1

$n \in \mathbb{N}$ $\quad n = 3711$

$3 \mid 3711$

$3 \mid 12$

$0, 1, 2, 3, \ldots, 9$

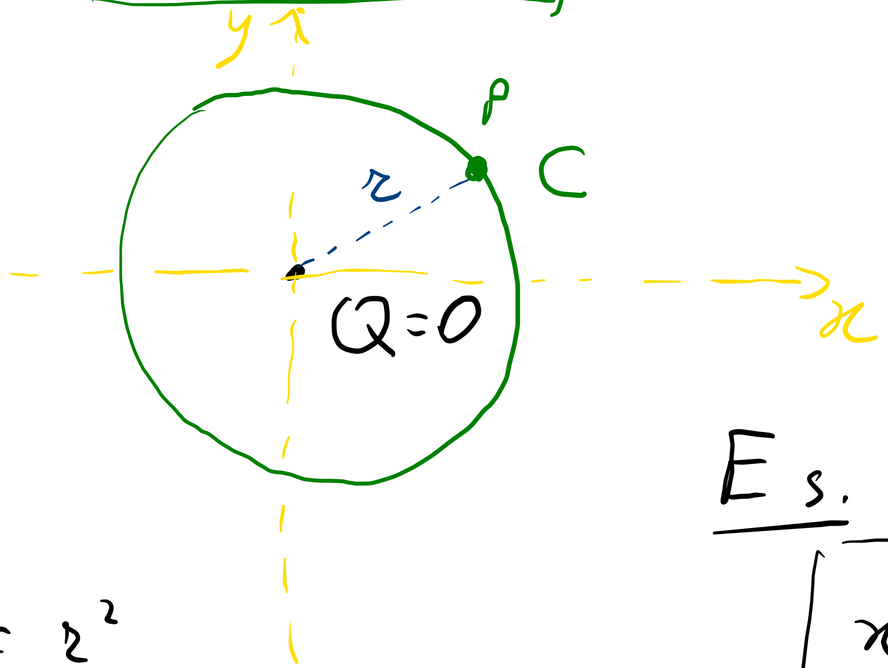
$10 = 3 \cdot 3 + 1$

$$3711 = 1 \cdot 10^0 + 1 \cdot 10 + 7 \cdot 10^2 + 3 \cdot 10^3$$

Resto: $1 + 1 \cdot 1 + 7 \cdot 1 \cdot 3 \cdot 1 = 1 + 1 + 7 + 3$

C circonferenza nel piano Euclideo π: luogo dei punti di centro Q e raggio $r>0$

$P \in \pi$ t.c. $\boxed{d(P,Q) = r}$



$O = Q = (0,0)$

$P = (x,y)$

$d(P,Q)^2 = r^2$

$C: \boxed{x^2 + y^2 = r^2}$

Es. $r = 1$

$$\boxed{x^2 + y^2 = 1}$$

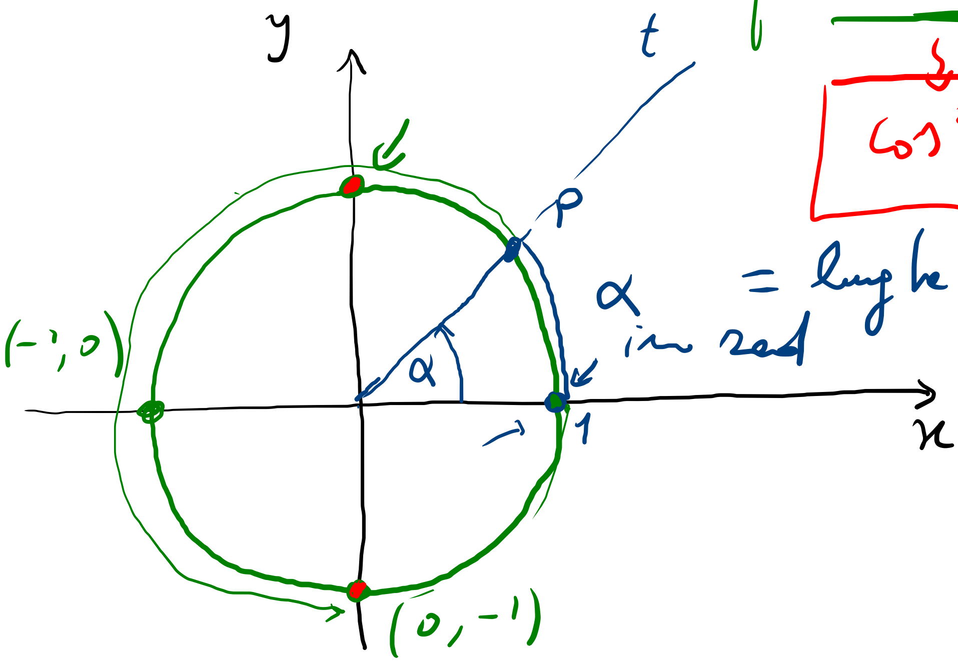
ang. giro $= 360°$

ang. retto $= 90°$

Grad. Cent.

ang. retto $= 100°$

Radianti   rad



$$\boxed{x^2 + y^2 = 1}$$

$$\Downarrow$$

$$\boxed{\cos^2\alpha + \sin^2\alpha = 1}$$

α = lunghezza arco in rad

Ang. giro $= 2\pi$ rad

ang. piatto π rad

ang. retto $\frac{\pi}{2}$

$$\alpha \rightsquigarrow P =: (\cos\alpha, \sin\alpha)$$

COORDINATE DI P

$\cos 0 = 1$

$\sin 0 = 0$

$\cos\frac{\pi}{2} = 0$, $\sin\frac{\pi}{2} = 1$

$\cos\pi = -1$

$\sin\pi = 0$

$\cos\frac{3\pi}{2} = 0$

$\sin\frac{3\pi}{2} = -1$

$$\boxed{\tan\alpha := \frac{\sin\alpha}{\cos\alpha}}$$

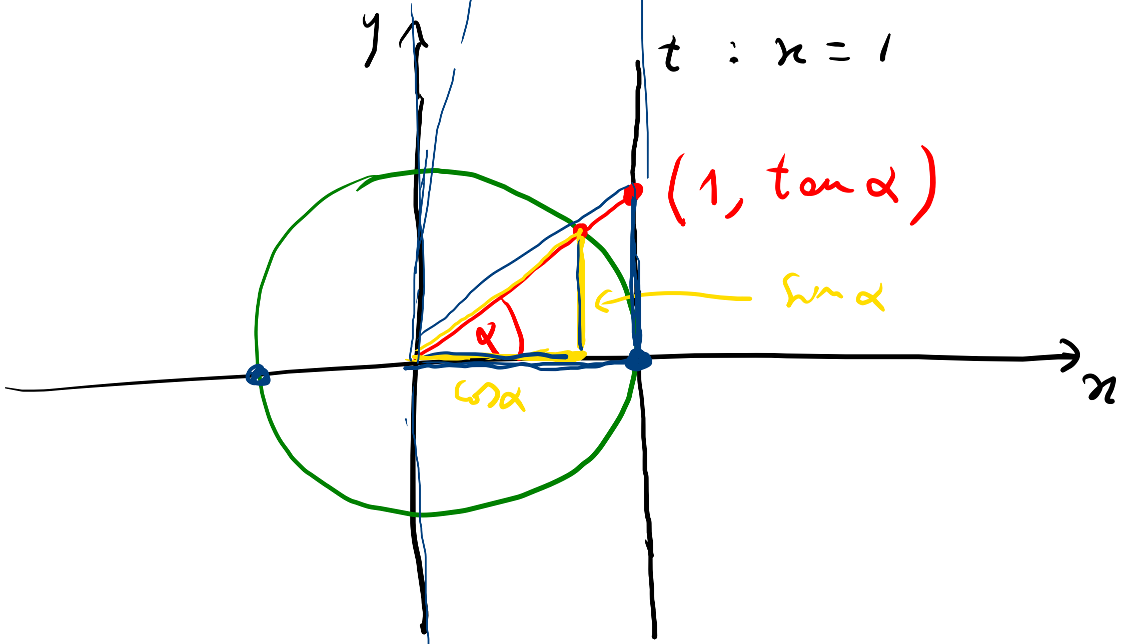
se $\cos\alpha \neq 0$

cioè $\alpha \neq \frac{\pi}{2}, \frac{3\pi}{2}$

$\alpha = \frac{\pi}{2} + k\pi, \; k \in \mathbb{Z}$

$$\cot\alpha := \frac{\cos\alpha}{\sin\alpha} = \frac{1}{\tan\alpha}$$

$\sin\alpha \neq 0$



Grafici

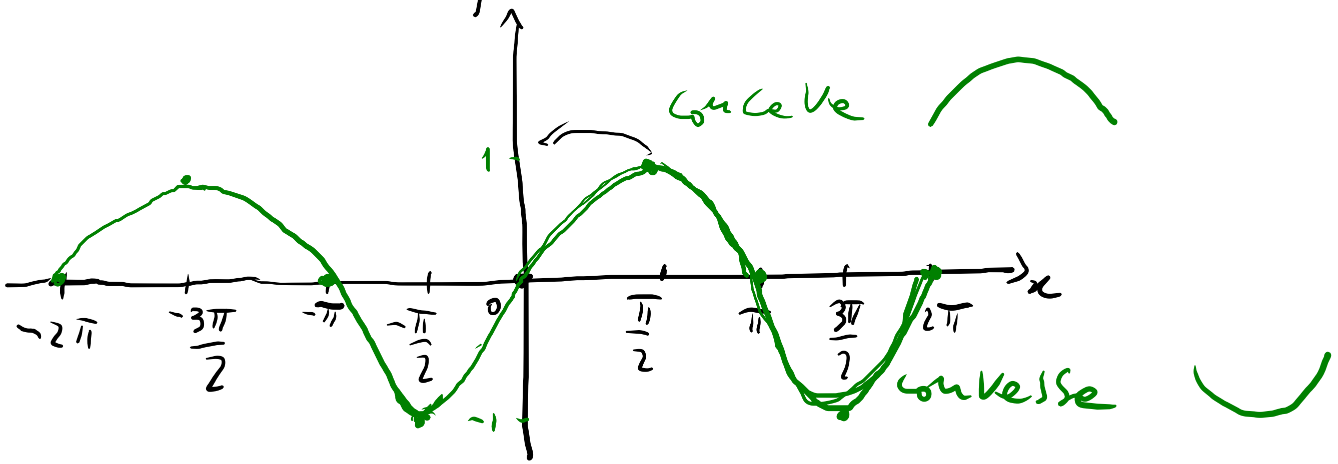
sen : $\mathbb{R} \to \mathbb{R}$

$\alpha \mapsto \operatorname{sen} \alpha$

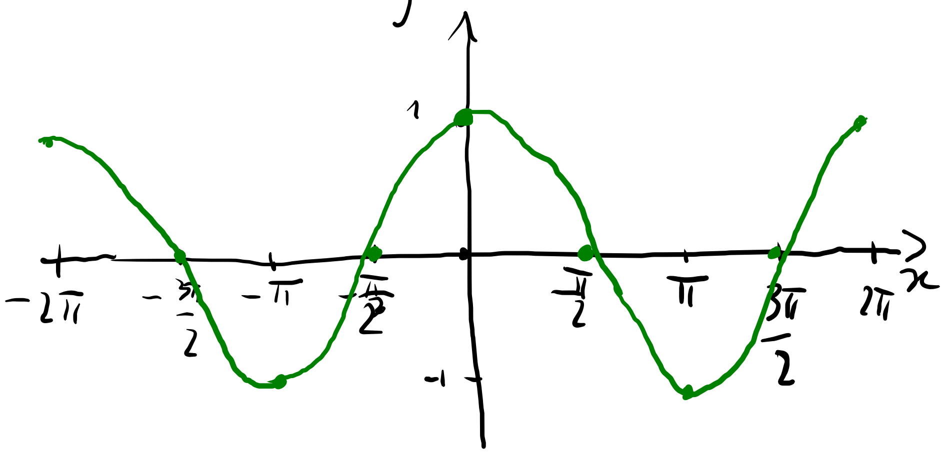
cos : $\mathbb{R} \to \mathbb{R}$

$\alpha \mapsto \cos \alpha$

sen



cos



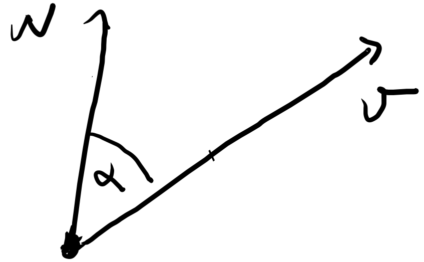
Vettori

$$\langle v, v\rangle = \|v\|^2 \geq 0$$

$$\cos 0 = 1$$

$$|x|$$

norma di v = lunghezza di v



$\|v\|$ = lunghezza

$$\boxed{\text{Prodotto scalare}}$$

$$v \cdot w = \langle v, w\rangle := \|v\| \cdot \|w\| \cdot \cos\alpha$$

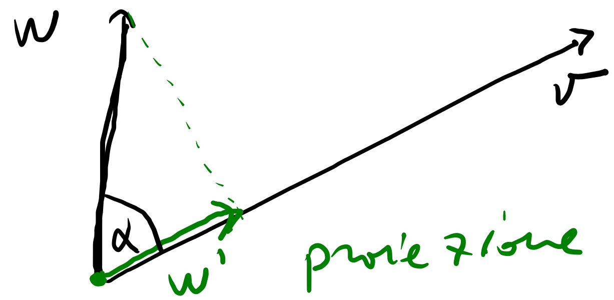
$$\|0\| = 0$$

$$\langle v, 0\rangle = 0$$

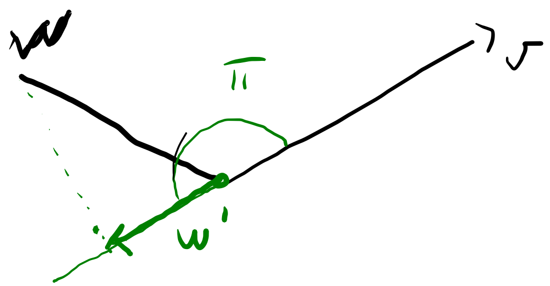
se $v \perp w$ allora $\langle v, w\rangle = 0$
$(\alpha = 90^\circ)$

$$v \cdot w = 0$$

Proprietà di $\langle \, , \, \rangle$

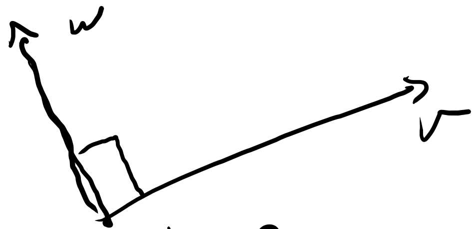


proiezione ortogonale di w lungo v

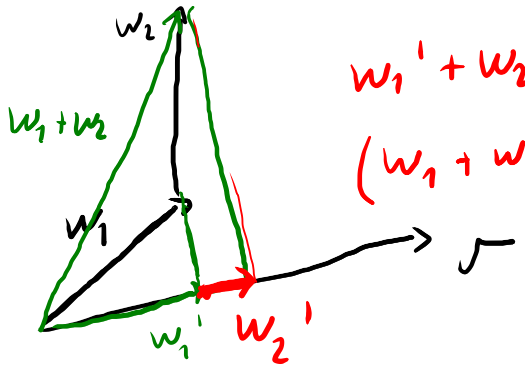


$$\|w'\| = \|w\| \, |\cos\alpha|$$

$$\underline{\langle v, w\rangle} = \|v\| \, \|w\| \cos\alpha = \underline{\langle v, w'\rangle} = \pm \|v\| \, \|w'\|$$



$w' = 0$



$w_1' + w_2' = (w_1 + w_2)'$

$$\langle v_1 + v_2, w\rangle = \langle v_1, w\rangle + \langle v_2, w\rangle$$

$$\langle \lambda v, w\rangle = \lambda \langle v, w\rangle$$

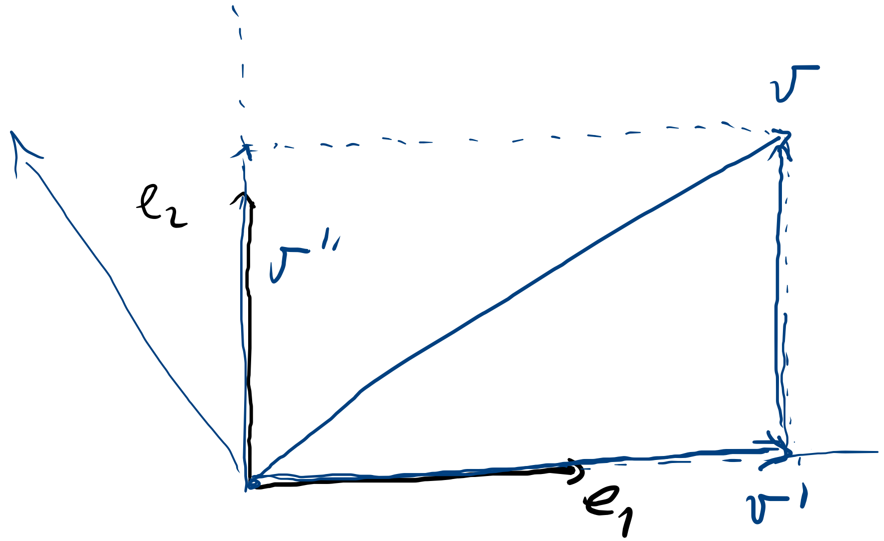
- $\langle v, w_1 + w_2\rangle = \langle v, w_1\rangle + \langle v, w_2\rangle$

$$\langle v, w\rangle = \langle w, v\rangle$$

$$\langle v, \lambda w\rangle = \lambda \langle v, w\rangle$$

e_1, e_2 ortogonali

$\|e_1\| = \|e_2\| = 1$



$v = v' + v'' = \underline{\lambda e_1 + \mu e_2}$ combinazione lineare di e_1 e e_2

$v' = \lambda e_1 , \quad \lambda \in \mathbb{R}$

$v'' = \mu e_2 , \quad \mu \in \mathbb{R}$

(λ, μ) componenti di v $\{e_1, e_2\}$ base per V
o coordinate

$v = \underline{\lambda_1 e_1 + \lambda_2 e_2}$

$w = \mu_1 e_1 + \mu_2 e_2$

$v + w = \underline{(\lambda_1 + \mu_1) e_1 + (\lambda_2 + \mu_2) e_2}$

$$\begin{cases} v = \lambda_1 e_1 + \lambda_2 e_2 \\ w = \mu_1 e_1 + \mu_2 e_2. \end{cases}$$

$$\alpha v = (\alpha \lambda_1) e_1 + (\alpha \lambda_2) e_2$$

$\alpha \in \mathbb{R}$

$$v \rightsquigarrow (\lambda_1, \lambda_2)$$

$$\alpha v \rightsquigarrow (\alpha \lambda_1, \alpha \lambda_2)$$

$$v + w \rightsquigarrow (\lambda_1 + \mu_1, \lambda_2 + \mu_2)$$

$\boxed{\{e_1, e_2\} \text{ base ortonormale}}$

$$\begin{cases} \langle e_1, e_1 \rangle = 1 \\ \langle e_2, e_2 \rangle = 1 \\ \langle e_1, e_2 \rangle = 0 = \langle e_2, e_1 \rangle \end{cases}$$

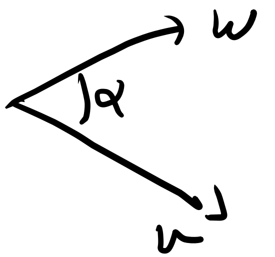
$$\boxed{\langle v, w \rangle} = \langle \lambda_1 e_1 + \lambda_2 e_2, \mu_1 e_1 + \mu_2 e_2 \rangle = \lambda_1 \mu_1 \langle e_1, e_1 \rangle + \cancel{\lambda_1 \mu_2 \langle e_1, e_2 \rangle} +$$

$$+ \cancel{\lambda_2 \mu_1 \langle e_2, e_1 \rangle} + \lambda_2 \mu_2 \langle e_2, e_2 \rangle =$$

$$\boxed{\langle v, w \rangle = \lambda_1 \mu_1 + \lambda_2 \mu_2} = \|v\| \|w\| \cos \alpha$$

$$\boxed{\cos \alpha = \frac{\langle v, w \rangle}{\|v\| \|w\|}}$$

$$\|v\| = \sqrt{\langle v, v \rangle} = \sqrt{\lambda_1^2 + \lambda_2^2}$$

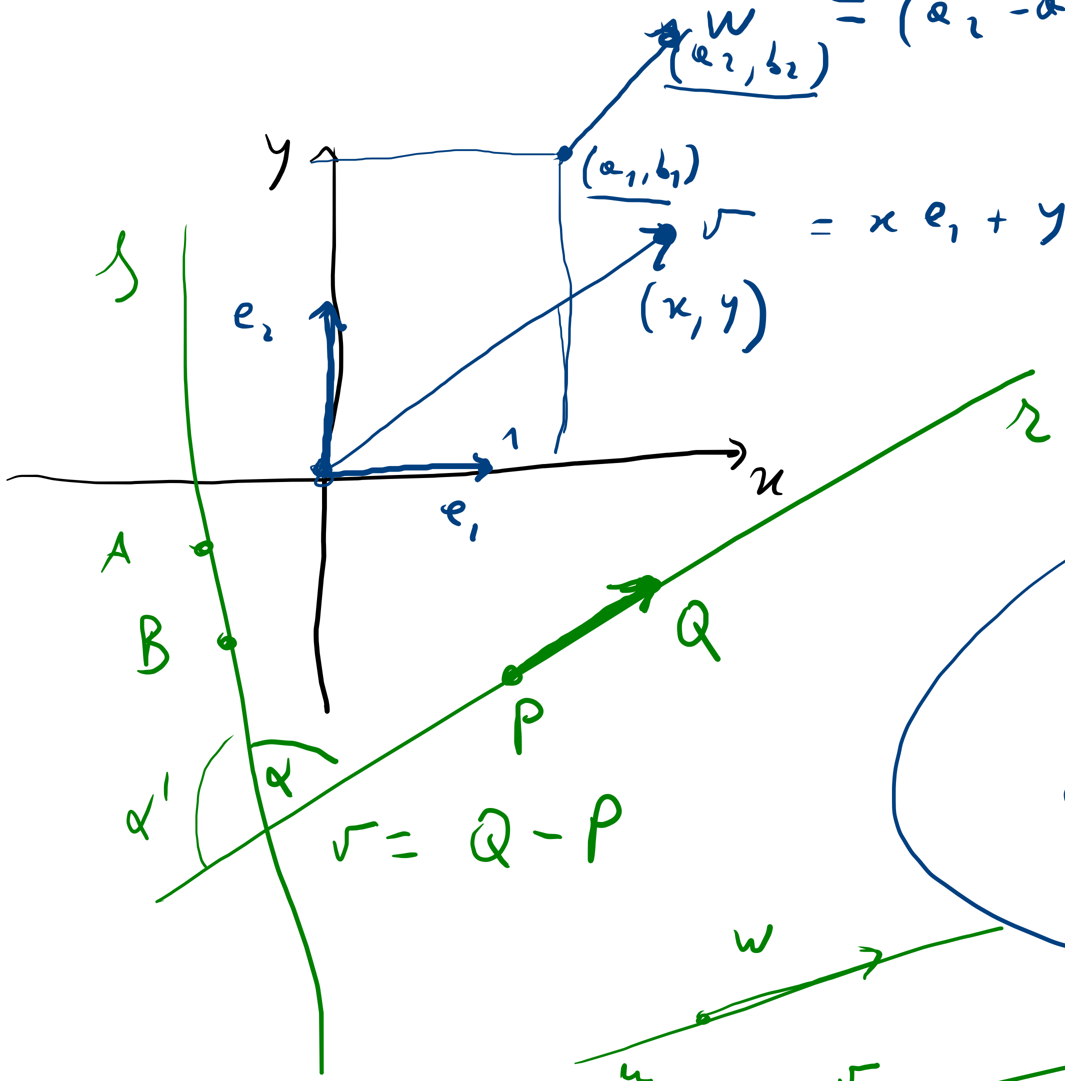


$w = (a_2 - a_1)\, e_1 + (b_2 - b_1)\, e_2$

$\Updownarrow$

$(a_2, b_2) - (a_1, b_1)$

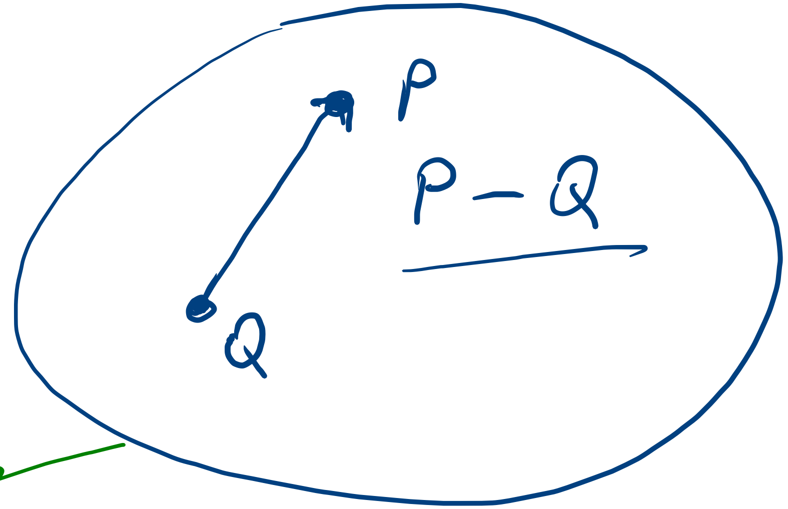
$v = x\, e_1 + y\, e_2$

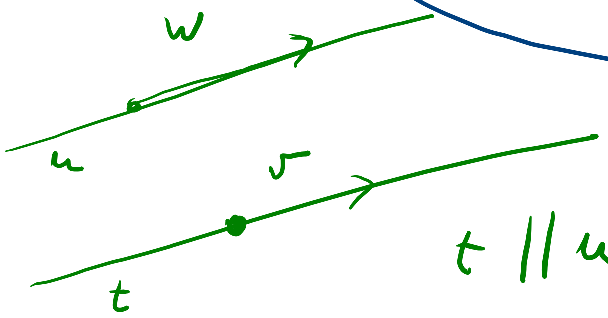


$w = A - B$

$v = Q - P$

$$\boxed{\cos\alpha = \frac{|\langle v, w \rangle|}{\|v\|\,\|w\|}}$$





$t \parallel u \iff w = \lambda v$ per un certo $\lambda \in \mathbb{R}$