Sommatorie

Dati n numeri $a_1, \dots, a_n$

$$\sum_{j=1}^{n} a_j = a_1 + \dots + a_n$$

Per $n=1$ ho solo a_1

$$\sum_{j=1}^{1} a_j = a_1$$

c'è la convenzione che se

$$\sum_{j=3}^{2} a_j = 0$$

o, più in generale,

se $n < m$, $\sum_{j=m}^{n} a_j = 0$

$\sum_{j=1}^{n} a_j$ $\cancel{=}$ può essere definito per induzione nel modo seguente:

$$\begin{cases} \sum_{j=1}^{1} a_j = a_1 \\ \text{dato } \sum_{j=1}^{n} a_j \text{ e dato } a_{n+1} \end{cases}$$

allora $\displaystyle\sum_{j=1}^{n+1} a_j \doteq \sum_{j=1}^{n} a_j + a_{n+1}$

$$\sum_{j=1}^{n} a_j = a_1 + \cdots + a_n$$

Dati $a_1, \dots, a_n \in \mathbb{R}$

$$\prod_{j=1}^{n} a_j = a_1 \cdots a_n$$

Proprietà delle sommatorie

1) $$\sum_{j=1}^{n} (a_j + b_j) = \sum_{j=1}^{n} a_j + \sum_{j=1}^{n} b_j$$

2) $$\sum_{j=1}^{n} c\, a_j = c \sum_{j=1}^{n} a_j$$

3) Sia $1 \leq m < n$, allora
$$\sum_{j=1}^{n} a_j = \sum_{j=1}^{m} a_j + \sum_{j=m+1}^{n} a_j$$

<u>Teor</u> (somma aritmetica)

$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$$

<u>Dim</u>

$(P_n)\quad \sum_{j=1}^{n} j = \frac{n(n+1)}{2}$. Induzione

1) $n=1$ $\qquad \sum_{j=1}^{1} j = j\big|_{j=1} = 1$

$$\frac{n(n+1)}{2}\Big|_{n=1} = 1 \quad ✓$$

2) Assumiamo (P_n) vera e dimostriamo (P_{n+1})

$$\sum_{j=1}^{n+1} j = \frac{(n+1)(n+2)}{2}$$

$$\sum_{j=1}^{n+1} j = \sum_{j=1}^{n} j + n+1 = \frac{n(n+1)}{2} + n+1 =$$

$$= (n+1)\left(\frac{n}{2}+1\right) = (n+1)\,\frac{n+2}{2}$$

$3+(n-2) = n+1$

$$1 + 2 + 3 + 4 + \dots + (n-3) + (n-2) + (n-1) + n$$

$4+n-3 = n+1$

$2+(n-1) = n+1$

$1+n = n+1$

Teor (Somma geom. di ragione r). Dato $r \in \mathbb{R}$, $r \neq 1$

$$\sum_{j=0}^{n} r^j = \frac{1-r^{n+1}}{1-r}$$