



Copertina

# Corso di Fisica dello Strato Limite Atmosferico

Analisi dimensionale e di scala  
e le  
grandezze scala tipiche  
dell'ABL

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## Sommario della lezione

- ➊ Fondamenti dell'analisi dimensionale e di scala per affrontare problemi fisici (alla lavagna)
- ➋ I modelli della realtà descrittivi ed esplicativi (alla lavagna)
- ➌ Il teorema di Buckingham o teorema  $\Pi$  (alla lavagna)
- ➍ Applicazioni dell'analisi di scala e confronto tra modello descrittivo e esplicativo dello stesso sistema fisico (alla lavagna)
- ➎ Esempi di grandezze scala utilizzate nello studio dello Strato Limite Atmosferico
- ➏ Bibliografia di riferimento e per approfondimenti.



## L'approccio esplicativo e la ricchezza delle grandezze utilizzate (lunghezza)

Length:

- $z$  = height above the surface  
 $h$  or  $z_i$  = depth of the boundary layer (or mixed layer)  
 $H$  = SBL integral length scale = heat-flux-history scale  
 $L$  =  $-[\overline{u'w'}_s^2 + \overline{v'w'}_s^2]^{3/4}/[k \cdot (g/\overline{\theta_v}) \cdot (\overline{w'\theta_v})]$  = Obukhov length  
 $L_L$  =  $-[\overline{u'w'}^2 + \overline{v'w'}^2]^{3/4}/[k \cdot (g/\overline{\theta_v}) \cdot (\overline{w'\theta_v})]$  = local Obukhov length  
 $h_e$  =  $u_* / f_c$  = Ekman layer depth  
 $\lambda_{\max}$  = Wavelength corresponding to peak in turbulence spectrum  
 $H$  = height of obstacle  
 $W$  = width of obstacle  
 $z_o$  = aerodynamic roughness length  
 $Z_s$  = scale of surface features or roughness



## L'approccio esplicativo e la ricchezza delle grandezze utilizzate (velocità)

<b>Velocity:</b>	$u_* = [\overline{u'w'}_s^2 + \overline{v'w'}_s^2]^{1/4}$	= friction velocity
	$w_* = [(g/\overline{\theta}_v) \cdot \overline{w'\theta_v}_s \cdot z_i]^{1/3}$	= convective velocity scale
	$w_{Lf} = [(g/\overline{\theta}_v) \cdot \overline{w'\theta_v} \cdot z]^{1/3}$	= local free convection velocity scale
	$u_L = [\overline{u'w'}^2 + \overline{v'w'}^2]^{1/4}$	= local (friction) velocity scale
	$V_B = [(g/\Delta\overline{\theta}_v_s) \cdot \overline{w'\theta_v} \cdot H]^{1/3}$	= SBL buoyancy velocity scale
	$V_M = (Z_s/\rho)^{1/2} [(\partial P/\partial x)^2 + (\partial P/\partial y)^2]_s^{1/4}$	= mechanical forcing scale
	$u_*^{ML} = u_*^2 / w_*$	= convective stress scale velocity
	$\overline{G}$ or $\overline{U}_g$	= geostrophic wind speed
	$\overline{G}_s$	= geostrophic wind speed at the surface
	$\overline{G}_{z_i}$	= geostrophic wind at the top of the boundary layer
	$\langle \overline{G} \rangle$	= geostrophic wind speed averaged over the boundary layer
	$\overline{U}$ or $\overline{M}$	= wind speed
	$\overline{M}_s$	= wind speed at the surface

$\overline{M}_{z_i}$  = wind speed at the top of the boundary layer  
 $\langle \overline{M} \rangle$  = wind speed averaged over the boundary layer

$(TKE)^{1/2}$  or  $\bar{e}^{1/2}$  = square root of turbulence kinetic energy  
 $(k z \epsilon)^{2/3}$  = dissipation velocity scale in the surface layer



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## L'approccio esplicativo e la ricchezza delle grandezze utilizzate (temperatura)

Temperature:  $\theta_*^{\text{ML}} = \overline{w' \theta'_v}_s / w_* =$  convective (ML) temperature scale

$\theta_*^{\text{SL}} = -\overline{w' \theta'_v}_s / u_* =$  surface-layer temperature scale

$\theta_{Lf} = \overline{w' \theta'_v} / w_{Lf} =$  local free-convection temperature scale

$\theta_L = -\overline{w' \theta'_v} / u_L =$  local temperature scale

$\theta_* = \overline{w' \theta'_v}_s / (\text{any other velocity scale})$

$\langle \overline{\theta_v} \rangle =$  mixed-layer average of  $\overline{\theta_v}$

$\Delta \theta_s = \langle \overline{\theta_v} \rangle - \overline{\theta}_{vs} =$  SBL surface cooling (inversion strength)



## L'approccio esplicativo e la ricchezza delle grandezze utilizzate (umidità)

**Moisture:**  $q_*^{\text{ML}} = \overline{w'q'_s} / w_*$  = convective (ML) humidity scale

$q_*^{\text{SL}} = -\overline{w'q'_s} / u_*$  = surface-layer humidity scale

$q_{Lf} = \overline{w'q'} / w_{Lf}$  = local free-convection humidity scale

$q_L = -\overline{w'q'} / u_L$  = local humidity scale

$q_* = \overline{w'q'_s} / (\text{any other velocity scale})$



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$q_L = -\overline{w'q'} / u_L$  = local humidity scale

$q_* = \overline{w'q'_s} / (\text{any other velocity scale})$



## L'approccio esplicativo e la ricchezza delle grandezze utilizzate (tempo)

Time:	
	$1/f_c$ = inertial period, where $f_c$ is the Coriolis parameter
	$1/N_{BV}$ = buoyant period, where $N_{BV}$ is the Brunt-Väisälä frequency
	$1/f_{\max}$ = eddy period, where $f_{\max}$ is the frequency at the peak in the turbulence spectrum
	$t_*^{ML}$ = $z_i / w_*$ = convective (ML) time scale
	$t_*^{SL}$ = $z / u_*$ = surface-layer time scale
	$x/\bar{U}$ = time required for wind to move distance $x$



## Il Richardson number e la stabilità del ABL

In ABL stabili, dove la convezione non è spontanea per il gradiente termico verticale è

$$\frac{\partial \theta}{\partial z} > 0$$

Energia cinetica turbolenta



Energia potenziale

$$\frac{\partial(TKE/m)}{\partial t} = Ad + M + B + Tr - \varepsilon$$

Diagram illustrating the terms in the turbulent kinetic energy equation:

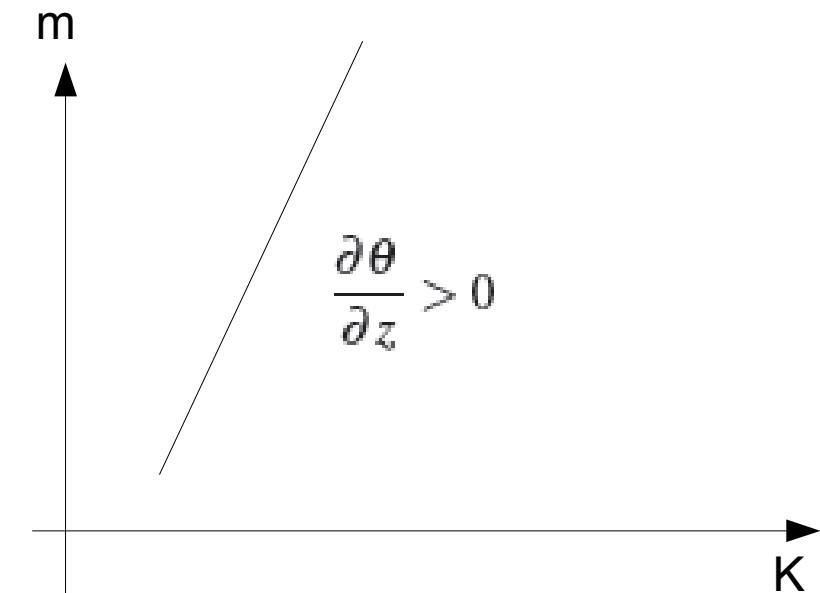
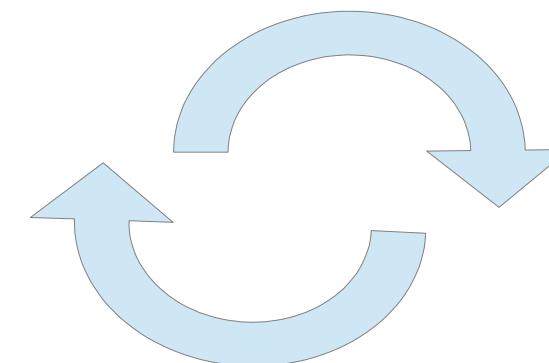
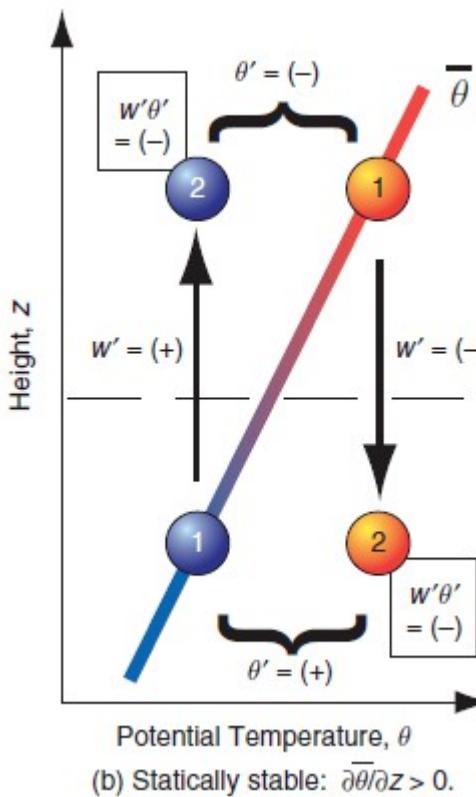
- Avvezione** (Ad): Represented by a blue box labeled  $M$ .
- Generazione meccanica (wind shear)**: Represented by a red box labeled  $B$ .
- Generazione o dissipazione per gravità (buoyancy)**: Represented by a red box labeled  $Tr$ .
- Dissipazione molecolare**: Represented by a black arrow pointing to the term  $\varepsilon$ .
- Trasporto turbolento**: Represented by a black arrow pointing to the term  $Tr$ .

## Il Richardson number e la stabilità del ABL

L'energia cinetica turbolenta viene utilizzata per spostare lungo la verticale masse d'aria in un ambiente stabile, quindi viene compiuto lavoro che va ad aumentare l'energia potenziale dell'aria

$$Ri = \frac{-B}{M} = \frac{\frac{g}{\bar{T}_v} \frac{\partial \bar{\theta}_v}{\partial z}}{\left( \frac{\partial \bar{u}}{\partial z} \right)^2 + \left( \frac{\partial \bar{v}}{\partial z} \right)^2}$$

Frequenza di Brunt Väisälä :





## Alcuni risultati sperimentali sul numero di Richardson

$$Ri = \frac{-B}{M} = \frac{\frac{g}{\bar{T}_v} \frac{\partial \bar{\theta}_v}{\partial z}}{\left( \frac{\partial \bar{u}}{\partial z} \right)^2 + \left( \frac{\partial \bar{v}}{\partial z} \right)^2}$$



(3.75). Laminar flow becomes turbulent when  $Ri$  drops below the critical value  $Ri_c = 0.25$ . Turbulent flow often stays turbulent, even for Richardson numbers as large as 1.0, but becomes laminar at larger values of  $Ri$ . The presence or absence of turbulence for  $0.25 < Ri < 1.0$  depends on the history of the flow: a behavior analogous to hysteresis. Flows for which  $Ri_c < 0.25$  are said to be *dynamically unstable*.



## Osservazione dell'instabilità di Kelvin-Helmholtz in atmosfera

When the shear in laminar flow across a density interface (e.g., between cold air below and warm air above) increases to the point at which the flow becomes dynamically unstable, the turbulence onset grows as a *Kelvin-Helmholtz (KH) instability* on the interface. First, small waves appear that grow in amplitude and curl over on themselves.





## Evoluzione dell'instabilità di Kelvin-Helmholtz

$t = 1$



$t = 2$



$t = 3$





## La velocità della scala convettiva e l'instabilità del ABL

In ABL instabili, dove la convezione è spontanea per il gradiente termico verticale è

$$\frac{\partial \theta}{\partial z} < 0$$

Energia cinetica turbolenta



Energia potenziale

$$\frac{\partial(TKE/m)}{\partial t} = Ad + M + B + Tr - \varepsilon$$

Avvezione      Generazione meccanica (wind shear)      Generazione o dissipazione per gravità (buoyancy)      Dissipazione molecolare

Trasporto turbolento





## ABL instabile: la velocità della scala convettiva

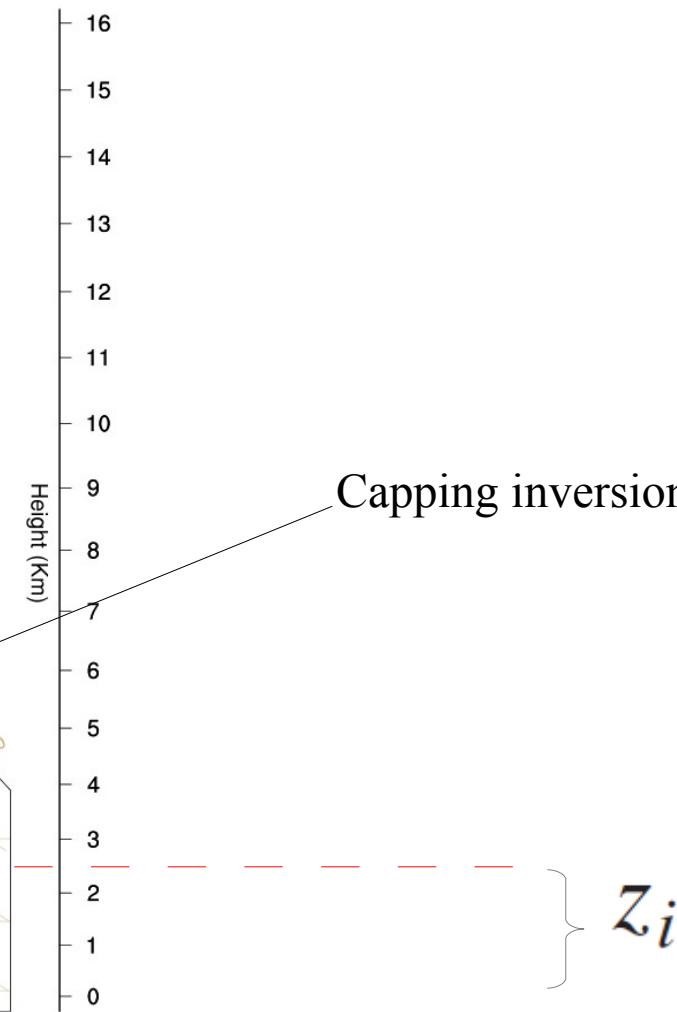
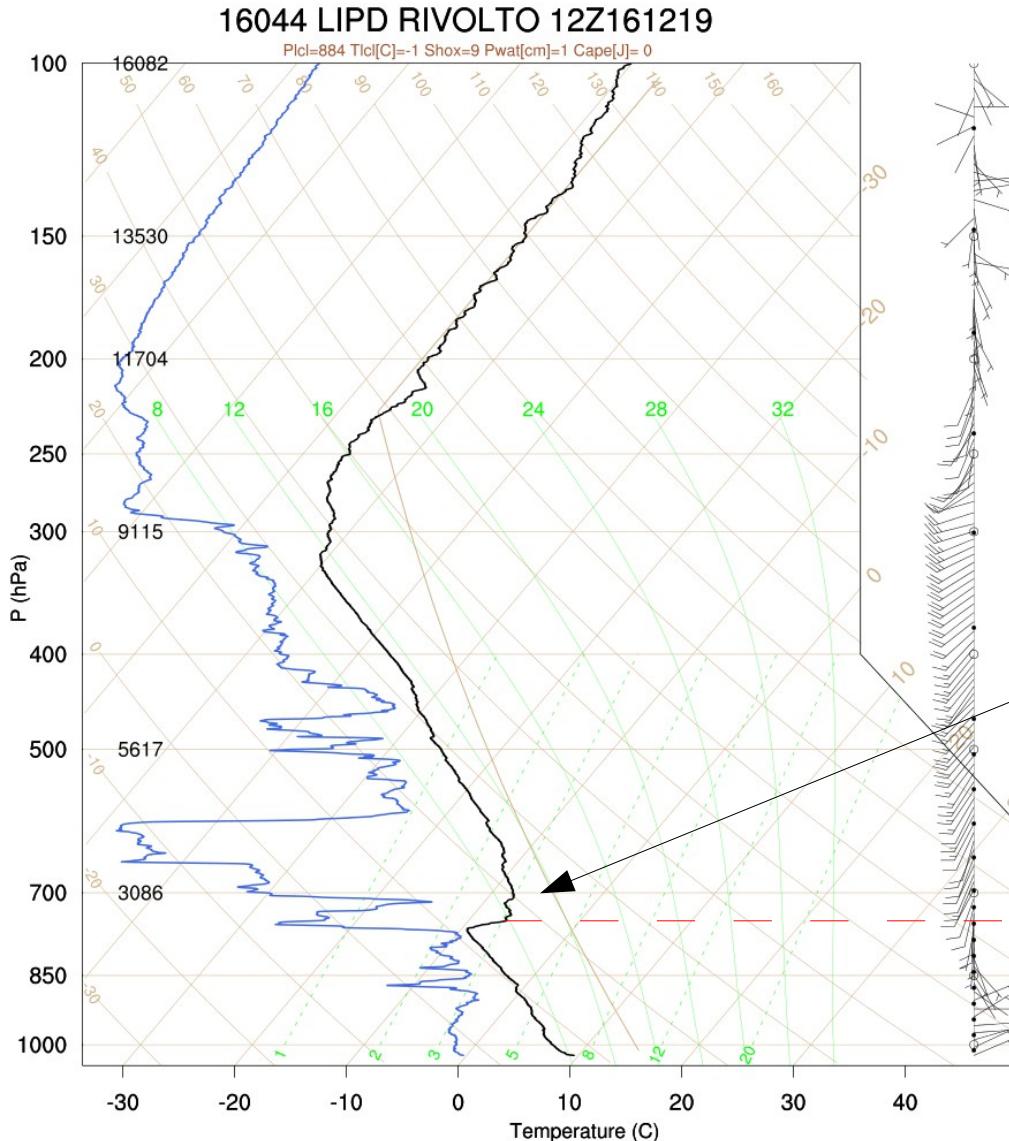
unstably stratified boundary layer is the *Deardorff velocity scale*

$$w_* = \left[ \frac{g \cdot z_i}{T_v} \overline{w' \theta'_s} \right]^{1/3} \quad (9.13)$$

where  $z_i$  is the depth of the boundary layer and the subscript  $s$  denotes at the surface. Values of  $w_*$  have been determined from field measurements and numerical simulations under a wide range of conditions. Typical magnitudes of  $w_*$  are  $\sim 1 \text{ m s}^{-1}$ , which corresponds to the average updraft velocities of thermals.

## ABL instabili; la capping inversion height

The altitude of the capping inversion,  $z_i$ , is the relevant length scale for the whole boundary layer for statically unstable and neutral conditions.





## La friction velocity e l'ABL neutro



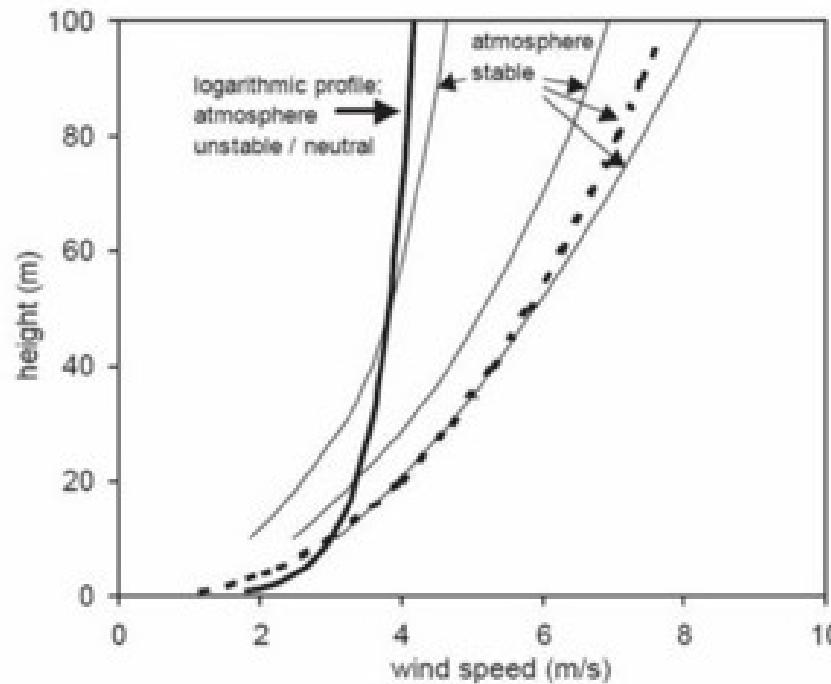
Another scale  $u_*$ , the *friction velocity*, is most applicable to statically neutral conditions in the surface layer, within which the turbulence is mostly mechanically generated. It is given by

$$u_* = \left[ \overline{u'w'}^2 + \overline{v'w'}^2 \right]^{1/4} = \left| \frac{\tau_s}{\rho} \right|^{1/2} \quad (9.14)$$

where  $\rho$  is air density,  $\tau_s$  is *stress* at the surface (i.e., drag force per unit surface area), and covariances  $\overline{u'w'}$  and  $\overline{v'w'}$  are the *kinematic momentum fluxes* (vertical fluxes of  $u$  and  $v$  horizontal momentum, respectively).

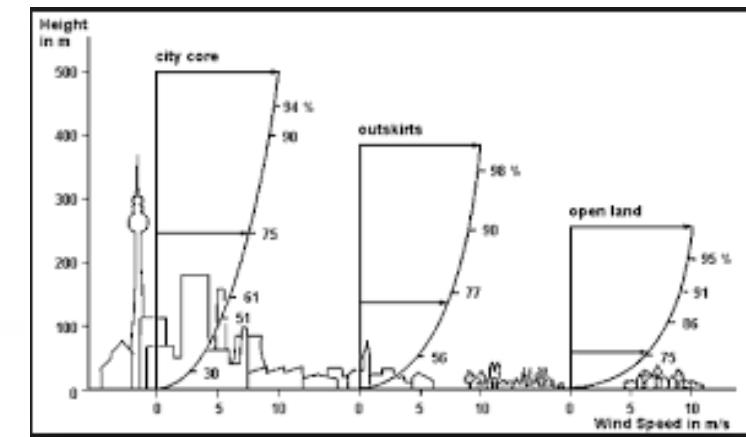


## Il profilo verticale dell'intensità del vento in ABL neutrali (ruolo della friction velocity)



$$u_z = \frac{u_*}{\kappa} \left[ \ln \left( \frac{z - d}{z_0} \right) + \psi(z, z_0, L) \right]$$

Where:



$u$  = windspeed ( $m s^{-1}$ )

$u^*$  = friction velocity ( $m s^{-1}$ )

$k$  = Von Karman's constant (0.4)

$z$  = height (m)

$d$  = zero-displacement height (m)

$z_0$  = roughness length

(after Oke, 1976)

$\psi(z, z_0, L)$  = stability term

## Lunghezze scala di particolare rilievo per il Surface Layer

bottom 5% of the boundary layer (referred to as the *surface layer*), an important length scale is the *aerodynamic roughness length*,  $z_0$ , which indicates the roughness of the surface (see Table 9.2)



**Table 9.2** The Davenport classification, where  $z_0$  is aerodynamic roughness length and  $C_{DN}$  is the corresponding drag coefficient for neutral static stability<sup>a</sup>

$z_0$ (m)	Classification	Landscape	$C_{DN}$
0.0002	Sea	Calm sea, paved areas, snow-covered flat plain, tide flat, smooth desert.	0.0014
0.005	Smooth	Beaches, pack ice, morass, snow-covered fields.	0.0028
0.03	Open	Grass prairie or farm fields, tundra, airports, heather.	0.0047
0.1	Roughly open	Cultivated area with low crops and occasional obstacles (single bushes).	0.0075
0.25	Rough	High crops, crops of varied height, scattered obstacles such as trees or hedgerows, vineyards.	0.012
0.5	Very rough	Mixed farm fields and forest clumps, orchards, scattered buildings.	0.018
1.0	Closed	Regular coverage with large size obstacles with open spaces roughly equal to obstacle heights, suburban houses, villages, mature forests.	0.030
$\geq 2$	Chaotic	Centers of large towns and cities, irregular forests with scattered clearings.	0.062

<sup>a</sup> From Preprints 12th Amer. Meteorol. Soc. Symposium on Applied Climatology, 2000, pp. 96–99.



## Lunghezze scala di particolare rilievo per il Surface Layer: la Obukhov length

roughness of the surface (see Table 9.2). For statically nonneutral conditions in the surface layer, there is an additional length scale, called the *Obukhov length*

$$L \equiv \frac{-u_*^3}{k \cdot (g/T_v) \cdot \left( \overline{w' \theta'} \right)_s}, \quad (9.15)$$

where  $k = 0.4$  is the von Karman constant. The absolute value of  $L$  is the height below which mechanically generated turbulence dominates.



## Sintesi sulle grandezze scala più importanti nei diversi regimi di ABL

Per gli ABL **instabili** le grandezze scala più importanti sono:

- La convective scale velocity  $w_*$
- La capping inversion  $z_i$

Per i surface layer **neutri** le grandezze scala più importanti sono:

- La friction velocity  $u_*$
- La roughness  $z_0$

Per i surface layer **non neutri** le grandezze scala sono:

- La friction velocity  $u_*$
- La roughness  $z_0$
- La lunghezza di Obukhov  $L$



## Bibliografia di riferimento

**The Atmospheric Boundary Layer**, Garratt J. R., Cambridge University Press -

**An Introduction to Boundary Layer Meteorology**, Stull R. B., Kluwer Academic Publisher

**Scaling**, Barenblatt, G. I. Cambridge University Press – 2003

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