Course of Geothermics

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Course Outline:

1. Thermal conditions of the early Earth and present-day Earth's structure
2. Thermal parameters of the rocks
3. Thermal structure of the lithospheric continental areas (steady state)
4. Thermal structure of the lithospheric oceanic areas
5. **Thermal structure of the lithosphere for transient conditions in various tectonic settings**
6. Heat balance of the Earth
7. Thermal structure of the sedimentary basins
8. Thermal maturity of sediments
9. Mantle convection and hot spots
10. Magmatic processes and volcanoes
11. Heat transfer in hydrogeological settings
12. Geothermal Systems

Effects of Surface Temperature Changes

The average surface temperature, T_0, can be estimated from:

$$T_0 = 3 + (T_{av.min} + T_{av.max})/2$$

$T_{av.min}$ = average annual minimum temperature
$T_{av.max}$ = average annual maximum temperature

- **Natural daily and seasonal temperature fluctuation propagate into the crust, but the effect decays exponentially with depth**

The effect of periodic surface heating is defined by: $T_\theta = T_0 \times \exp(-\varepsilon z) \sin(\omega t - \varepsilon z)$

The equation describes the departure (T_θ) from a mean value of T at a specific depth, z, and time t, resulting from a surface heating cycle with amplitude T_0 and frequency ω ($\omega = 2\pi/P$, P=period).

$\varepsilon = (\pi/P\kappa)^{1/2}$ ε=Thermal property of the medium T_0=amplitude of the surface temperature cycle

$\sin(\omega t - \varepsilon z)$ =time lag between the temperature perturbation at the surface and at depth

$\exp(-\varepsilon z)$ = decay in the amplitude of the temperature perturbation with depth

$z = 2\pi/\varepsilon = 2\pi/(\pi/P\kappa)^{1/2} = (4\pi P\kappa)^{1/2}$ The depth at which the T fluctuation is in phase with the surface cycle (**$\varepsilon z = 2\pi$**), defines z_{wl}

z_{wl}=Effective wavelength for a temperature cycle near the surface of the Earth

The magnitude of the temperature perturbation at a depth of one effective wavelength is given by:

$$exp(-\varepsilon z_{wl}) = \exp(-2\pi) = 0.0019$$

Effects of Surface Temperature Changes

$$(\partial T/\partial z)_a = (\partial T/\partial z) + (\partial T_\theta/\partial z)$$

$(\partial T/\partial z)_a$ = **apparent thermal gradient**

$$(\partial T_\theta/\partial z) = T_0 \times (-\varepsilon)\exp(-\varepsilon z) \times [\sin(\omega t - \varepsilon z) + cos(\omega t - \varepsilon z)]$$

$(\partial T_\theta/\partial z)$ = **magnitude of perturbation**

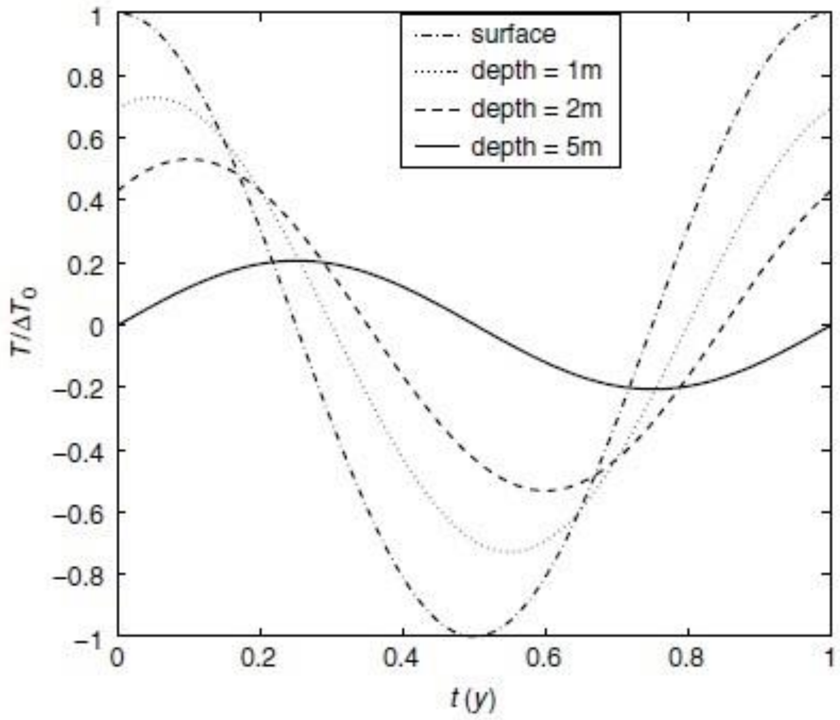
Maximum disturbance is attained when: $\sin(\omega t - \varepsilon z) = \cos(\omega t - \varepsilon z)$ for $t_{\max} = (\pi n + \pi/4 + \varepsilon z)/\omega$

$$|(\partial T_\theta/\partial z)|_{\max} = T_0 \times (-\varepsilon) \times \exp(-\varepsilon z) \times [\sin(\pi n + \pi/4) + \cos(\pi n + \pi/4)] = T_0(\varepsilon)\exp(-\varepsilon z) \times [\sqrt{2}]$$

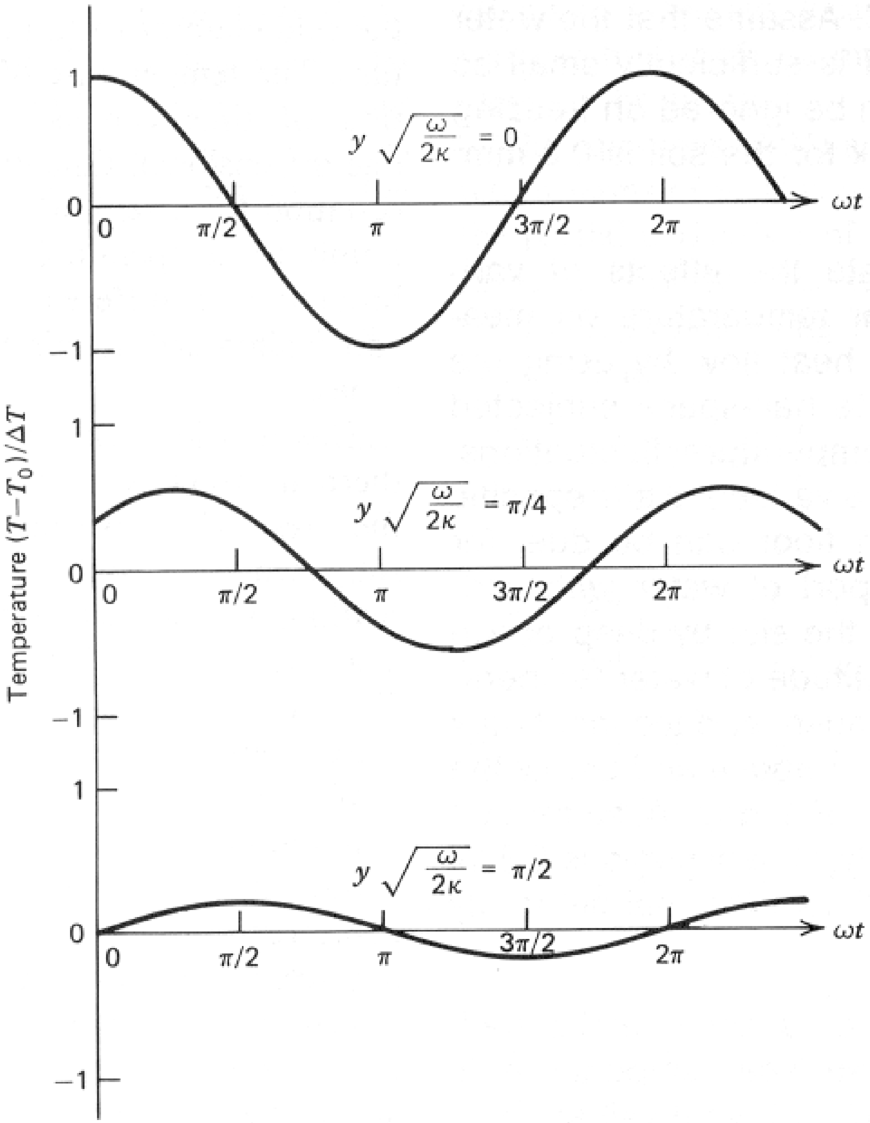
The threshold depth, z_{min}, at which the maximum departure from mean gradient is no longer significant:

$$|(\partial T_\theta/\partial z)|_{\max} = 0.01 \times |(\partial T/\partial z)| = T_0(\varepsilon)\exp(-\varepsilon z_{\min}) \times [\sqrt{2}]$$

$$z_{\min} = -\frac{1}{\varepsilon}\ln\left|\frac{0.01}{T_0\varepsilon\sqrt{2}}\frac{\partial T}{\partial z}\right|$$



Effects of Surface Temperature Changes



- **Surface temperature variations produces a phase variation and a decrease of the amplitude with increasing depth**

Effects of Surface Temperature Changes

- The effect in depth of surface temperature changes depends (1) on the magnitude of the temperature step, (2) the time since the event, and (3) the thermal diffusivity of the ground.

Climatic changes can be modelled as discrete events, each with an associated step function in surface temperature:

$$T_\theta = T_0 \times \mathrm{erfc}[z/(2\sqrt{\kappa t})]$$

T_θ = departure from original equilibrium temperature at depth z and time t after an instantaneous change in surface temperature of T_0

The effect of more than one temperature step (climatic event) is found by: $T_\theta = \Sigma T_{\theta i}$

The change in thermal gradient β due to a change in surface temperature, T_0, is: $\Delta\beta = -T_0 \times [(\pi\kappa t)^{-1/2} \times \exp(-z^2/(4\kappa t))]$

The effect of more than one event can be found: $\Delta\beta = \Sigma\Delta\beta_i$

We can define a skin depth L at which the amplitude of the T variations is $1/\varepsilon$ of that at the surface of the Earth:

$L=1/\varepsilon$ $$L = \sqrt{\frac{2\kappa}{\omega}}$$ $\kappa=10^{-6}\mathrm{m}^2\mathrm{s}^{-1}$

skin depth (L) for the daily T variation ($\omega=7.27\times10^{-5}\mathrm{s}^{-1}$) is less than 20 cm
skin depth (L) for the yearly T variation ($\omega= 2\times10^{-7}\mathrm{s}^{-1}$) is 3.3 m
skin depth (L) for an ice age (10^5 yr) T variation ($\omega=1.99\times10^{-12}\mathrm{s}^{-1}$) is >1km

Steady *vs* Transient Geotherms

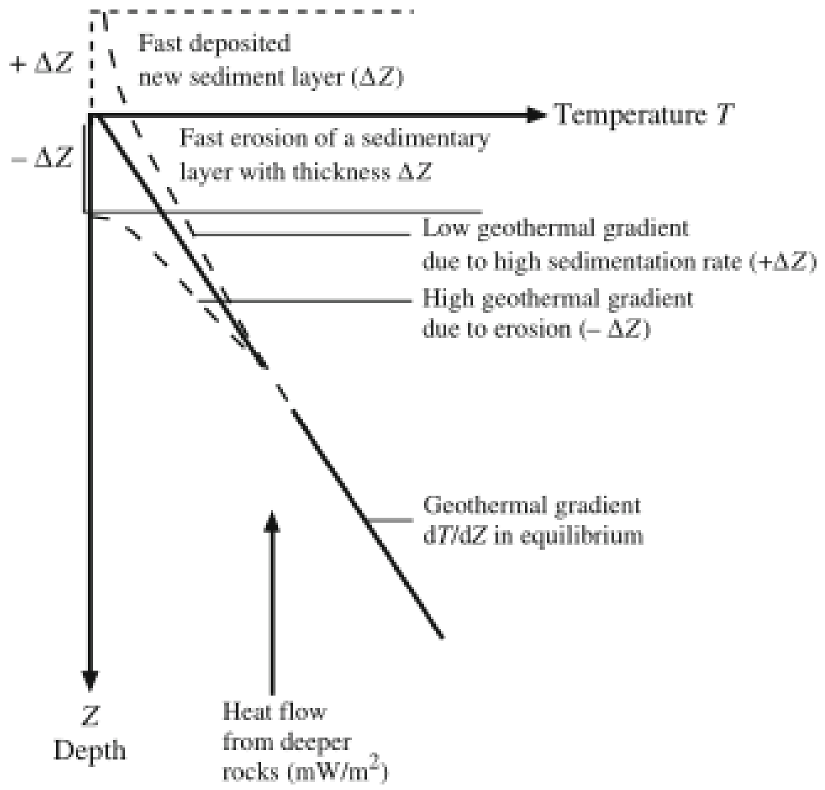
Heat flow > 90 mWm^2 imply melting in the crust or a weak lithospheric mantle
(other heat transport mechanisms are effective in tectonic active areas)

Crustal thickness variations imply changes of crustal heat production and deformation
(change of temperature distribution)

- Erosion or crustal extension initially cause steeper geotherms and enhanced heat flux and later the reduced crustal thickness and possible injection of basaltic melts (depleted in radioelements) leads to a lower heat flux than initial.
- Crustal thickening causes the geothermal gradient and the heat flux to decrease at first and then to increase due to higher crustal heat production (e.g., Tibet and Alps).
- Heat flux may record shallow processes such as the cooling of recently emplaced plutons. The anomalously high heat flux in the Basin and Range Province (about 110 mWm^2) and the high elevation (about 1750 m) is consistent with an extension of 100% and presence of shallow magma intrusions.

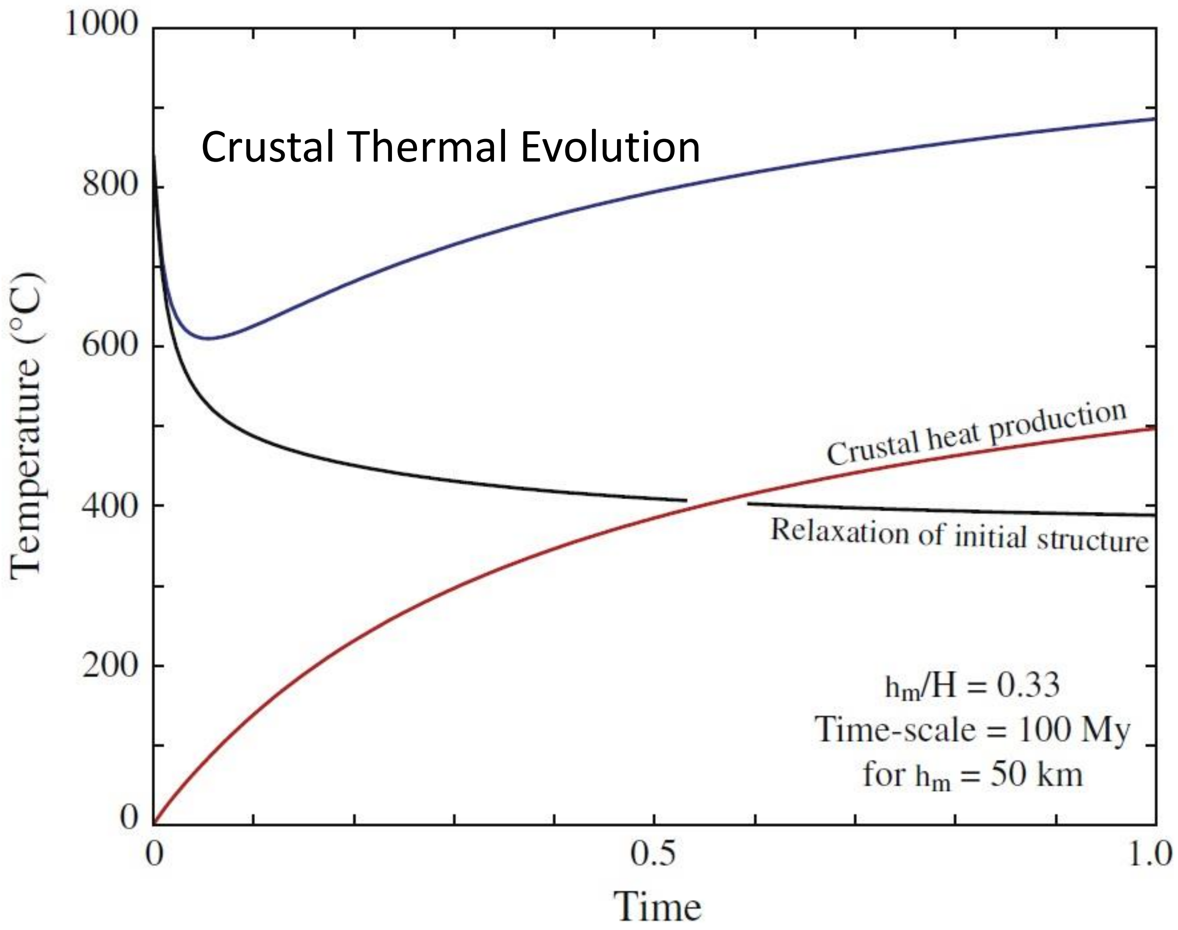
Steady *vs* Transient Geotherms

- Erosion or crustal extension cause initially steeper geotherms and enhance heat flux and when these transient effects decay the reduced crustal thickness leads to a lower heat flux.
- Sedimentation or crustal thickening causes the geothermal gradient and heat flux to decrease at first and then to increase due to higher crustal heat production.
- Other transient conditions are produced by crustal melting in the upper crust modify the vertical distribution of radioelements.



Crustal temperature return to equilibrium with local heat sources in < 100 Myr, while thick lithosphere last ~ 500 Myr

Post-Orogenic Thermal Evolution



- **Postorogenic thermal evolution is sum of two components:**

$$T(z,t) = T_i(z,t) + T_r(z,t)$$

T_i accounts for diffusive relaxation of the initial thermal structure $T_0(z)$, such that the initial condition is $T_i(z,0)=T_0(z)$.
T_r accounts for crustal heat production.

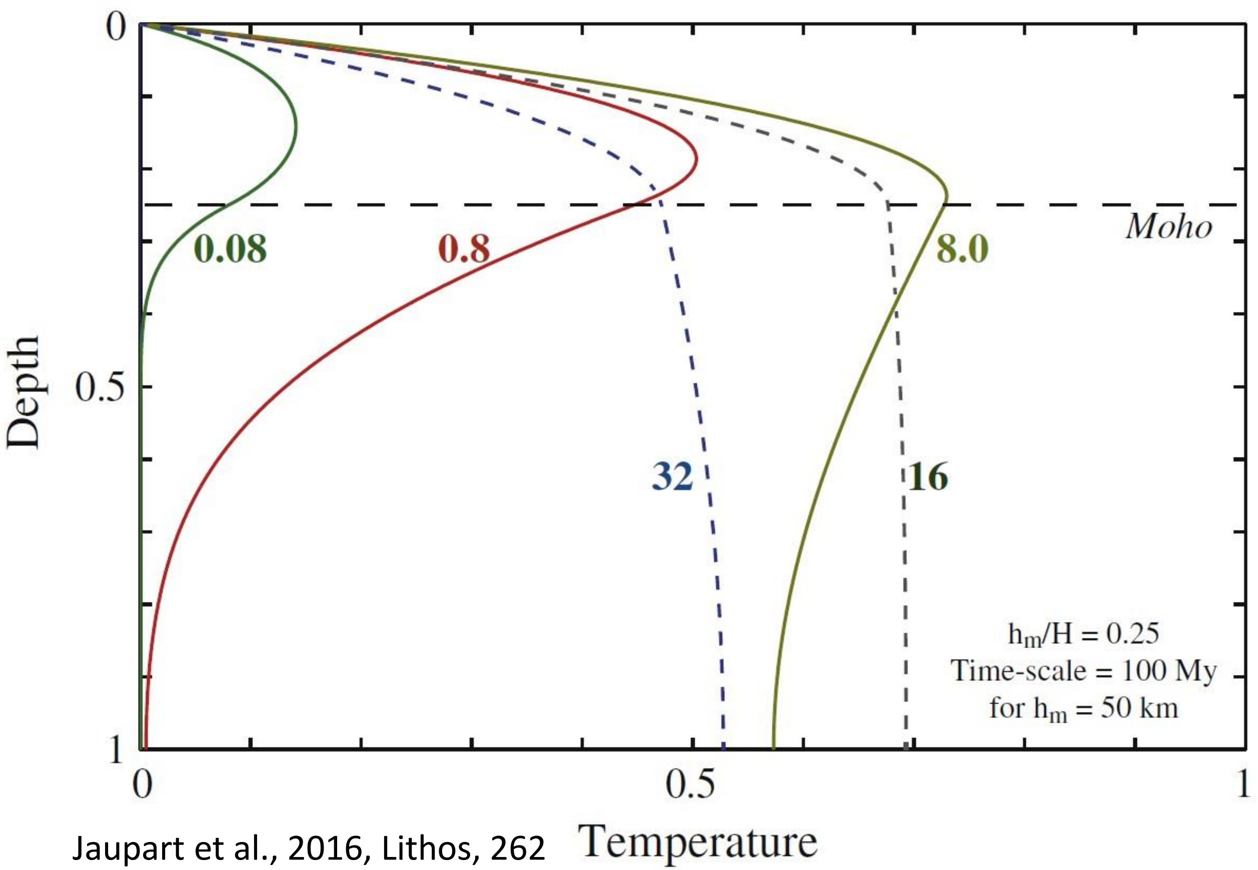
Post-accretion thermal evolution of crust with an initial temperature anomaly confined to a lower crustal layer (850 °C between depths of 35 km and 50 km, $z=0.8\times h_m$). Crustal thickness h_m=50 km, lithospheric thickness H=150 km, Ac=1.5 µWm^{-3}.

Jaupart et al., 2016, Lithos, 262

- The thermal evolution depends on heat loss through the surface, which acts in opposite senses for the two components *Ti* and *Tr*: It accelerates cooling and the thermal relaxation of the initial anomaly, but it slows down heating by crustal heat production.
- Starting from an initial “hot geotherm”, one observes an initial cooling phase that gets interrupted by radiogenic heating.
- Heating by crustal heat sources overwhelms the initial cooling after ~10 Myr and temperatures in the lower crust rise to values > 850 °C after about 60 My.
- This time lag would be shorter for a smaller initial thermal perturbation and higher heat production, and it would be longer for a smaller heat production.

Post-Orogenesis Thermal Evolution

Heating of the crust and lithosphere by crustal heat sources (T_r)



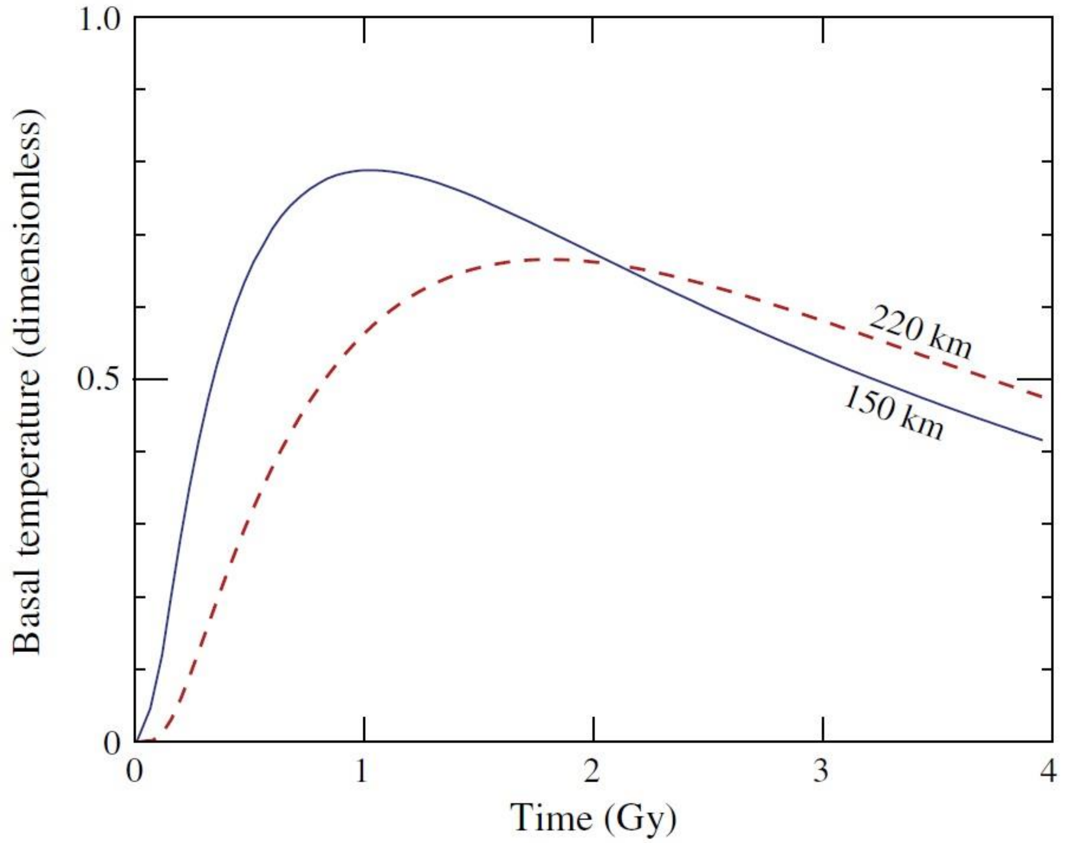
Jaupart et al., 2016, Lithos, 262

Heat production decays according to the $A(t)=A_0\exp(-t/\tau_r)$, τ_r=3.4 Gy. T has been scaled to $A_0 h_m^2/(2\lambda)$ (Moho T for a uniform and steady crustal heat production equal to A_0).
The labels refer to times scaled to the crustal diffusive time-scale, ≈100 My for 50 km thick crust.

- In a first phase, temperatures rise steadily everywhere with a peak just above the Moho (the lithospheric mantle lagging behind the radioactive crust).
- The second phase is characterized by the slow evolution of the lithospheric root towards thermal equilibrium with the crustal heat sources.
- Afterwards, there is a phase of secular cooling due to the rundown of radioactivity.

Post-Orogenesis Thermal Evolution

Variation of *T* at the base of the lithosphere due to crustal heat production



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T has been scaled to $A_0 h_m^2/(2\lambda)$ (Moho *T* for a uniform and steady crustal heat production equal to A_0).

Jaupart et al., 2016, Lithos, 262

- Heat production reduces in the lithosphere following closely an exponential decay with time constant τ_{radio}≈3.4 Gyr, which is not much larger than the diffusive relaxation time. Therefore, heat production decreases whilst lithospheric *T* is catching up with the deep crust.
- Once secular quasi equilibrium conditions have been attained, *T* decreases everywhere due to radioactive decay. Lithospheric *T* peaks at a late time, which increases with increasing lithosphere thickness.
- Changes in the amount and/or vertical distribution of crustal heat sources that are induced by an orogenic event are rapidly translated into the thermal structure of the crust, but can only affect the deep lithosphere after a long time lag.
- The crust and its thick lithospheric root may remain thermally and mechanically decoupled for longer than the time between two orogenic events.

Cooling of rocks in proximity of intrusions (Step-shaped temperature distributions)

- **For the thermal modeling of intrusions it is possible to assume that their emplacement is infinitely rapid, compared to the time of the subsequent thermal equilibration (*instantaneous heating model*).**

T_i = temperatures of the intrusion
T_b= temperatures of the host rock

If we choose a one dimensional coordinate system in which the origin $z = 0$ is exactly at the contact of the model intrusion, then the initial and boundary conditions can be:

Initial Conditions: $T = Ti$ for all $z > 0$ and $T=Tb$ for all $z<0$ at $t=0$

Boundary Conditions: $T = Ti$ for all $z=+\infty$ and $T=Tb$ for all $z=-\infty$ at $t>0$

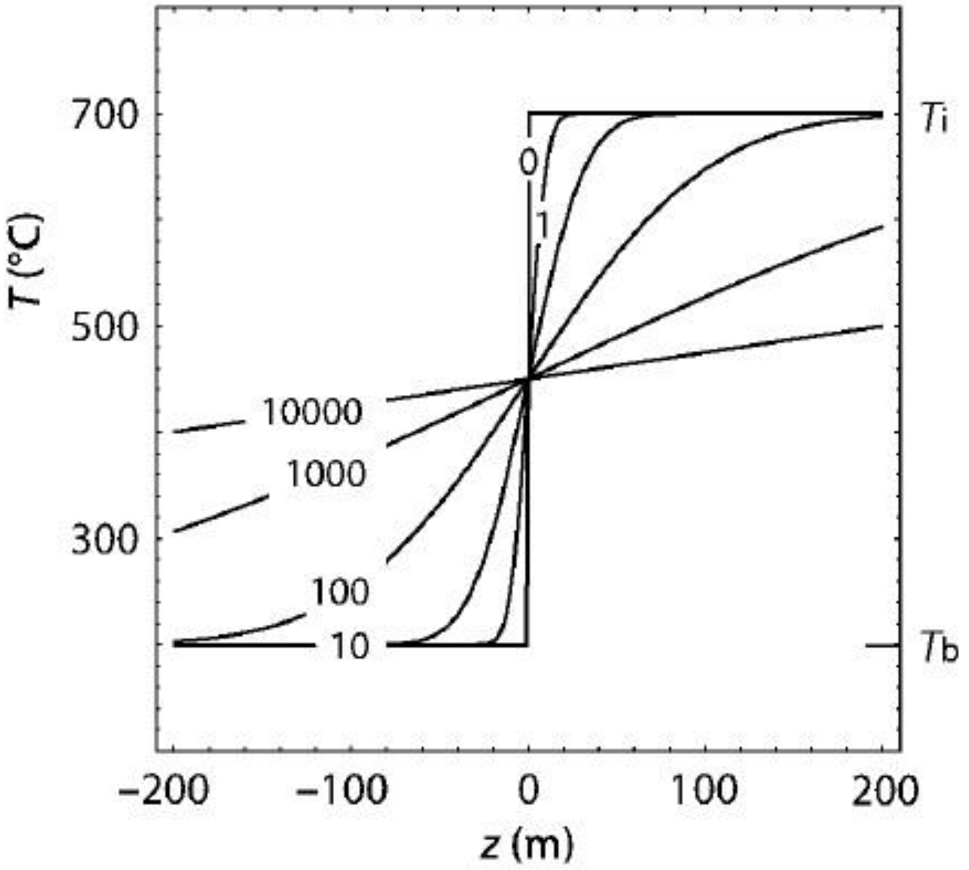
$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2} \qquad T = T_b + \frac{(T_i - T_b)}{2}\left(1 + \text{erf}\left(\frac{z}{\sqrt{4\kappa t}}\right)\right)$$

In another coordinate system in which the coordinate origin is located at a distance l from the temperature step:

Initial Conditions: $T = Ti$ for all $z > l$ and $T=Tb$ for all $z<l$ at $t=0$

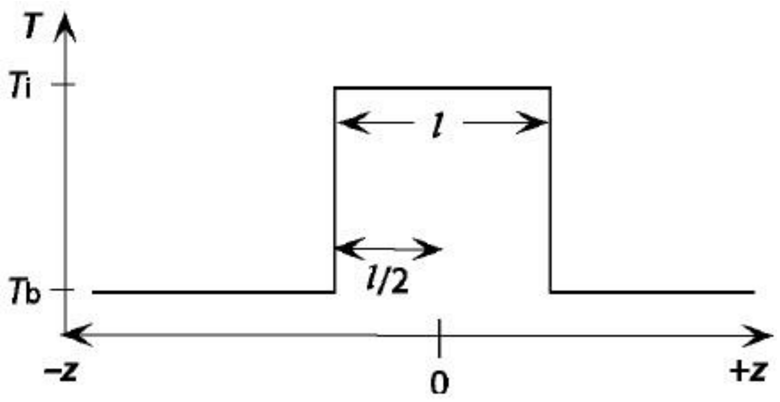
$$T = T_b + \frac{(T_i - T_b)}{2}\left(1 + \text{erf}\left(\frac{z-l}{\sqrt{4\kappa t}}\right)\right)$$

Temperature profiles at different times (in years) after the intrusion event



- Thermal evolution on both sides of the mean temperature between T_i and T_b develops symmetrically.

Cooling of rocks in proximity of intrusions (Step-shaped temperature distributions)



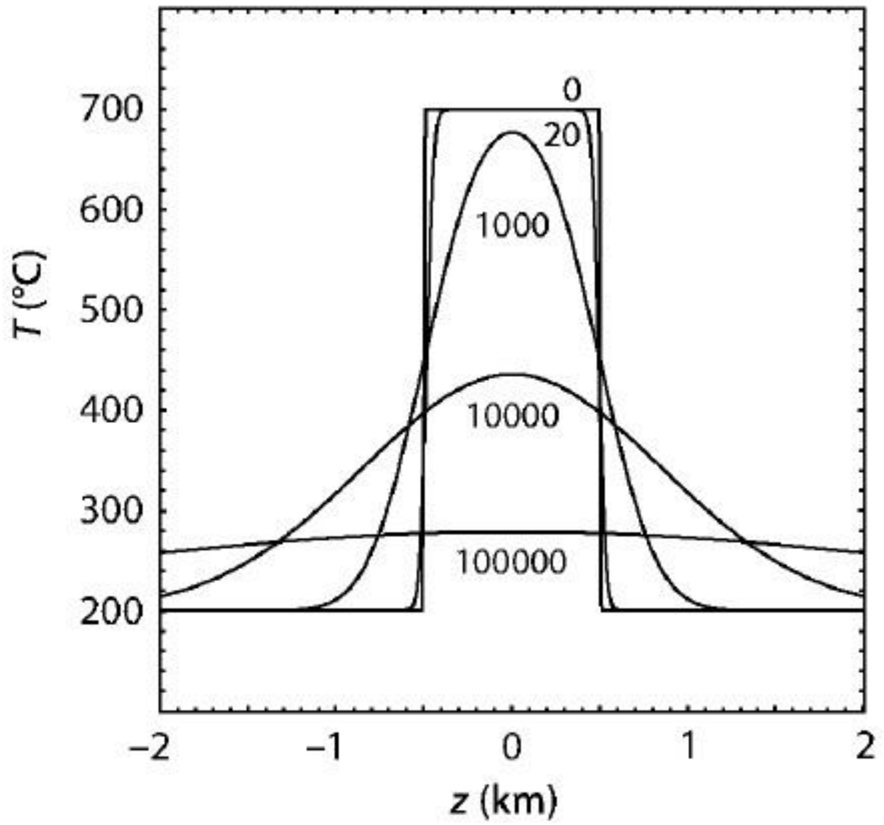
For a coordinate system with its origin in the center of a dike with the thickness *l*, the initial conditions may be described by:

$T=T_i$ for $-(l/2) < z < (l/2)$

$T=T_b$ for $(l/2) < z < -(l/2)$

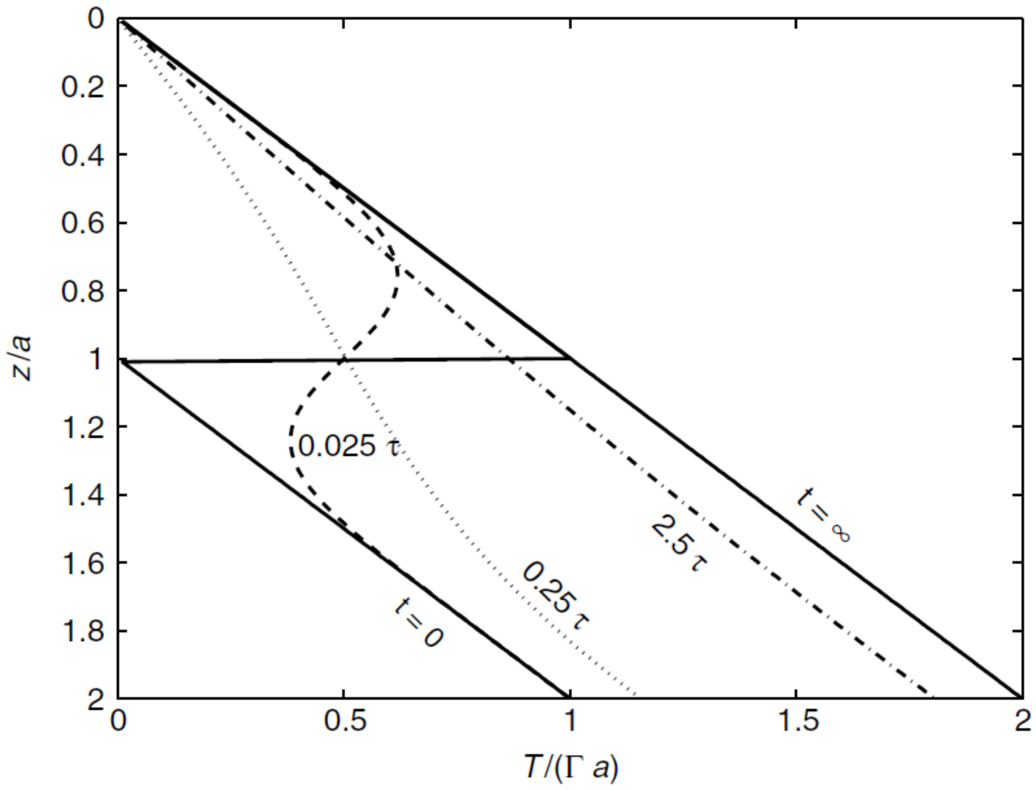
$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}$$

$$T = T_\mathrm{b} + \frac{(T_\mathrm{i} - T_\mathrm{b})}{2} \left(\mathrm{erf}\left(\frac{0.5l - z}{\sqrt{4\kappa t}} \right) + \mathrm{erf}\left(\frac{0.5l + z}{\sqrt{4\kappa t}} \right) \right)$$



Transient Effects
Effect of Overthrusting: Stacking of Two Slabs

- Underthrusting of a cold slab under the lithosphere will initially cool the lithosphere, followed by thermal re-equilibration.
- Thermal re-equilibration after the stacking of two slabs requires a time more than twice the thermal time constant $\tau = a^2/\kappa$.
  E.g.: For a 100 km thick slab with $\tau \approx$300My, thermal re-equilibration takes >2.5 x τ ≈ 750 Myr.



$T(z,t=0)=0$ for $z<a$ $T(z,t=0)=\Delta T=-\Gamma a$ for $a<z<2a$.

$$T(z,t)=\Delta T\sum_{n=1}^{\infty}\frac{(-)^n}{k_n}\sin(k_n z/a)\sin(k_n)\exp(-k_n^2\kappa t/a^2) \quad z<a \qquad k_n=(2n-1)\pi/4$$

$$T(z,t)=\Delta T\sum_{n=1}^{\infty}\frac{(-)^n}{k_n}\cos(k_n(z-2a)/a)\cos(k_n)\exp(-k_n^2\kappa t/a^2) \quad a<z<2a$$

During the time of re-equiliabration, the temperature in the overriding slab is lower than it was initially:

$$\frac{q(t)}{\lambda\Gamma}=\left(1-\sum_{n=1}^{\infty}(-)^n\sin(k_n)\exp(-\kappa k_n^2 t/a^2)\right)$$

z=depth
a=thickness of the slab (100 km)
Γ= geothermal gradient
Γλ=fixed heat flux at the base

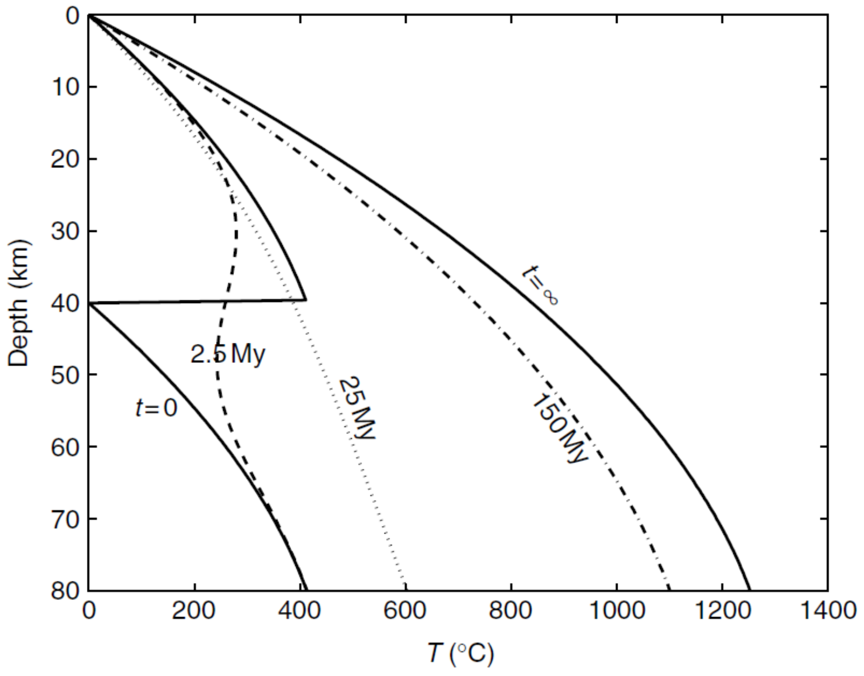
$\tau = a^2/\kappa$ τ=317 Myr

t=0.025τ=8 Myr

Temperature starts to increase again at ~0.25τ

Transient Effects
Effect of Overthrusting: Crustal Scale Thrusting

- During continental collision, one crustal block can thrust over another
- The increase in temperature following the superposition of two crustal blocks can be ~800 K for *A* of 0.8 mWm^3
- It requires more than 25 Myr for $T > T_0$



When one block overrides another one, both with the same thickness *a*, and with uniform heat generation *A* and assuming no heat flux at the base (Q_m=0), *T*(*z*) is:

$$T = T_0 + \frac{(Q_m + Ay_c)}{K} y - \frac{A}{2K} y^2$$ or $$T(z) = \frac{2Aaz}{\lambda} - \frac{Az^2}{2\lambda}$$ (initial steady state conditions)

The initial temperature perturbation and the transient temperature are:

$$\Delta T(z,t=0) = -\Gamma z = \frac{-Aaz}{\lambda}, \ 0 < z < a$$

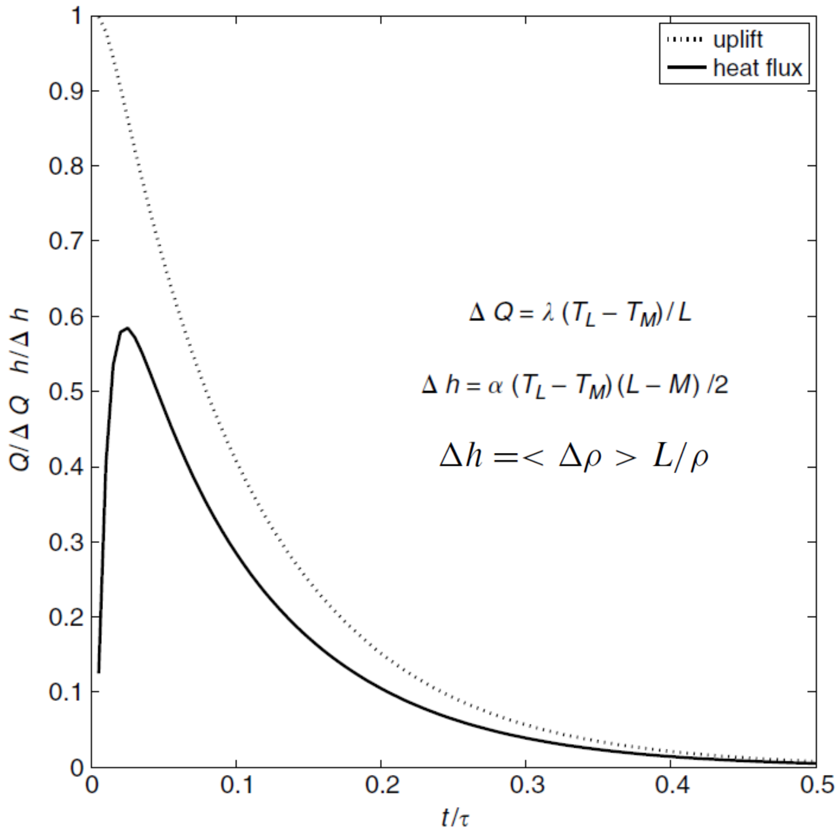
$$T(z,T) = \frac{\Gamma a}{2} \sum_{n=1}^{\infty} \frac{(-)^n}{k_n} \sin(k_n) \sin(k_n z/a) \exp(-k_n^2 \kappa t/a^2) + \Gamma a \sum_{n=1}^{\infty} \frac{(-)^n}{k_n^2} \cos(k_n) \sin(k_n z/a) \exp(-k_n^2 \kappa t/a^2)$$

$$\Delta T(z,t=0) = -\frac{3\Gamma a}{2} = \frac{-3Aa^2}{2\lambda}, \ a < z < 2a$$

$$T(z,t) = \frac{\Gamma a}{2} \sum_{n=1}^{\infty} \frac{(-)^n}{k_n} \cos(k_n) \cos(k_n(z-2a)/a) \exp(-k_n^2 \kappa t/a^2) + \Gamma a \sum_{n=1}^{\infty} \frac{(-)^n}{k_n^2} \sin(k_n) \cos(k_n(z-2a)/a) \exp(-k_n^2 \kappa t/a^2)$$

with $k_n = (2n-1)\pi/4$

Mantle Delamination



T_M= Moho *T*
T_L= *T* at the base of the lithosphere
L=thickness of the lithosphere
M or z_m= *Moho depth*

Thermal perturbation in the mantle lithosphere and transient temperature perturbation are:

$$T(z,t=0) = (T_L - T_M)\frac{L-z}{L-z_M},\ z_M < z < L,$$

$$T(z,t) = (T_M - T_L)\sum_{n=1}^{\infty}\left(\left(\frac{z_M}{L}-1\right)\cos\frac{n\pi z_M}{L} - \frac{1}{(n\pi)}\sin\frac{n\pi z_M}{L}\right)\times\frac{1}{n\pi}\sin\frac{n\pi z}{L}\exp(-n^2\pi^2\kappa t/L^2)$$

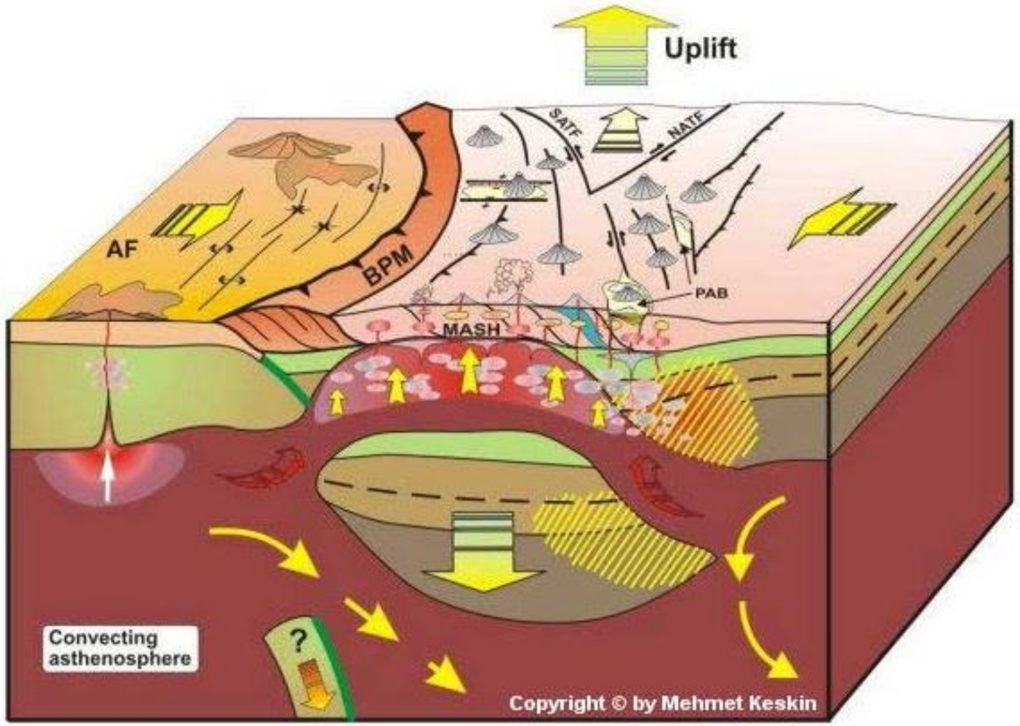
Surface heat flux and average change in density of a lithosoheric column are:

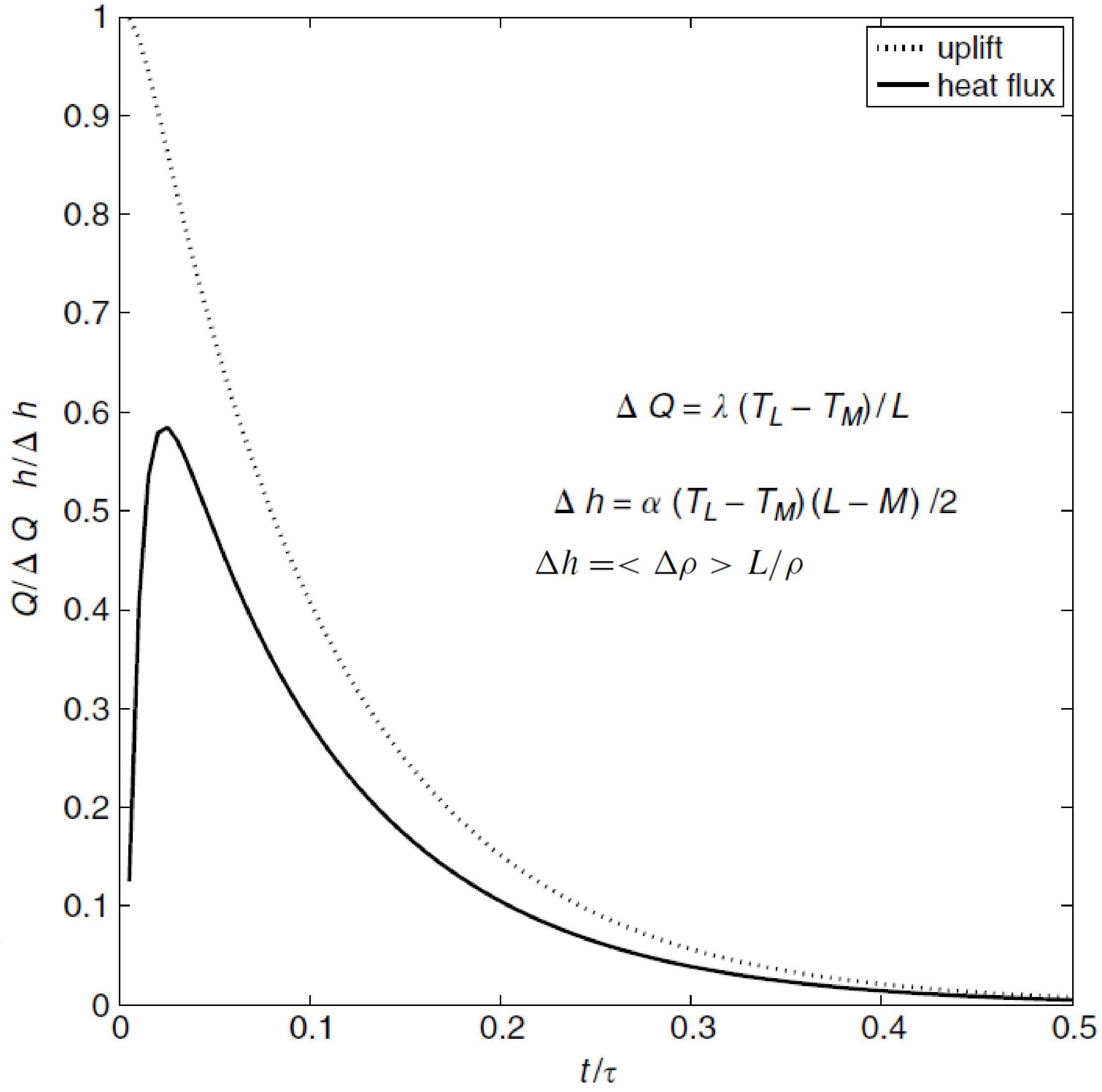
$$q(z,t) = \frac{\lambda(T_M - T_L)}{L}\sum_{n=1}^{\infty}\left(\left(\frac{z_M}{L}-1\right)\cos\frac{n\pi z_M}{L} - \frac{1}{n\pi}\sin\frac{n\pi z_M}{L}\right)\times\exp(-n^2\pi^2\kappa t/L^2)$$

$$\frac{<\Delta\rho>}{\rho} = \frac{\alpha}{2}(T_L - T_M)\frac{(L-z_M)}{L}\sum_{n=1}^{\infty}\frac{2}{(2n-1)^2\pi^2}\times\left(\left(\frac{z_M}{L}-1\right)\cos\frac{(2n-1)\pi z_M}{L} - \frac{1}{(2n-1)\pi}\sin\frac{(2n-1)\pi z_M}{L}\right)\times\exp(-(2n-1)^2\pi^2\kappa t/L^2)$$

Mantle Delamination

The rapid peeling off of the mantle lithosphere and its replacement by asthenospheric material cause rapid plateau uplift.

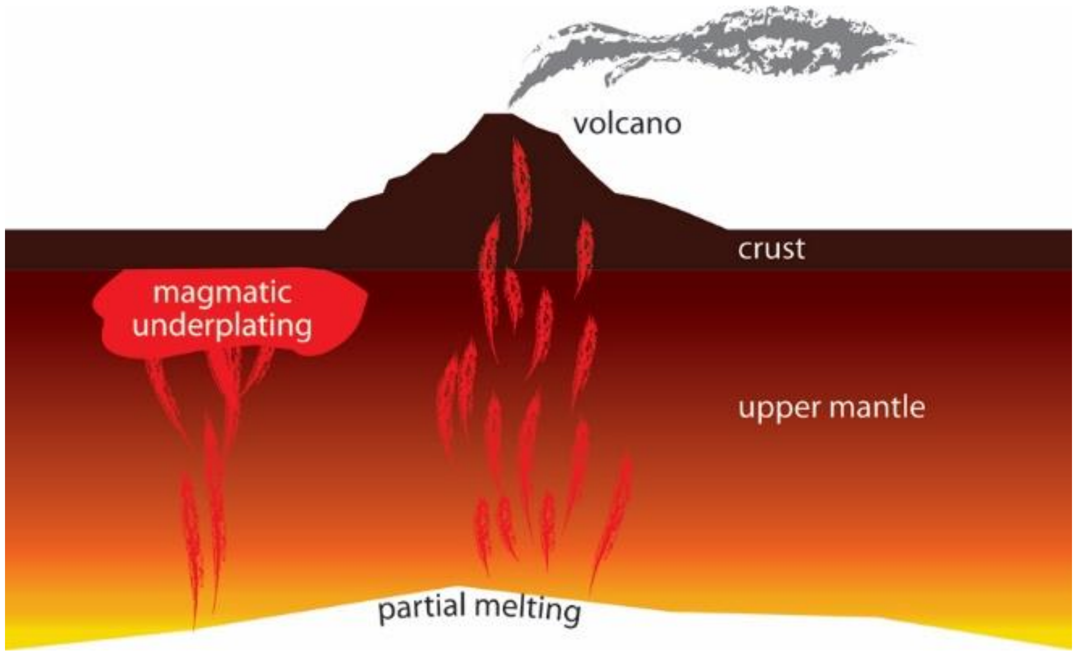




- With Moho temperature of ~ 600°C and a 150 km thick lithosphere, an uplift ≈ 1.1 km can be achieved almost instantly.
- Relaxation of the topography to half the initial level requires ~ 80 My.
- The peak in surface heat flux perturbation lags 25 My behind the uplift and is 7.5 mW $m^{-2} \approx 0.5 \times \lambda \times (T_M - T_L)/L$.

τ=714 Myr

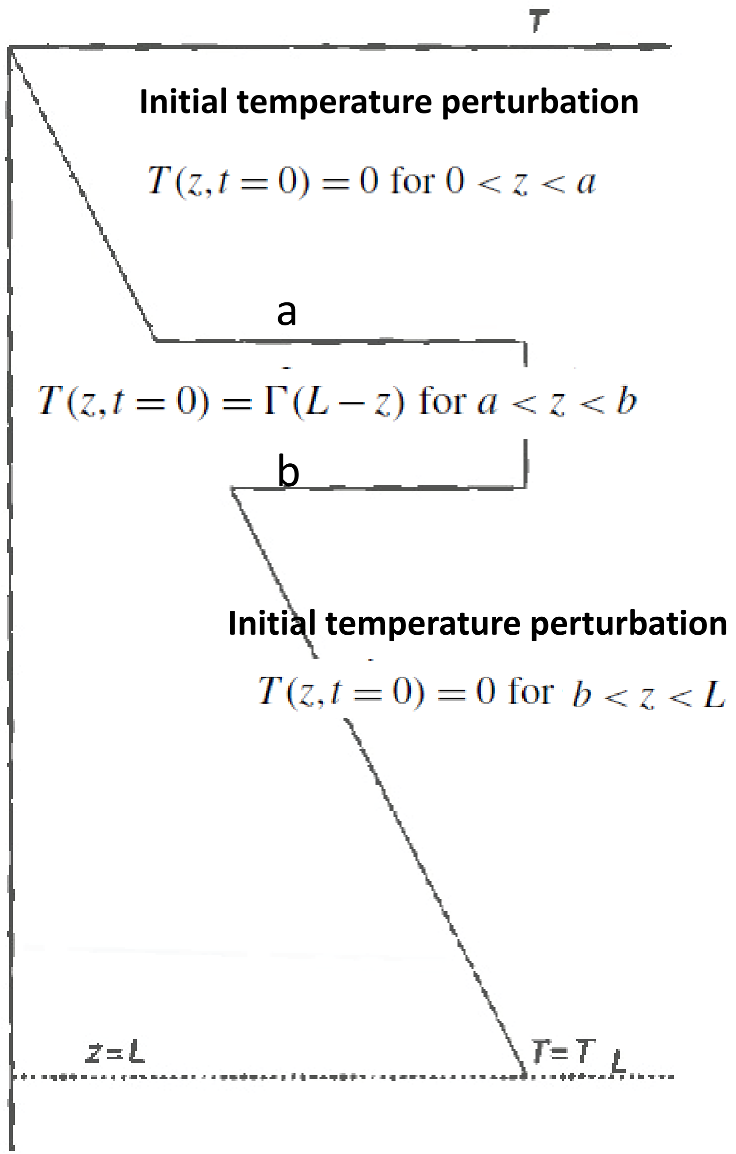
Magmatic Underplating



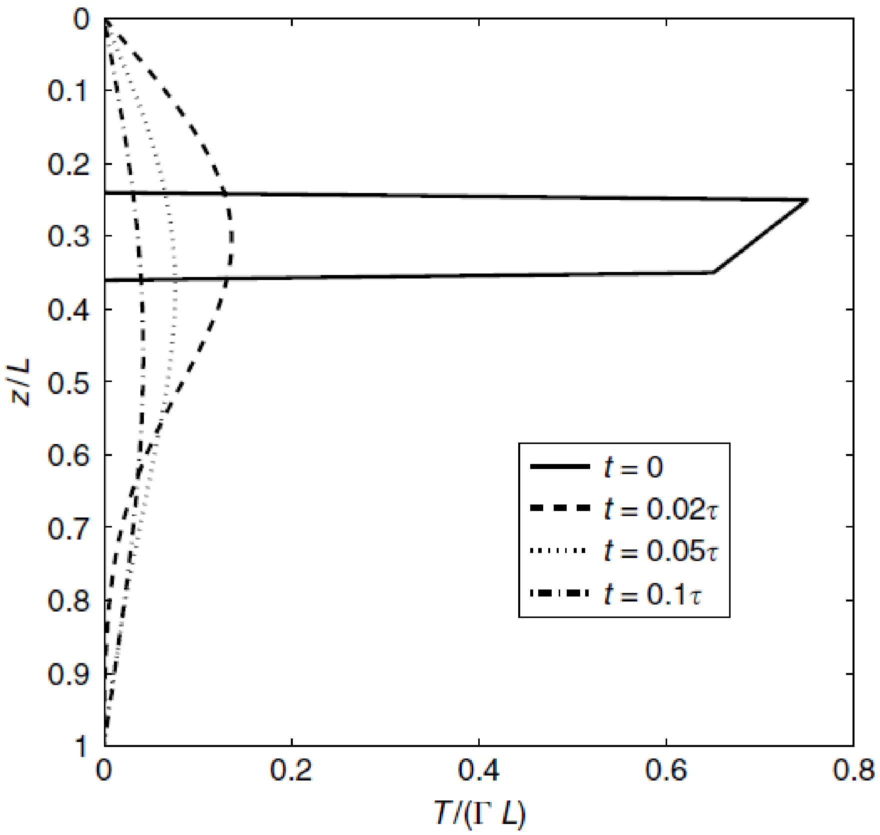
$$T(z,t) = \sum_{n=1}^{\infty} A_n \sin(n\pi z/L) \exp(-n^2\pi^2 t/\tau)$$

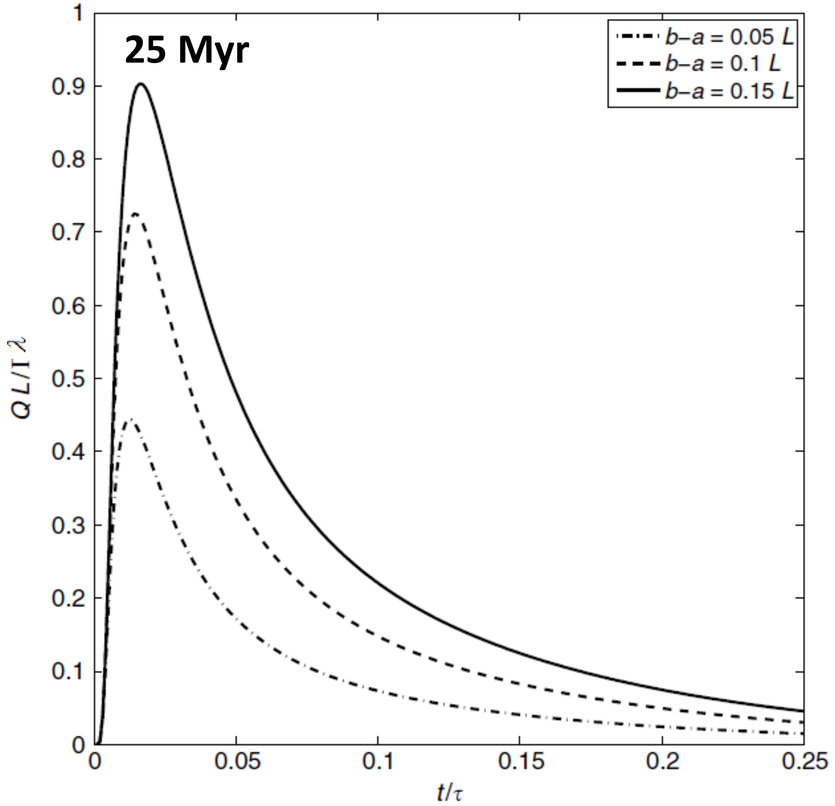
$$\frac{A_n}{2\Gamma} = \frac{(L-a)(\cos(n\pi a/L) - (L-b)\cos(n\pi b/L))}{n\pi} + \frac{L^2}{n^2\pi^2}(\sin(n\pi a/L) - \sin(n\pi b/L))$$

$$q(t) = \frac{\lambda}{L} \sum_{n=1}^{\infty} A_n n\pi \exp(-n^2\pi^2 t/\tau)$$



Magmatic Underplating





The transient surface heat flux reaches its peak a short time after underplating occurs (≈25 My, depending on the intrusion depth) and its amplitude can be significant.

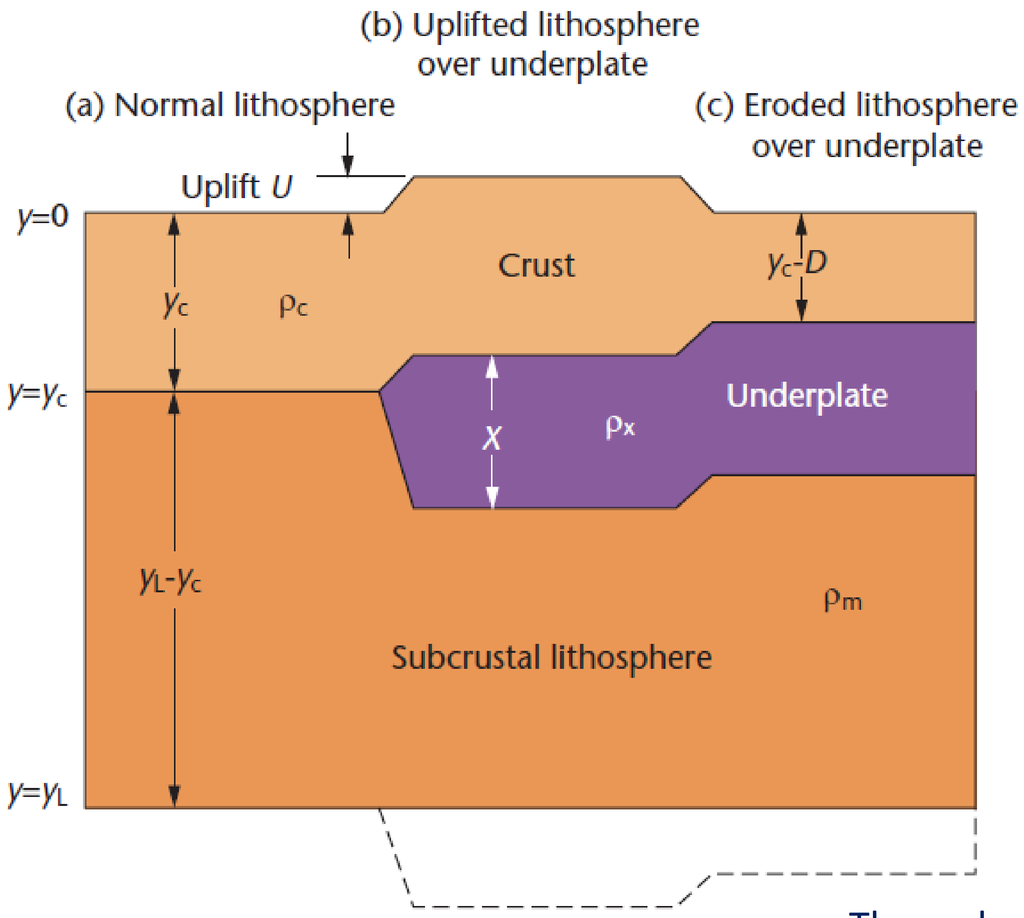
t_{max} = time taken to reach the maximum temperature at dimensionless distance: $y^* = y/a$

Maximum temperature $T^*_{max} \approx \sqrt{\frac{2}{\pi e}} \frac{1}{y^*}$ a=half-width of the body $t^*_{max} \approx \frac{y^{*2}}{2}$

Temperature perturbation decays quite rapidly and is large only close to the intruding layer

Magmatic Underplating

- The density of igneous rocks, generated by adiabatic decompression of the mantle, ranges between 2990 and 3070 kgm^{-3} (< 3300 kgm^{-3}). Thus, the replacement of the lithospheric mantle with these igneous rocks causes uplift.
- The amplitude of the thermal uplift following underplating depends on the thickness of the layer, but it is usually modest. For an excess temperature of 600 K, the surface uplift will be 18×10^{-3} times the layer thickness (e.g., the underplated layer must be at least 60 km thick to cause an uplift of the order of 1000 m).



$$h(t)=\alpha\int_0^L T(z,t)dz = 2\alpha L\sum_{n=1}^{\infty}\frac{A_{2n-1}}{2n-1}\exp(-(2n-1)^2\pi^2 t/\tau)$$

h(t)=thermal uplift

Uplift due to the density difference

Pressure at the depth y_L:

Normal Lithosphere

$$y_c\rho_c g+(y_L-y_c)\rho_m g$$

Uplifted Lithosphere

$$y_c\rho_c g + X\rho_x g+(y_L-y_c-X+U)\rho_m g$$

$$U = X\frac{(\rho_m-\rho_x)}{\rho_m} = X\left(1-\frac{\rho_x}{\rho_m}\right)$$

(Uplifted lithosphere)

$$D = X\left(\frac{\rho_m-\rho_x}{\rho_m-\rho_c}\right)$$

(in case of erosion y_c=y_c-D)

The rock uplift for an underplate thickness of 5 km is one tenth of the underplate thickness

Solutions of the Diffusion Equation

In summary, types of solutions of the diffusion equation $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$ **are**:

1. Solutions that may be found by integration. These are *steady state* problems in which it is possible to assume: $\mathrm{d}T/\mathrm{d}t = 0$
2. Solutions containing an error function. These are found for problems that have their boundary condition at infinity (e.g., to describe the thermal evolution of intrusions << than the thickness of the crust or their distance to the earths surface).
3. Solutions containing Fourier series. These are found for time dependent problems with spatially fixed boundary conditions.

For the third case, let us assume the following boundary conditions: $T = 0$ at $x = 0$ at time $t > 0$. $T = 0$ at $x = l$ at time $t > 0$.

A general function that satisfies the condition for which the first time derivative is directly proportional to the second spatial derivative (diffusion equation), with κ proportionality constant, and the boundary conditions has the form:

$$T = \sum_{n=0}^{\infty} a_n \mathrm{e}^{b_n t} \sin\left(\frac{n\pi x}{l}\right)$$

a_n and b_n constants

- The solution contains an exponential function of time and a sine-function of *x*
- The boundary conditions at $x = 0$ and $x = l$ are always satisfied as the sine-function is always zero at these two values of *x.*
- The solution contains an infinite sum is a generalization. If a single term of the infinite sum satisfies the diffusion equation, so will the infinite sum of a series of terms.

Solutions of the Diffusion Equation

For a single term of the infinite sum, the time derivative of $T = \sum_{n=0}^{\infty} a_n \mathrm{e}^{b_n t} \sin\left(\frac{n\pi x}{l}\right)$ gives: $\frac{\partial T}{\partial t} = ab\mathrm{e}^{bt} \sin\left(\frac{n\pi x}{l}\right)$

The spatial derivatives are: $\frac{\partial T}{\partial x} = \frac{n\pi a}{l} \mathrm{e}^{bt} \cos\left(\frac{n\pi x}{l}\right)$ as well as: $\frac{\partial^2 T}{\partial x^2} = -\frac{n^2 \pi^2 a}{l^2} \mathrm{e}^{bt} \sin\left(\frac{n\pi x}{l}\right)$

- The first derivative with respect to *t,* will always be proportional to its second derivative with respect to *x* (the condition of the diffusion equation is met).

The condition of proportionality between the first time derivative and second spatial derivative is satisfied if the constant *b* has the value:

$$b = -\kappa \frac{n^2 \pi^2}{l^2}$$

The values for the constants a_n can be determined from the initial conditions. At time $t = 0$, $e^{bt} = 1$:

$T(x, 0) = f(x) = \sum_{n=0}^{\infty} a_n \sin\left(\frac{n\pi x}{l}\right)$ the coefficients a_n can be determined from the integral: $a_n = \frac{2}{l} \int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right) \mathrm{d}x$

(example of Fourier series)

References

Main Readings:

Books:

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Article

- Jaupart et al., 2016. Radiogenic heat production in the continental crust. Lithos 262, 398–427.