$$\sum_{j=0}^{n} r^j = \frac{1-r^{n+1}}{1-r} \qquad P_n$$

Dim. per induzione

1) Verificare che P_0 è vera ✓

$$\sum_{j=0}^{0} r^j = r^0 = 1$$

$$\frac{1-r^{0+1}}{1-r} = \frac{1-r}{1-r} = 1$$

2) Assumiamo

$$\sum_{j=0}^{n} r^j = \frac{1-r^{n+1}}{1-r} \qquad P_n$$

e dimostriamo

$$\sum_{j=0}^{n+1} r^j = \frac{1-r^{n+2}}{1-r} \qquad P_{n+1}$$

$$\sum_{j=0}^{n+1} r^j = \sum_{j=0}^{n} r^j + r^{n+1} = \frac{1-r^{n+1}}{1-r} + r^{n+1}$$

$$= \frac{1-r^{n+1}+r^{n+1}(1-r)}{1-r}$$

$$= \frac{1-\cancel{r^{n+1}}+\cancel{r^{n+1}}-r^{n+2}}{1-r}$$

$$= \frac{1-r^{n+2}}{1-r}$$

Coefficienti binomiali

Per ogni $n \in \mathbb{N} \cup \{0\}$ definiamo

$$n! = \begin{cases} 1 & \text{se } n = 0 \\ 1 \ldots n & \text{se } n \in \mathbb{N}. \end{cases}$$

n fattoriale

Es. $1! = 1$

$2! = 1 \cdot 2 = 2$

$3! = 1 \cdot 2 \cdot 3 = 6$

$4! = 1 . 2 . 3 . 4 = 3! . 4 = 24$

Dati $0 \le k \le n$, n e k interi,

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Siano $n > k > 1$, osserviamo

$$\frac{n!}{k!\,(n-k)!} = \frac{n(n-1)\cdots(n-k+1)\overbrace{\cancel{(n-k)(n-k-1)\cdot\cdots 2\cdot 1}}^{(n-k)!}}{k!\,\cancel{(n-k)!}}$$

$$= \frac{n(n-1)\cdots(n-k+1)}{k!}$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} \qquad n \geq k \geq 0$$

$$\binom{n}{0} = \frac{n!}{0!\;n!} = \frac{\cancel{n!}}{1\;\cancel{n!}} = 1$$

$$\binom{n}{1} = \frac{n!}{(n-1)!} = \frac{n\;\cancel{(n-1)!}}{\cancel{(n-1)!}}$$

$$= n$$

$$\binom{n}{k} = \frac{n!}{k!\;(n-k)!}$$

$$\binom{n}{n-k} = \frac{n!}{(n-k)!\;(n-(n-k))!}$$

$$= \frac{n!}{(n-k)!\;\;k!}$$

$$\binom{n}{k} = \binom{n}{n-k} \qquad n$$

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\begin{cases} \binom{n}{0} = 1 \\ \binom{n}{n} = 1 \end{cases} \qquad \begin{cases} \binom{n}{1} = n \\ \binom{n}{n-1} = n \end{cases}$$

<u>Teor</u> (Newton) $\forall\, a, b \in \mathbb{R}$ e per $n \in \mathbb{N}$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

$$= \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$\boxed{\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}}$$

$$(a+b)^1 = a+b = \binom{1}{0}a + \binom{1}{1}b$$

$$\begin{array}{ccccccccccccc}
 & & & & & & 1 & & & & & & \\
 & & & & & 1 & & 1 & & & & & \\
 & & & & 1 & & 2 & & 1 & & & & \\
 & & & 1 & & 3 & & 3 & & 1 & & & \\
 & & 1 & & 4 & & 6 & & 4 & & 1 & & \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1
\end{array}$$

Numeri Complessi.

$\mathbb{N} \quad \mathbb{Z} \quad \mathbb{Q} \quad \mathbb{R} \quad \mathbb{C}$

$$\underline{\mathbb{C}} = \{ (x,y) : x, y \in \mathbb{R} \} = \mathbb{R} \times \mathbb{R}$$
$$= \underline{\mathbb{R}^2}$$

In $\mathbb{C}$ introduciamo una somma ed un prodotto

$$(x,y) + (u,v) = (x+u, y+v)$$
$$(x,y) \cdot (u,v) = (xu - yv, xv + yu)$$

$$(x,y)+(u,v)=(x+u,\ y+v)$$
$$(x,y)(u,v)=(xu-yv,\ xv+yu)$$

$$(x,0)+(u,0)=(x+u,\ 0)$$
$$(x,0)(u,0)=(xu,\ 0)$$

Se noi identifichiamo $\mathbb{R}$ con l'asse delle x

$$x \longrightarrow (x,0)$$
$$x_1+x_2 \longrightarrow (x_1+x_2,\ 0)$$
$$x_1 x_2 \longrightarrow (x_1 x_2,\ 0)$$

$$(x,y)(u,v) = (xu - \cancel{yv}, xv + \cancel{yu})$$

$$x(u,v) = (xu, xv)$$

$$(x,0)(u,v) = (xu, xv)$$

D'ora innanzi i vettori

$$(x,0) = x \,.$$

$$(x,0) + (u,v) = (x+u, v)$$

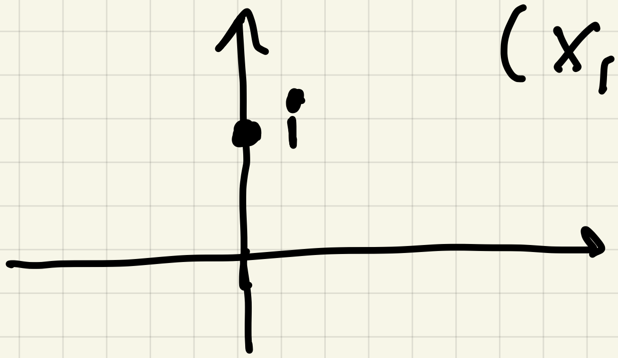
$$x + (u,v) = (x+u, v)$$

$$(x,y) = (x,0) + (0,y)$$

$$= x + y(0,1) \qquad i = (0,1)$$

$$= x + y\, i$$

$$= x + iy = (x,y)$$



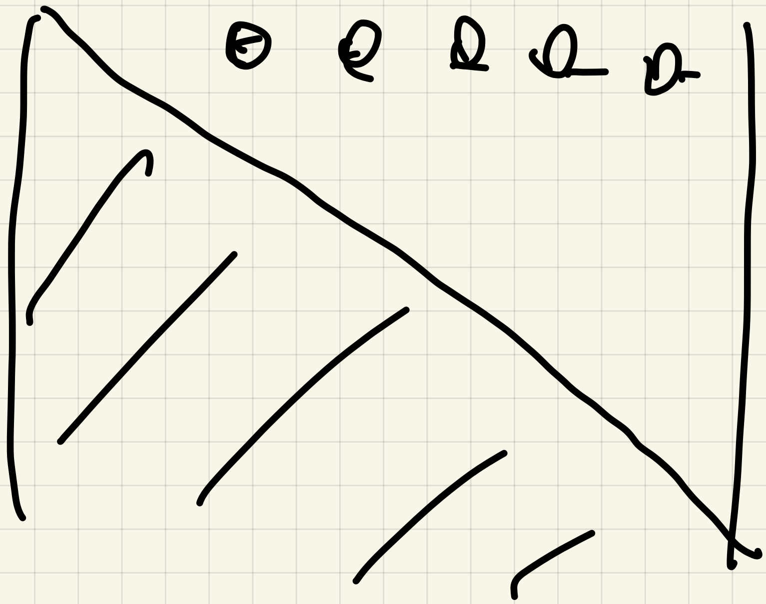
$(x,y)(u,v)=$
$=(xu-yv,\ xv+yu)$

$$i^2 = i\ i = (0,1)(0,1) = (-1,0) = -1$$

$$i^2 = -1$$

$$(x,y) = x+iy = z \qquad z$$

L'equazione $z^2=-1$ ha soluzione tra i numeri complessi.



$$z^2 + 1 = 0$$

$$z = \pm i$$