Condensed Matter Physics I
I test - 22 November 2011
(2 hours 30')

- Solve all the exercises, corresponding to a total maximum score of 36. If the score is between 33 and 36 it is considered equal to 30/30 *cum laude*, if it is between 30 and 32 it is considered equal to 30/30.
- Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.

Exercise 1: *Crystalline structures*

Consider the following lattices:

1. Side-centered cubic: a simple cubic lattice with additional points in the centers of the vertical faces.
2. Base-centered cubic: a simple cubic lattice with additional points in the centers of the horizontal faces of the cubic cell.
3. Edge-centered cubic: a simple cubic lattice with points added at the center of each edge joining nearest neighbors.

For each of the above mentioned lattices: (*i*) draw the structure; (*ii*) determine whether it is simply a Bravais lattice or a Bravais lattice with a basis and, in the latter case, how many atoms there are in the basis; (*iii*) write a possible choice of the primitive vectors and, when needed, the basis.

Exercise 2: *Diffraction: Atomic form factors and Structure factors*

1. Calculate the geometrical structure factor $S(\mathbf{K})$ for KCl which has a rocksalt structure (with $\mathbf{K}$ we indicate a reciprocal lattice vector), showing that for a proper choice of the origin, it is real.
2. Since K^+ and Cl^- are isoelectronic, they have a very similar atomic form factor. As a first approximation, consider them exactly equal. Write $S(\mathbf{K})$ under this approximation. Specify which are its possible values and to which ($\mathbf{K}$ vectors correspond.
3. Determine the Bravais lattice structure that has the same geometrical structure factor and comment on the result. (*The answer will be considered complete if also a formal rationale is given.*)

Exercise 3: *Free electrons model*

Consider two metals, M_1 and M_2, whose electron densities can be treated with free-electron gas models. Suppose that the metal M_2 has an electron density which is twice the one of as metal M_1. Justifying your answer, determine:

1. What is the electric conductivity of metal M_2 relative to M_1, assuming the relaxation time τ is the same for both?
2. What is the electronic specific heat c_v (i.e., heat capacity per unit volume) of M_2 relative to M_1 in the Drude (a) and in the Sommerfeld (b) model?
3. What is the Hall coefficient of M_2 relative to M_1, i.e., the transverse electric induced by a given magnetic field and a longitudinal current?

(Note: if not justified, the answer will NOT be considered valid, even if it is correct).

Exercise 4: *Friedel oscillations*

We studied the ground state of 1D electron gas with periodic boundary conditions and with hard walls. We found that in the latter case each electron wave-function is of the type $\psi_k(x) \propto \sin(kx)$ and as a result of the sum over all the electrons, the electron density $n_{1D}(x)$ shows oscillations on length scales of order $1/k_F$. Consider here the ground state of a 3D electron gas with Fermi momentum k_F confined between two planar walls perpendicular to the x direction, say, and with periodic boundary conditions in y and z. Consider the thermodynamic limit ($L_x, L_y, L_z \to \infty$ but n constant). *(Note: you are encouraged to use the thermodynamic limit since the beginning, and not only at the end as we did in class.)*

1. Write the solution $\psi_{\mathbf{k}}(\mathbf{r})$ *(you should show that it is* $\propto \sin(k_x x)e^{ik_y y + ik_z z}$*; give the complete expression, specifying* k_x*,* k_y*,* k_z*, the normalization contant..)*
2. Write the expression of the electron density $n_{3D}(x)$ at distance x from a planar wall.
3. Graph this function (in a universal way ?) and compare with $n_{1D}(x)$.