)$ and make a plot.

2. Consider now a 3D system with simple cubic structure with lattice parameter a, whose energy band is described by:

$$E(\mathbf{k}) = -\gamma cos(ak_x).$$

   Also in this case write explicitly the expression for the density of states $g(E)$ and make a plot. Is there any difference in the expression of $g(E)$ with respect to the 1D case? If any, specify it.

3. Consider now a 3D system with tetragonal Bravais lattice, with primitive cell vectors along $\hat{x}$, $\hat{y}$, $\hat{z}$ of length a, b, b respectively, with $a << b$. The energy band is described by:

$$E(\mathbf{k}) = -\left[\gamma cos(ak_x) + \gamma' cos(bk_y) + \gamma' cos(bk_z)\right].$$

   Do you expect $\gamma << \gamma'$ or viceversa? Justify your answer.

4. When the band is half-filled, which is the value of the Fermi energy?

5. Make a plot of the section of the first Brillouin zone and of the Fermi surface in the $(k_z = \pi/2b)$ plane.

6. In the same plane, make a schematic plot of other constant energy surfaces.

NOTE: *Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.*