Image Processing for Physicists

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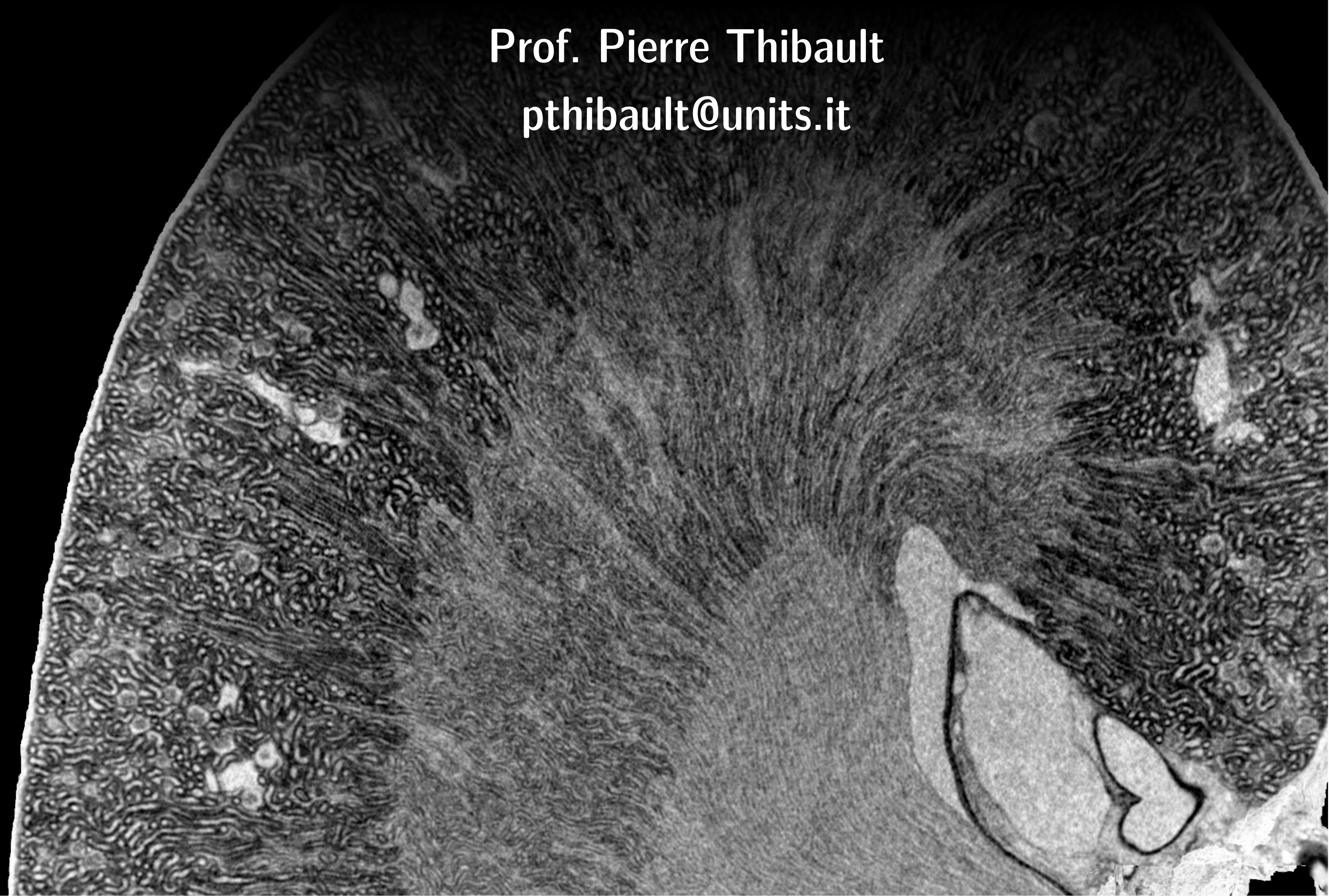


Overview

- coordinate transformations
  - translation, rotation, shear, ...

Geometry

- intensity transformations
  - normalization, gamma, thresholding, ...

- image analysis using morphological operations
  - dilation, erosion, opening, closing, ...

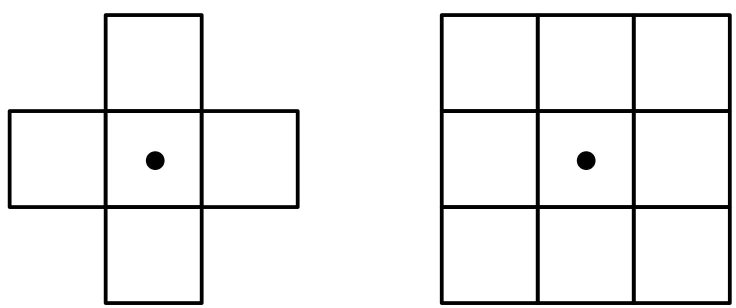
binary images

- image segmentation
  - by morphology, intensity, region, ...

General image transformations

- coordinate transformations → pixel positions (→ interpolation)
- intensity transformations
- pixel-wise transformations
- neighborhood transformations

morphology

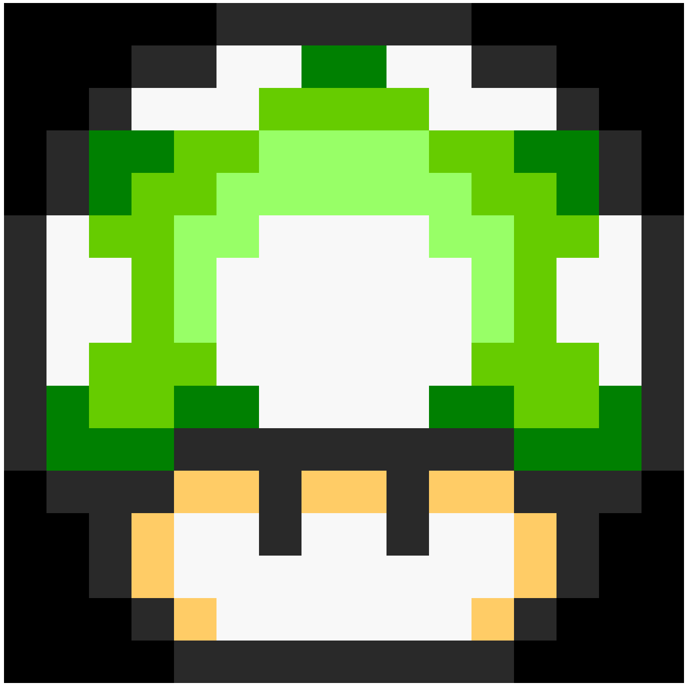


4-neighborhood 8-neighborhood

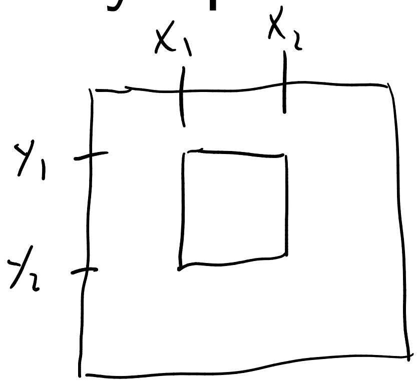
General image transformations

- images as an array

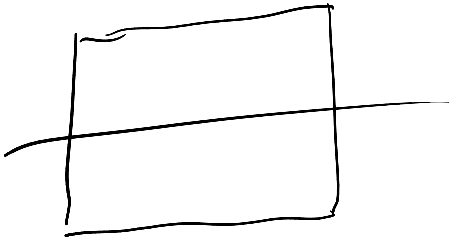
$M \times N$ ($\times$ channels)

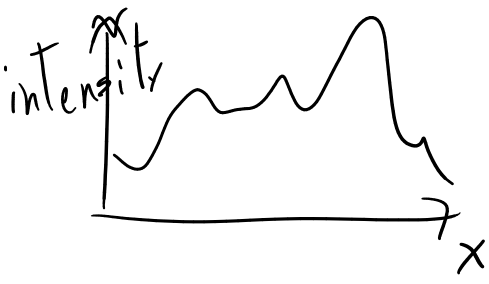


- sub array operations



- line extractions





General image transformations

- element wise addition

$$I = I_1 + I_2$$

$$I(m,n) = I_1(m,n) + I_2(m,n)$$

- element wise multiplication

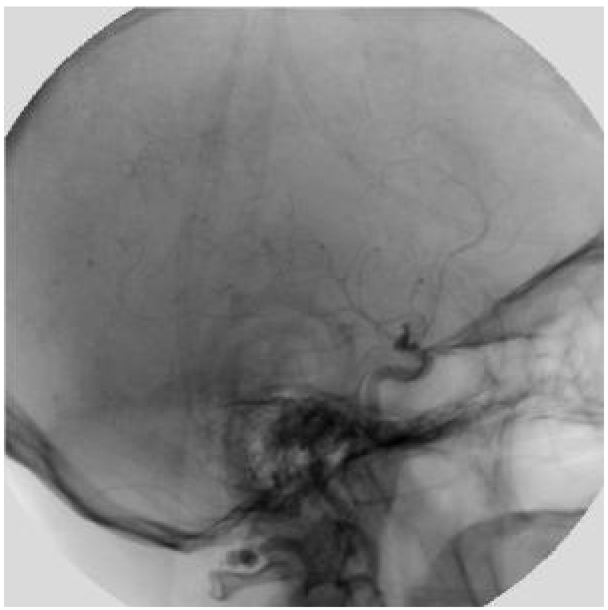
$$I = I_1 \cdot I_2$$

$$I(m,n) = I_1(m,n) \cdot I_2(m,n)$$

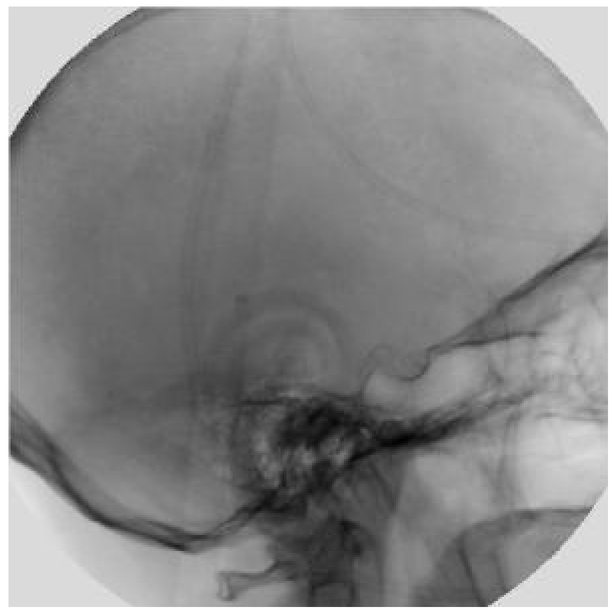
Image Subtraction Example

motion artifact

- Digital Subtraction Angiography
- Xray images before/after contrast agent



-



=

Live or contrast image

Mask image

DSA image

Source: Gonzales, Digital Image Processing

Image Addition Example

- Add multiple noisy images of same object
- (More on noise in later lectures)

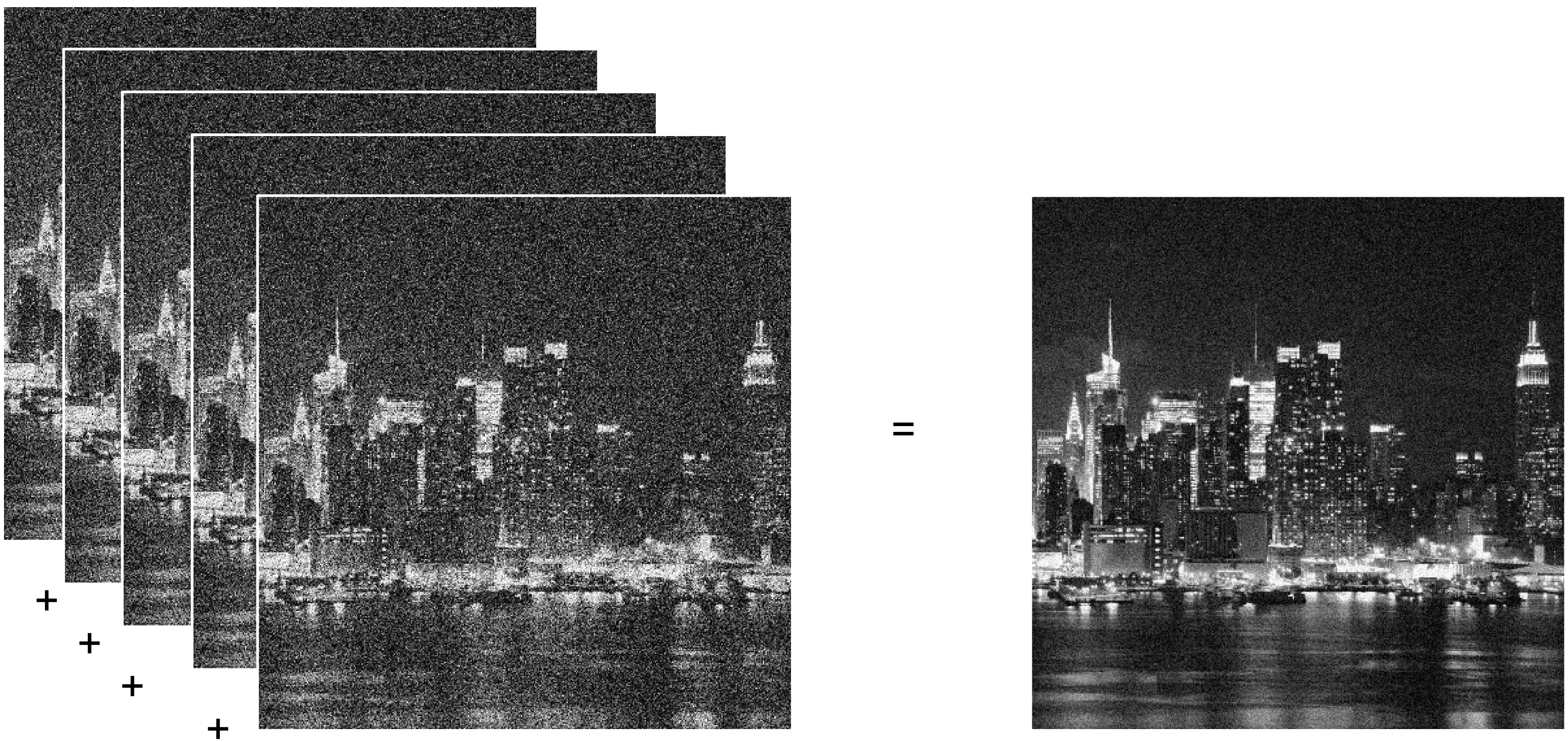


Image Multiplication Example

a b c

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Source: Gonzales, Digital Image Processing

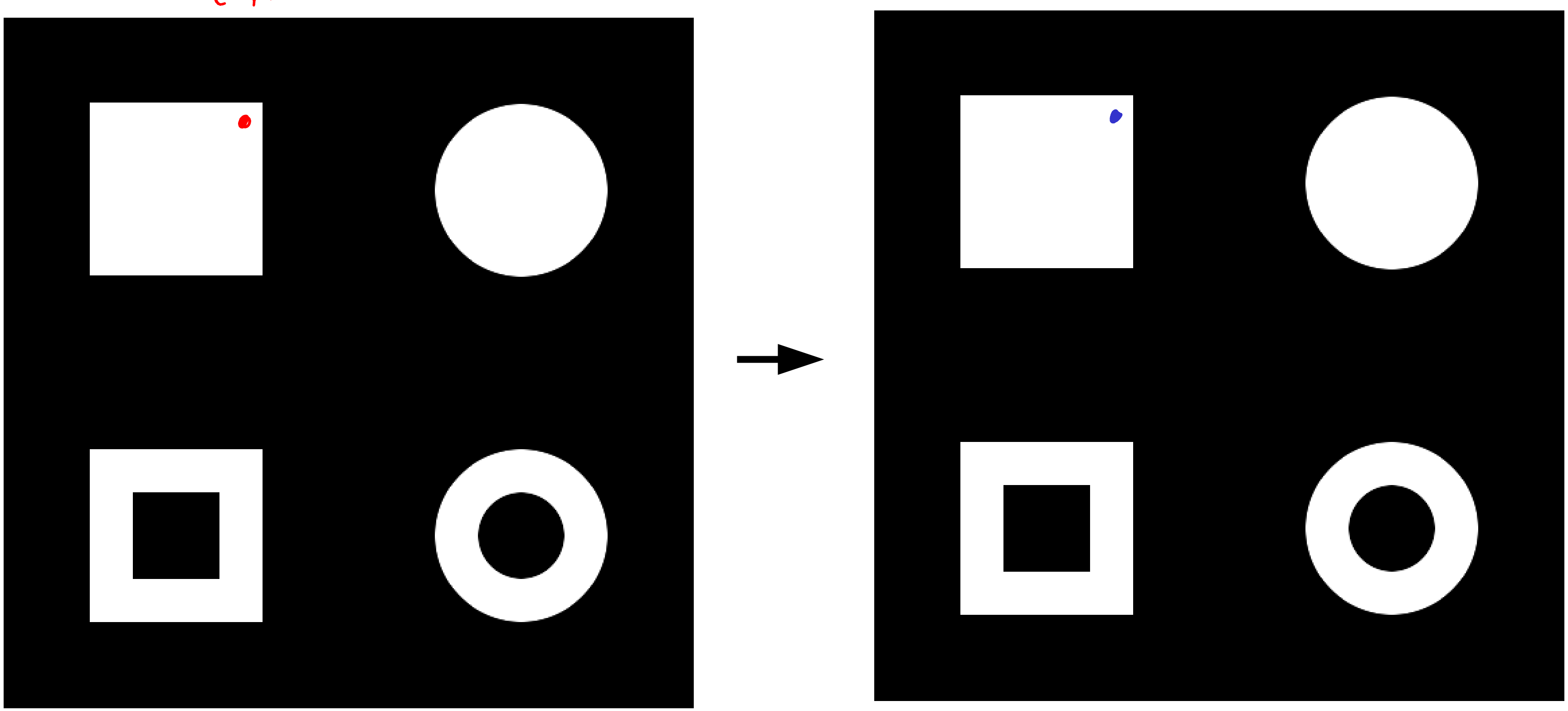
$\boxed{\text{Affine}}$ transformations

- identity

$$\boxed{\begin{pmatrix} x' \\ y' \end{pmatrix}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \boxed{\begin{pmatrix} x \\ y \end{pmatrix}}$$

(x, y)

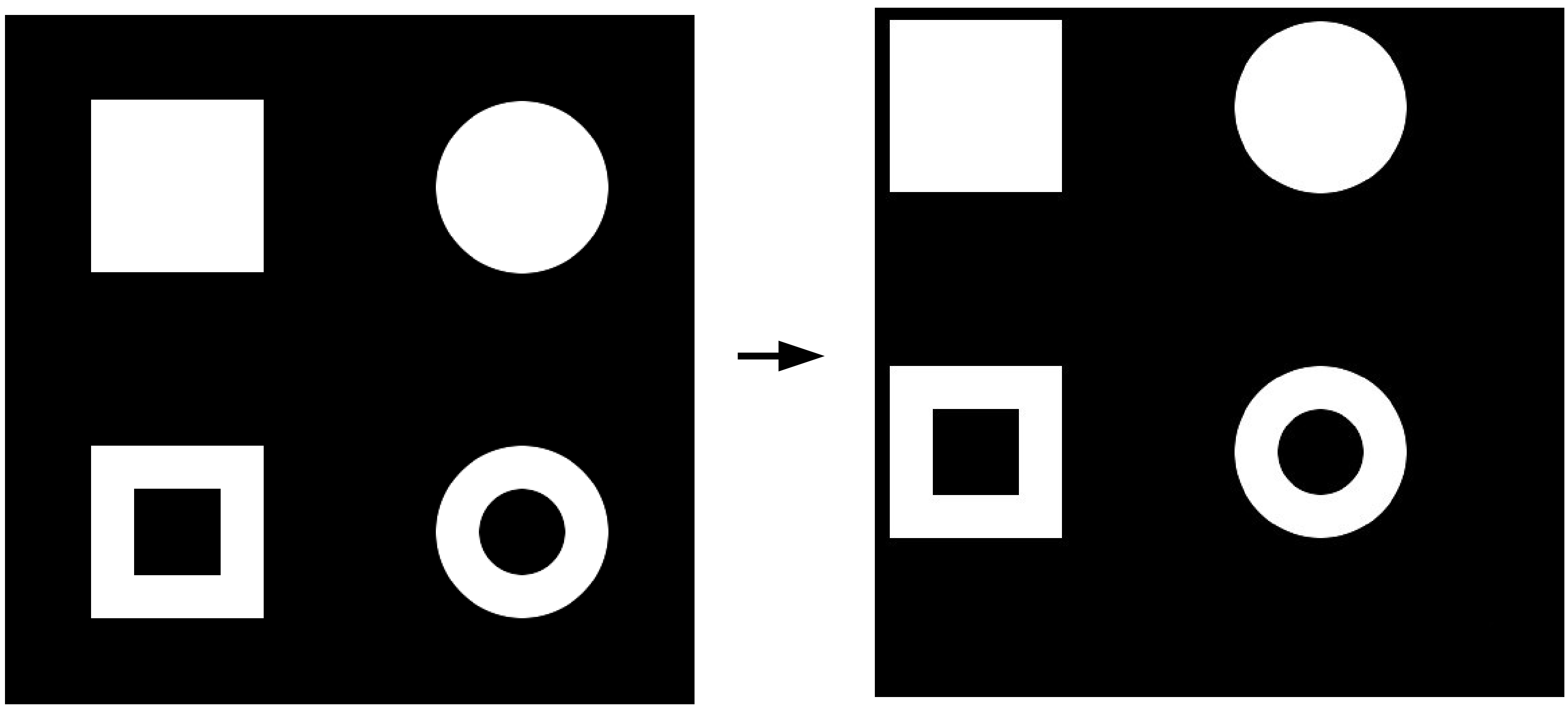
(x', y')



Affine transformations

- translation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \overbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}^{\text{transformation matrix}} \begin{pmatrix} x \\ y \end{pmatrix} + \overbrace{\begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}}^{\text{offset}}$$

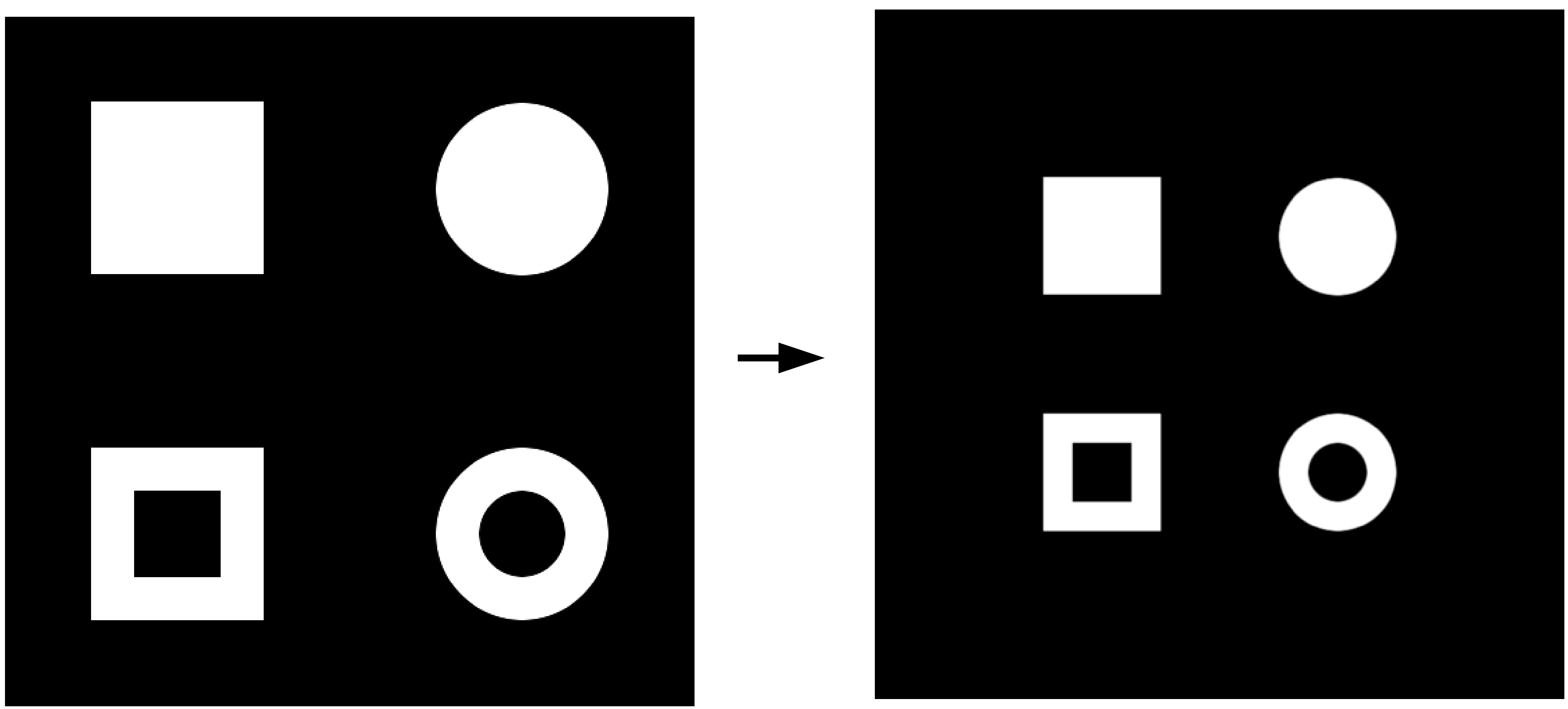


Affine transformations

- scaling

$$\begin{vmatrix} x' \\ y' \end{vmatrix} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{vmatrix} x \\ y \end{vmatrix}$$

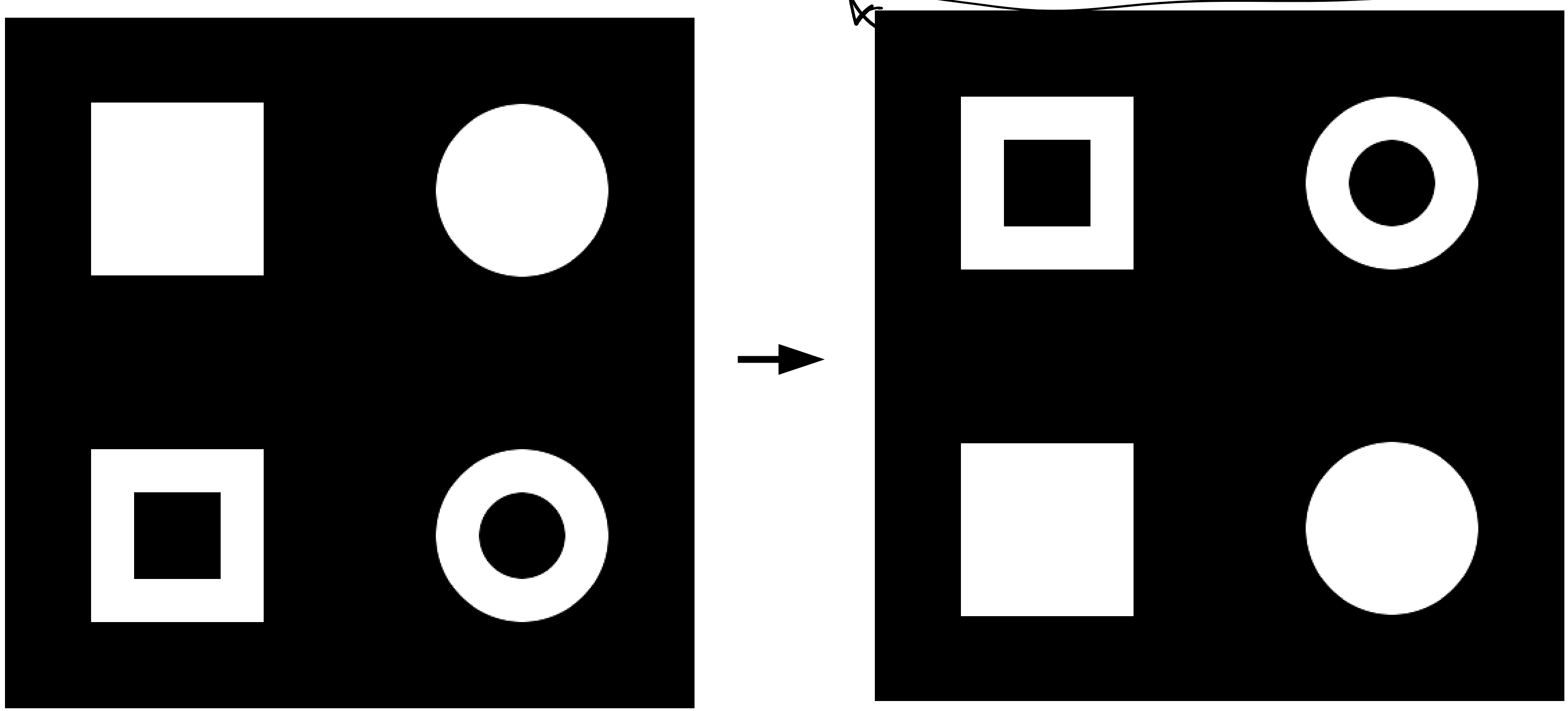
here $a = b = \frac{1}{2}$



Affine transformations

- reflections

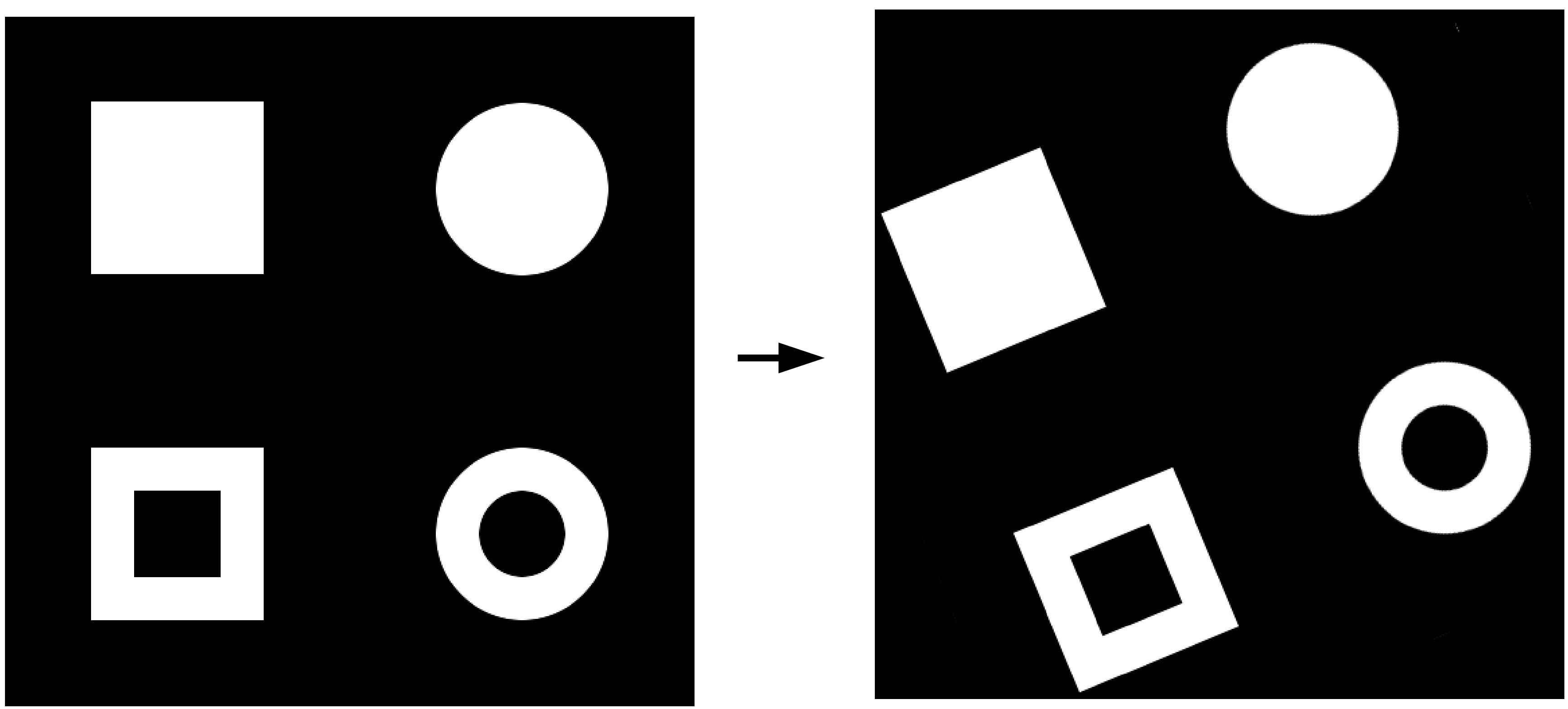
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Affine transformations

- rotation

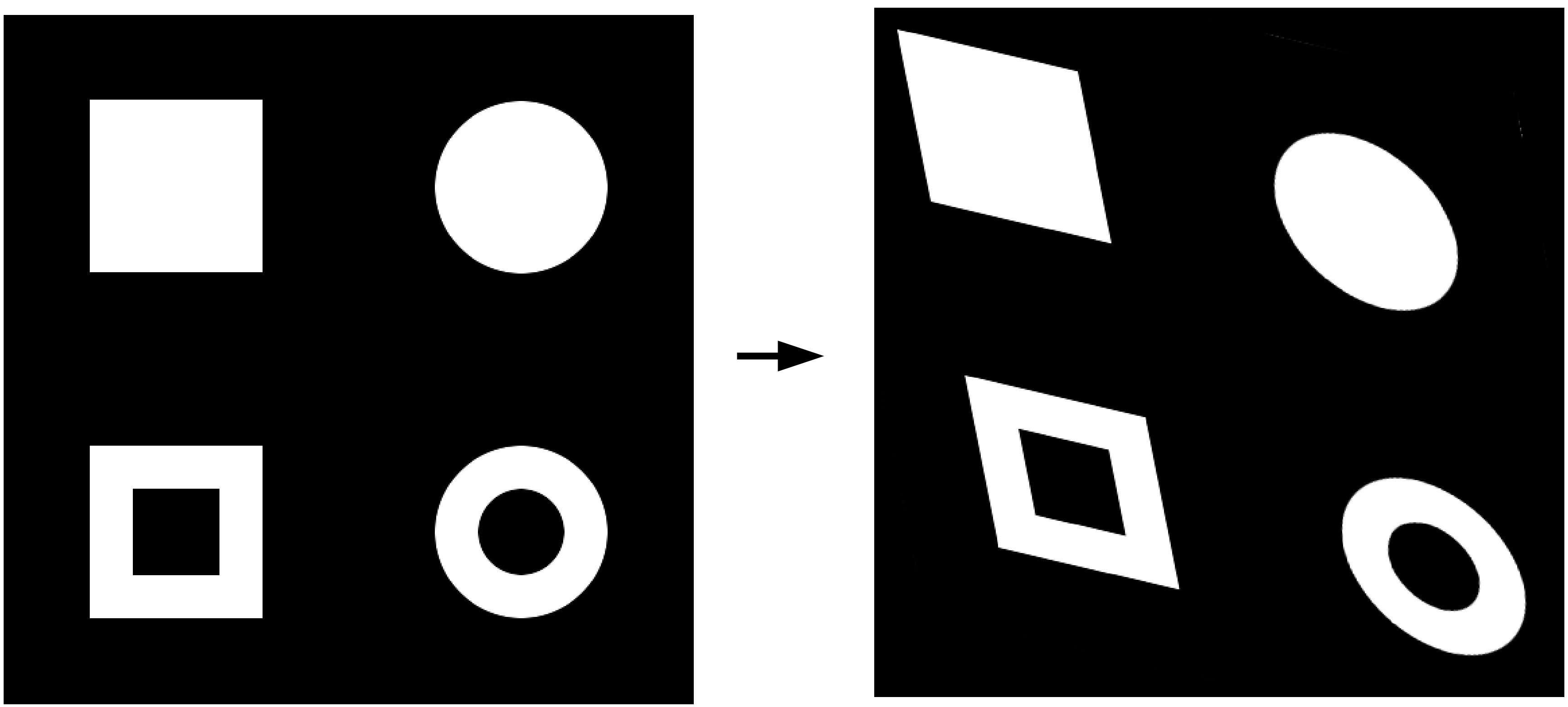
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Affine transformations

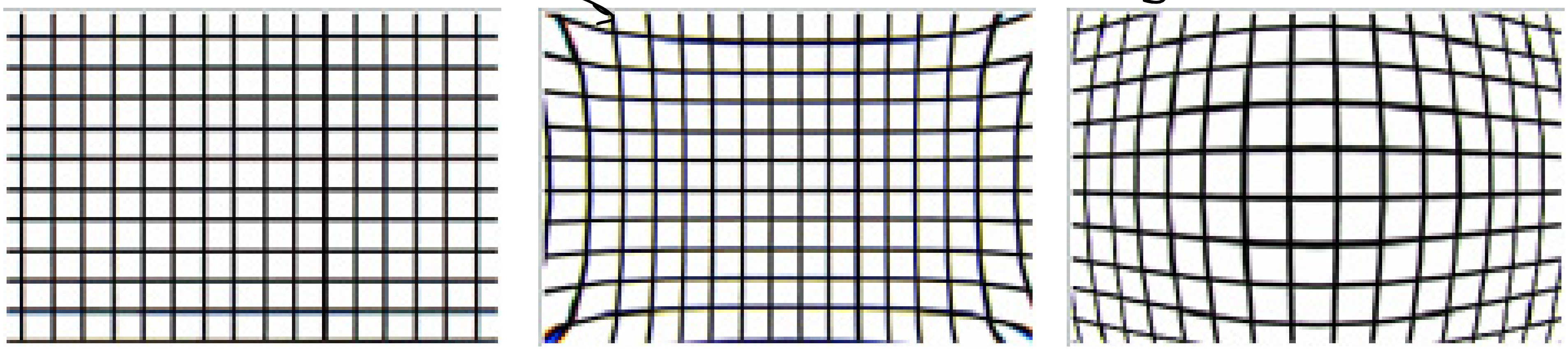
- shear

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Nonlinear coordinate transformation

- pincushion and barrel distortion

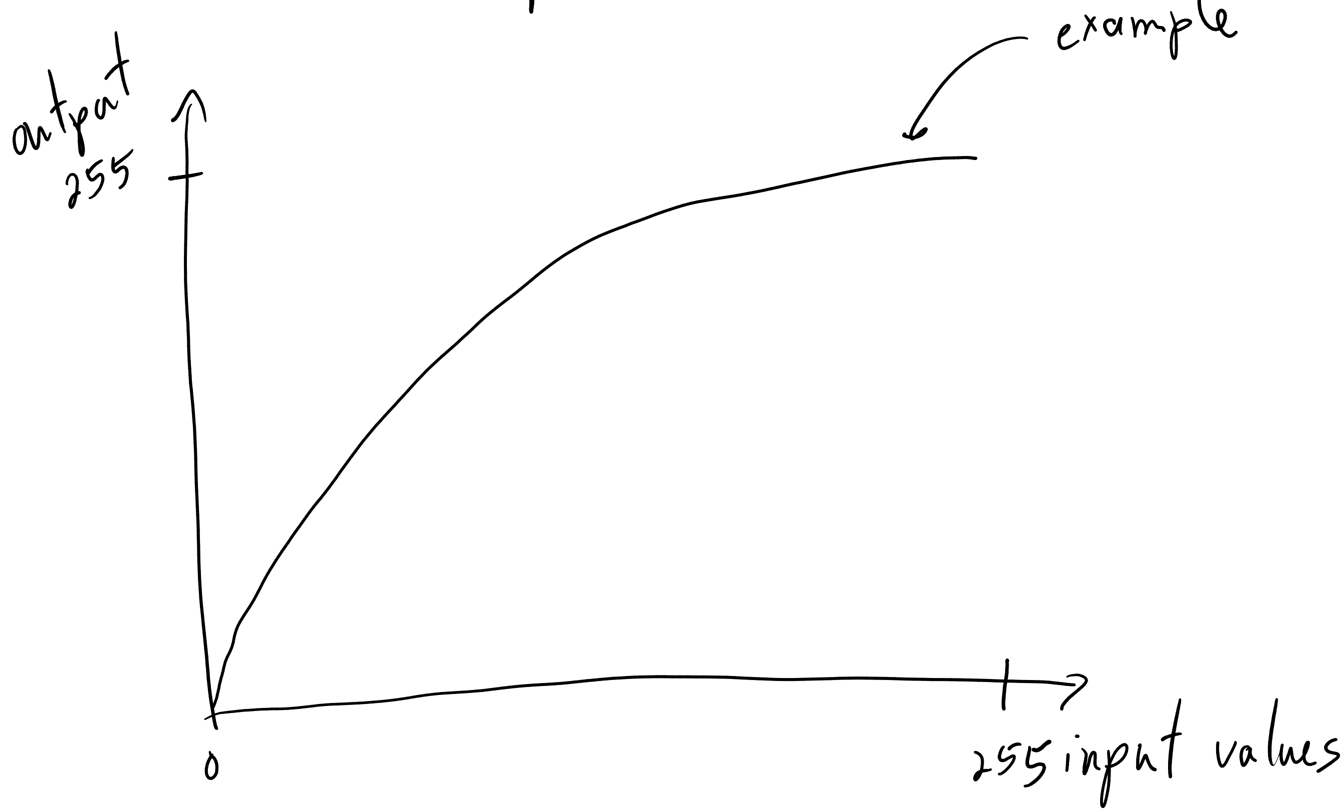


- mapping depends on radial distance from centre

$$\boxed{\begin{aligned} x' &= x_0 + ax + by \\ y' &= y_0 + cx + dy \\ & \text{Affine} \end{aligned}} \quad \begin{aligned} &+ ex^2 + fxy + gy^2 + \ldots \\ &+ hx^2 + ixy + jy^2 + \ldots \end{aligned}$$

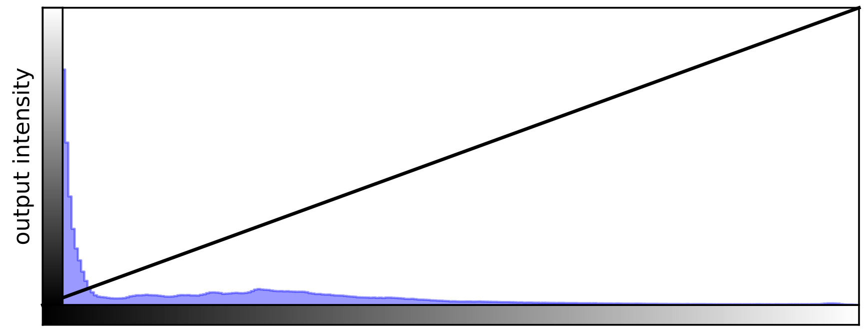
Intensity mapping

Transformation of pixel values

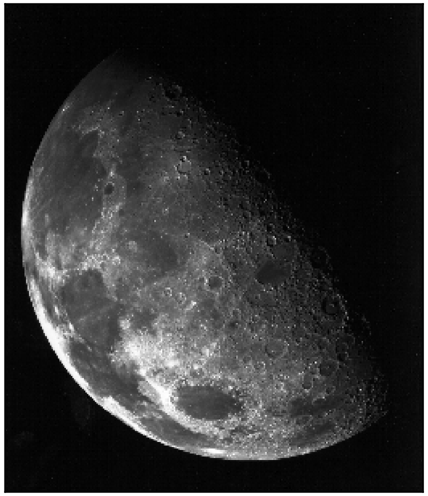


Intensity mappings

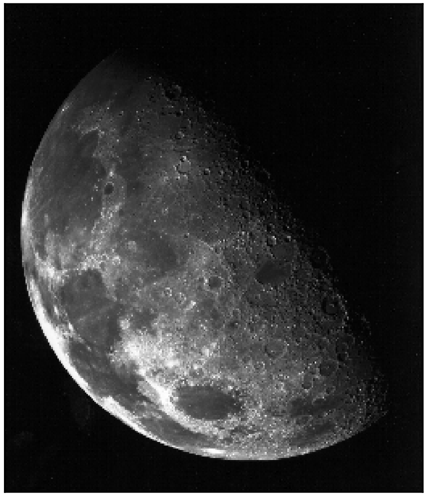
Identity



output = input

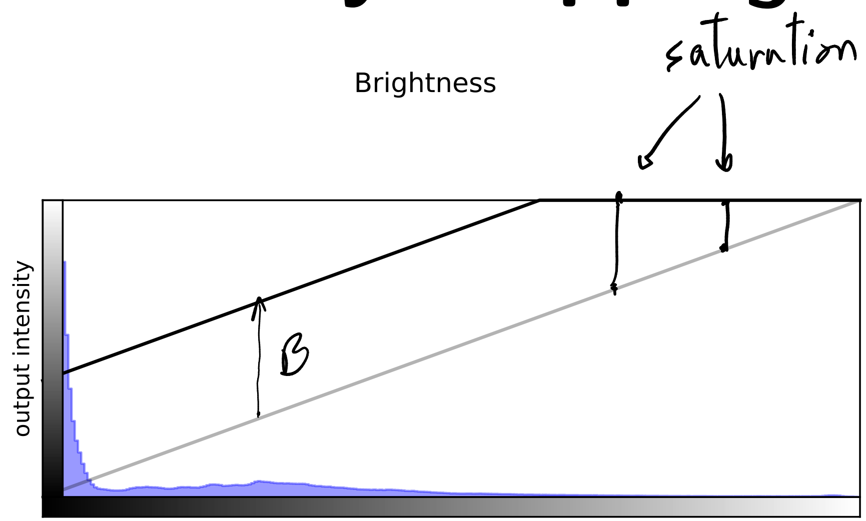


original

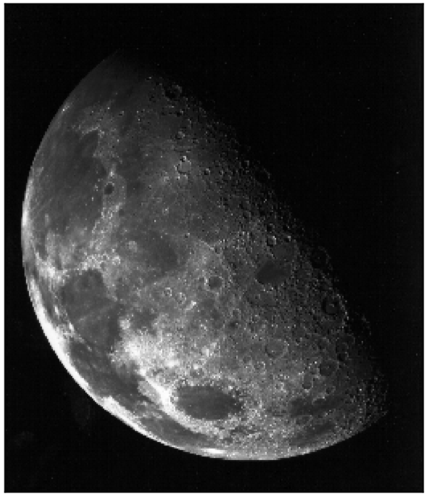


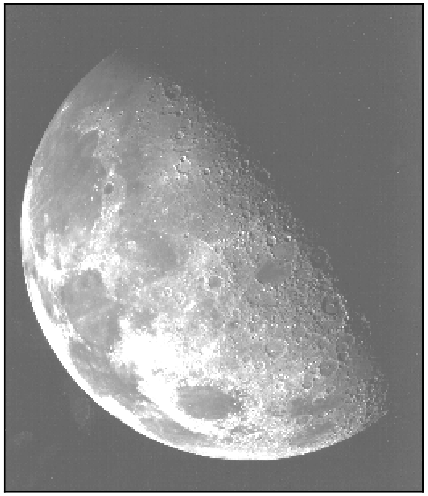
remapped

Intensity mappings



output = input + B


original


remapped

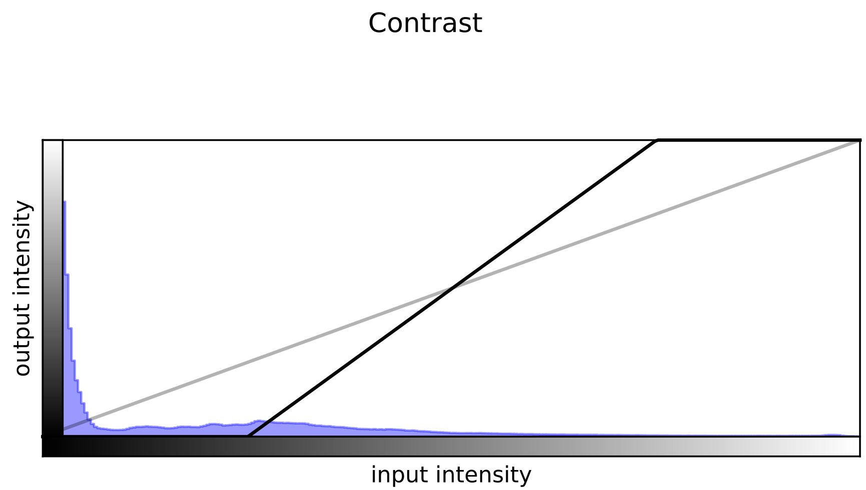
Intensity mappings

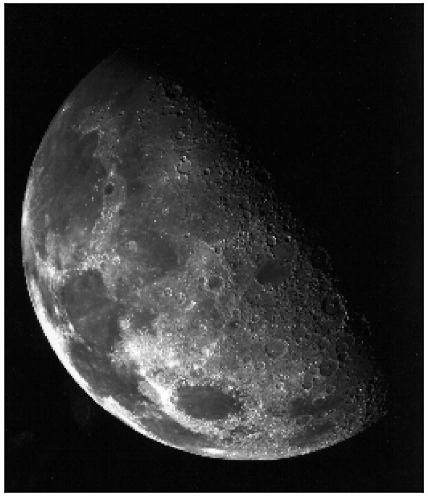
middle of dynamic range

output =

C(input − 128)

+ 128

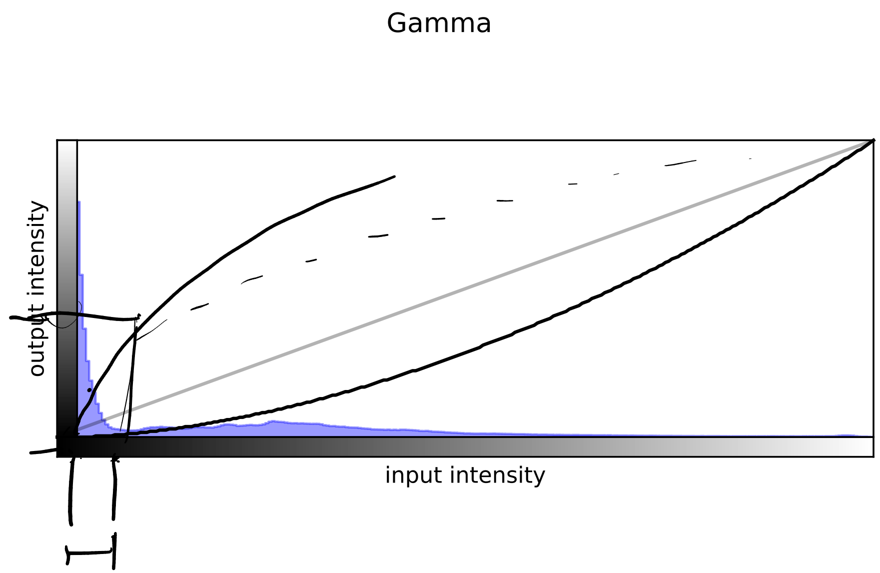




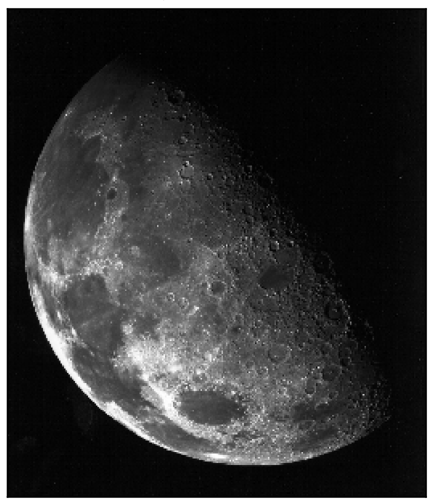
original

remapped

Intensity mappings



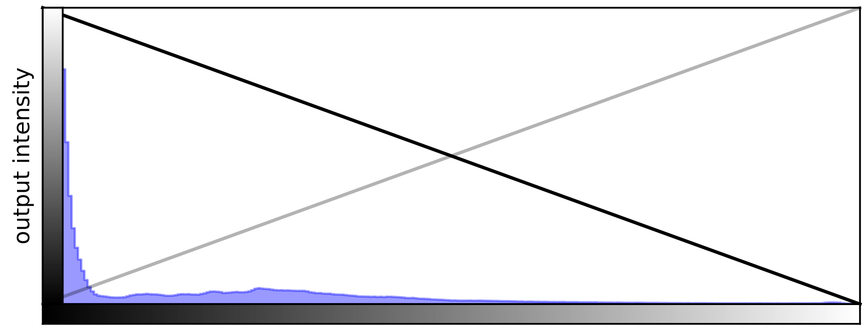
$$\text{output} = 255 \cdot \left(\frac{\text{input}}{255}\right)^{\gamma}$$


original


remapped

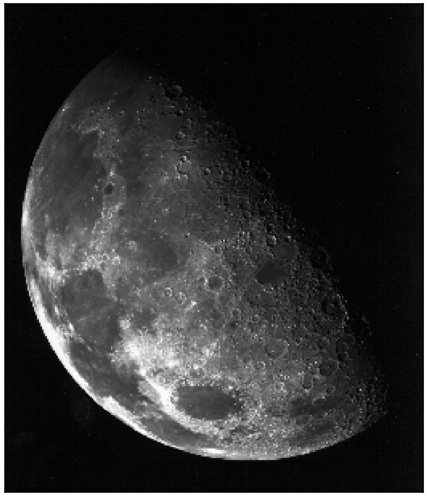
Intensity mappings

Inversion

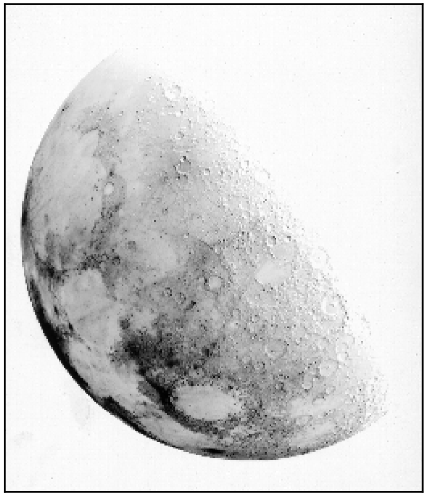


output=

255 - input



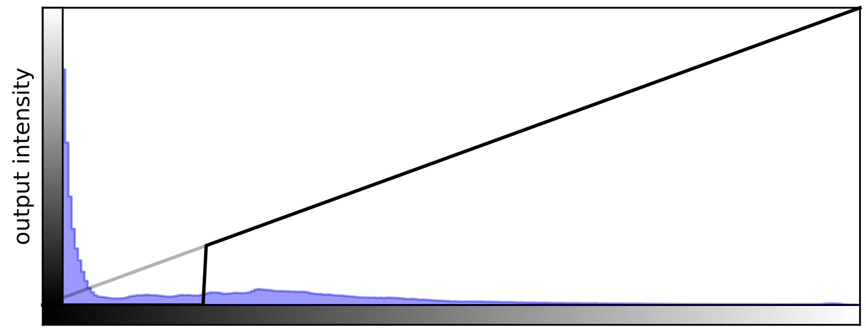
original

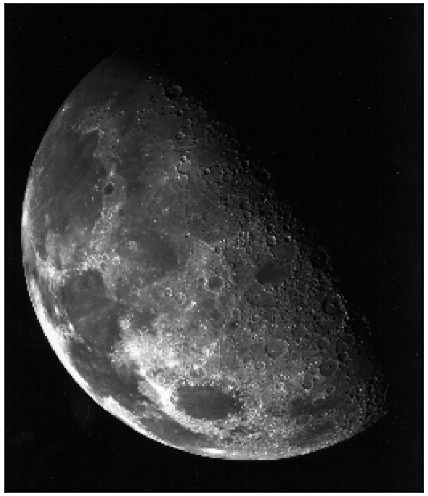


remapped

Intensity mappings

Threshold



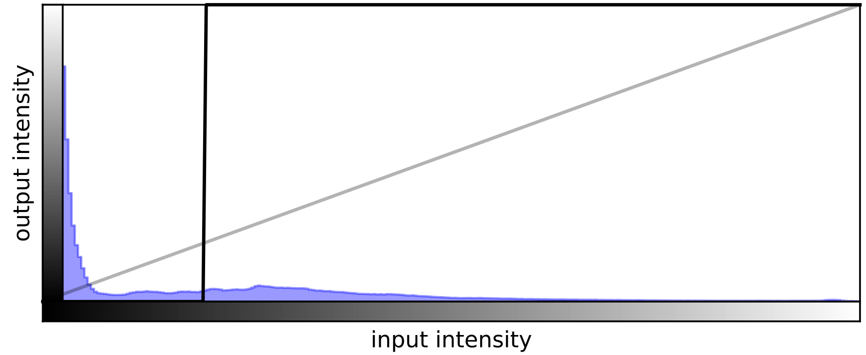


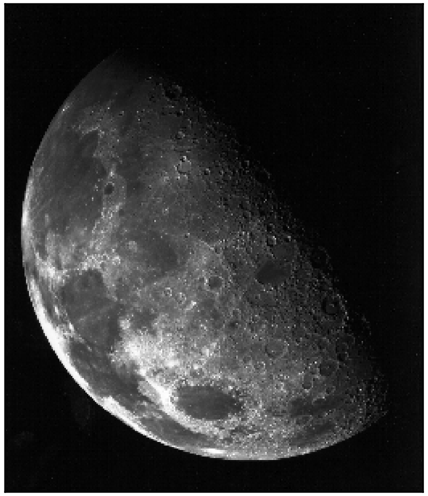
original

remapped

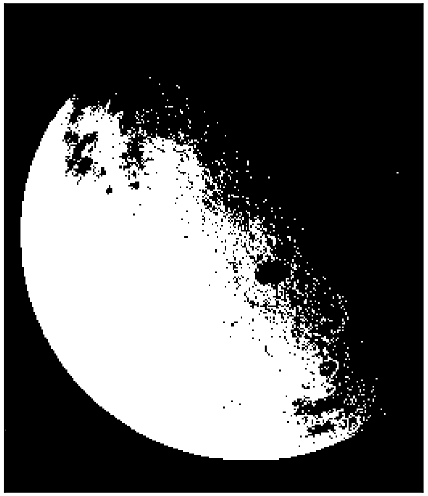
Intensity mappings

Binary threshold





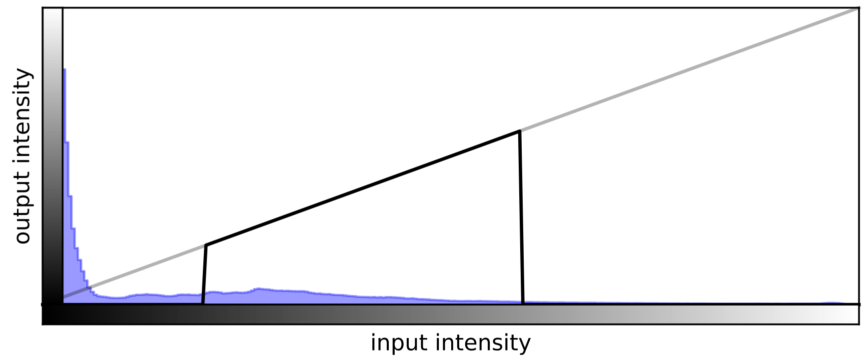
original

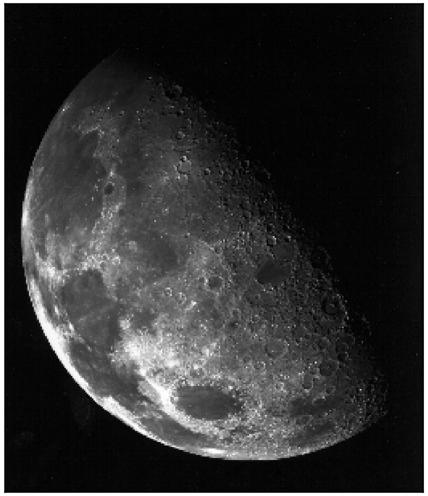


remapped

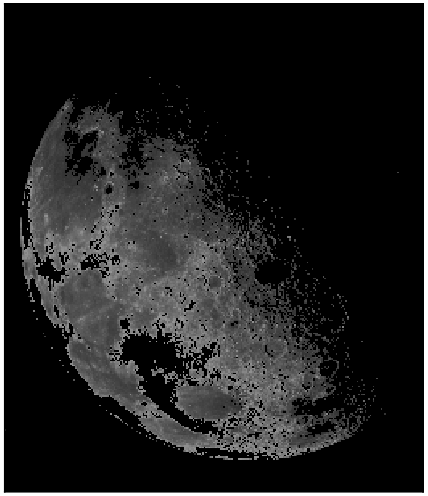
Intensity mappings

Window





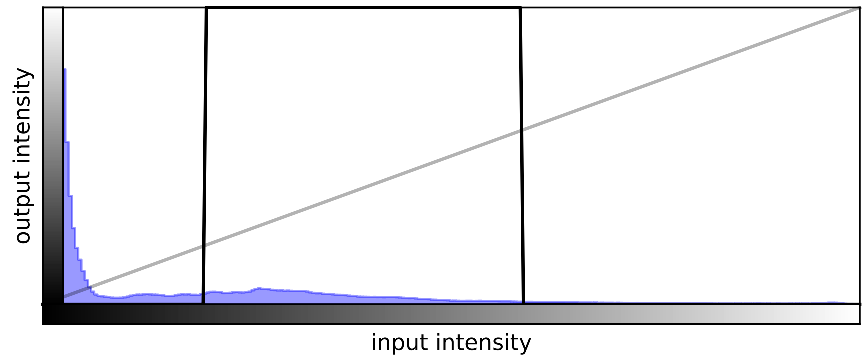
original

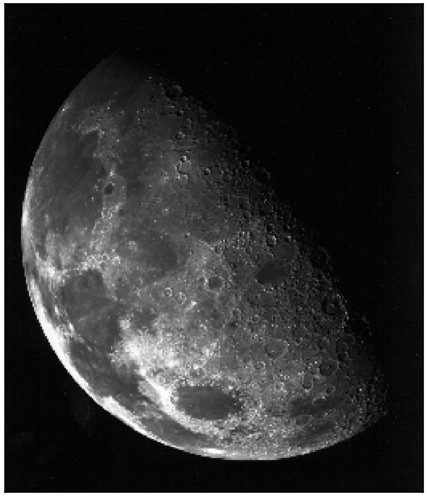


remapped

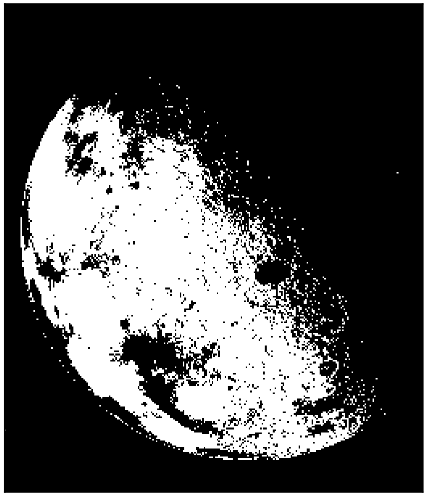
Intensity mappings

Binary window





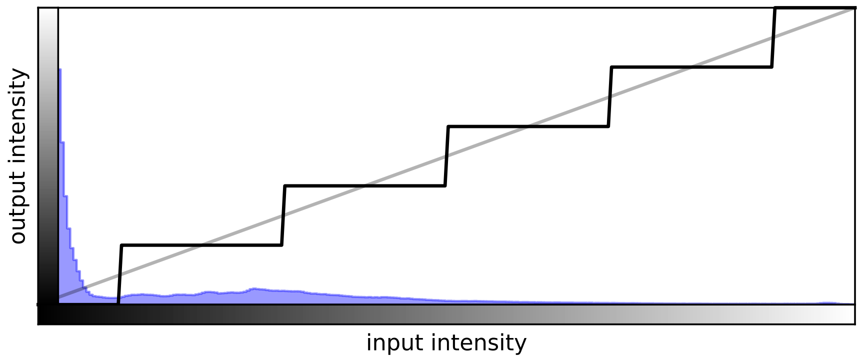
original

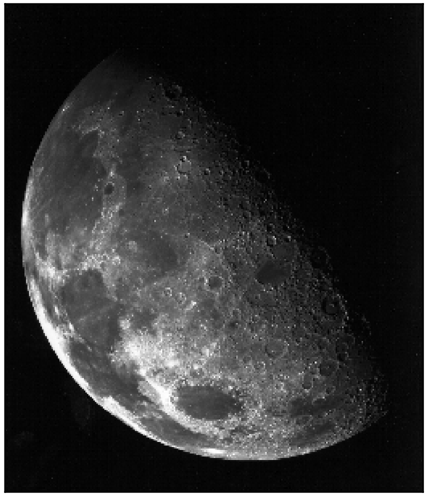


remapped

Intensity mappings

Posterization





original

remapped

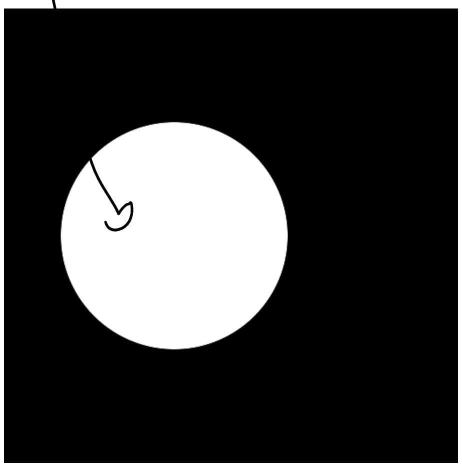
Morphological operations

- analyze morphology of image structures
  - based on set theory and topology
- extract image information
  - shape
  - size
  - connectivity
  - number
  - boundary
- mostly on binary images

Set operations

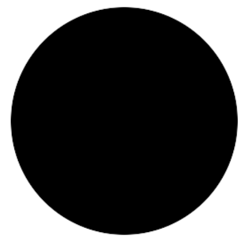
elements in the set

binary mask
A



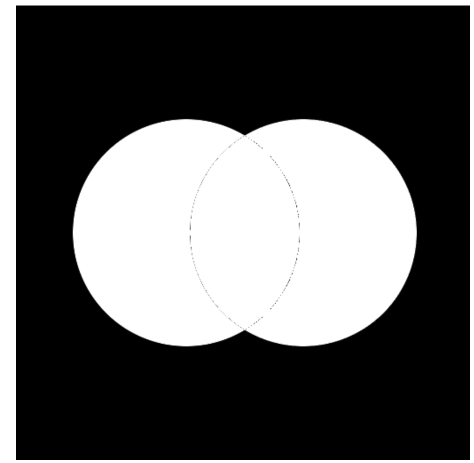
complement (background)
A^c

"not"



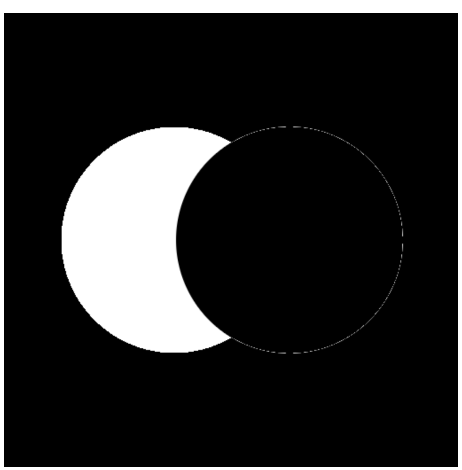
union
$A \cup B$

"or"



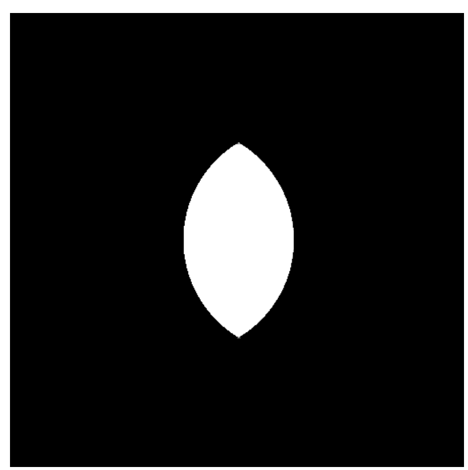
difference
$A \setminus B$

"and not"



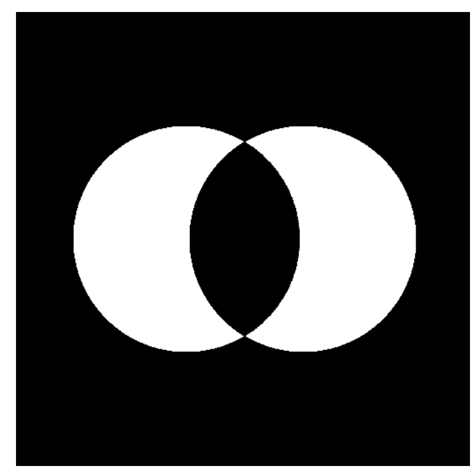
intersection
$A \cap B$

"and"



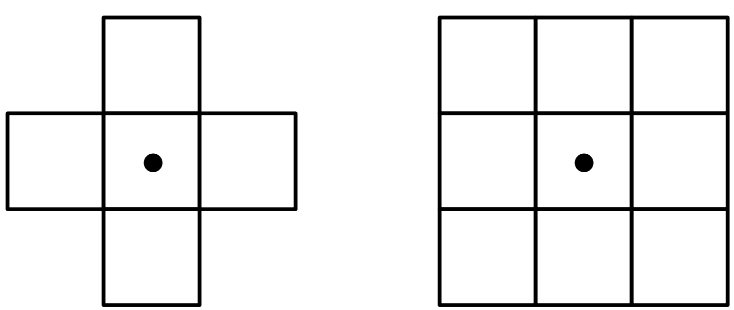
symmetric difference
$A \Delta B$

"xor"



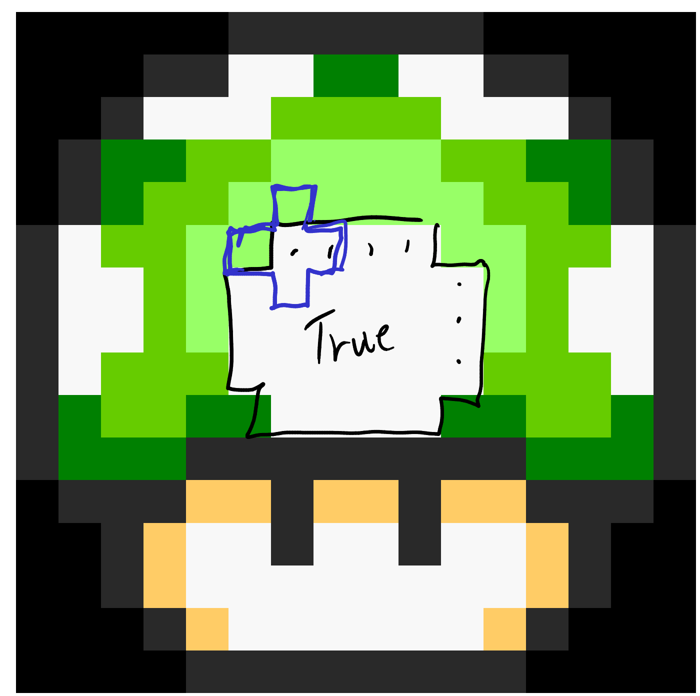
Structuring elements

- small bit mask to probe the image
- scan origin of SE over image
- check overlap between SE and image
- set pixel(s) to zero (or one)



4-connectivity SE

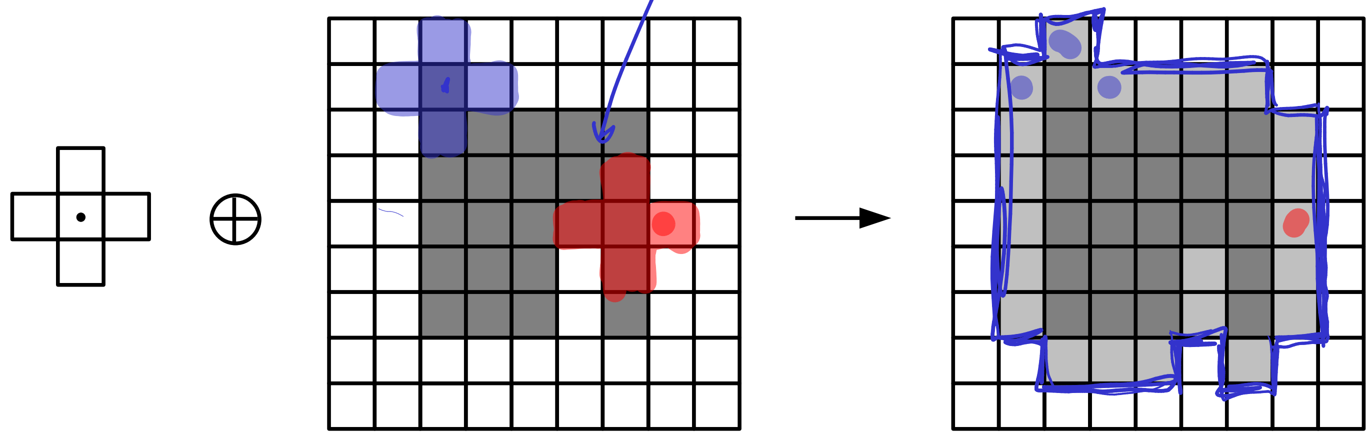
8-connectivity



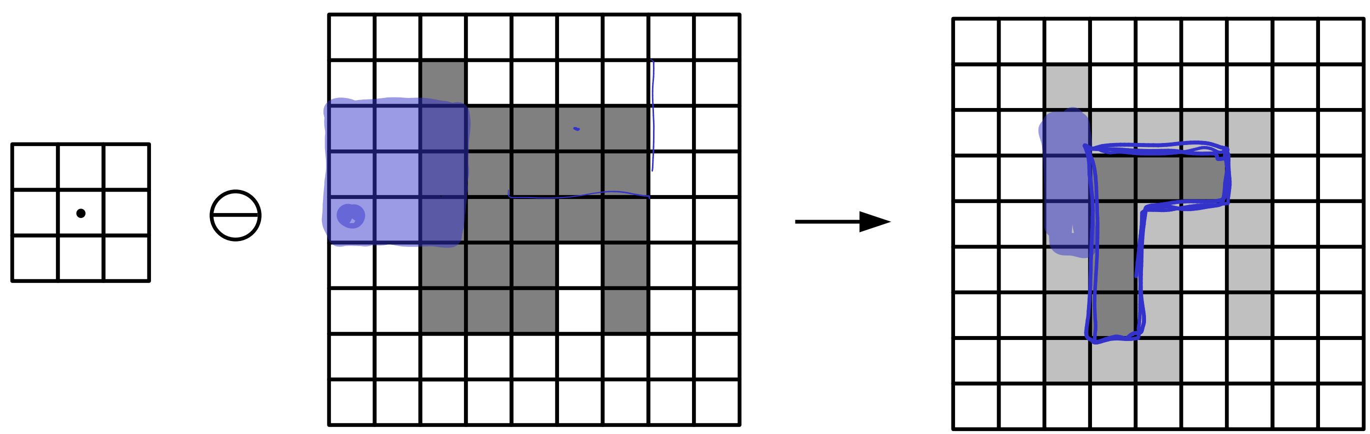
Basic operations

- Dilation: expand region

True

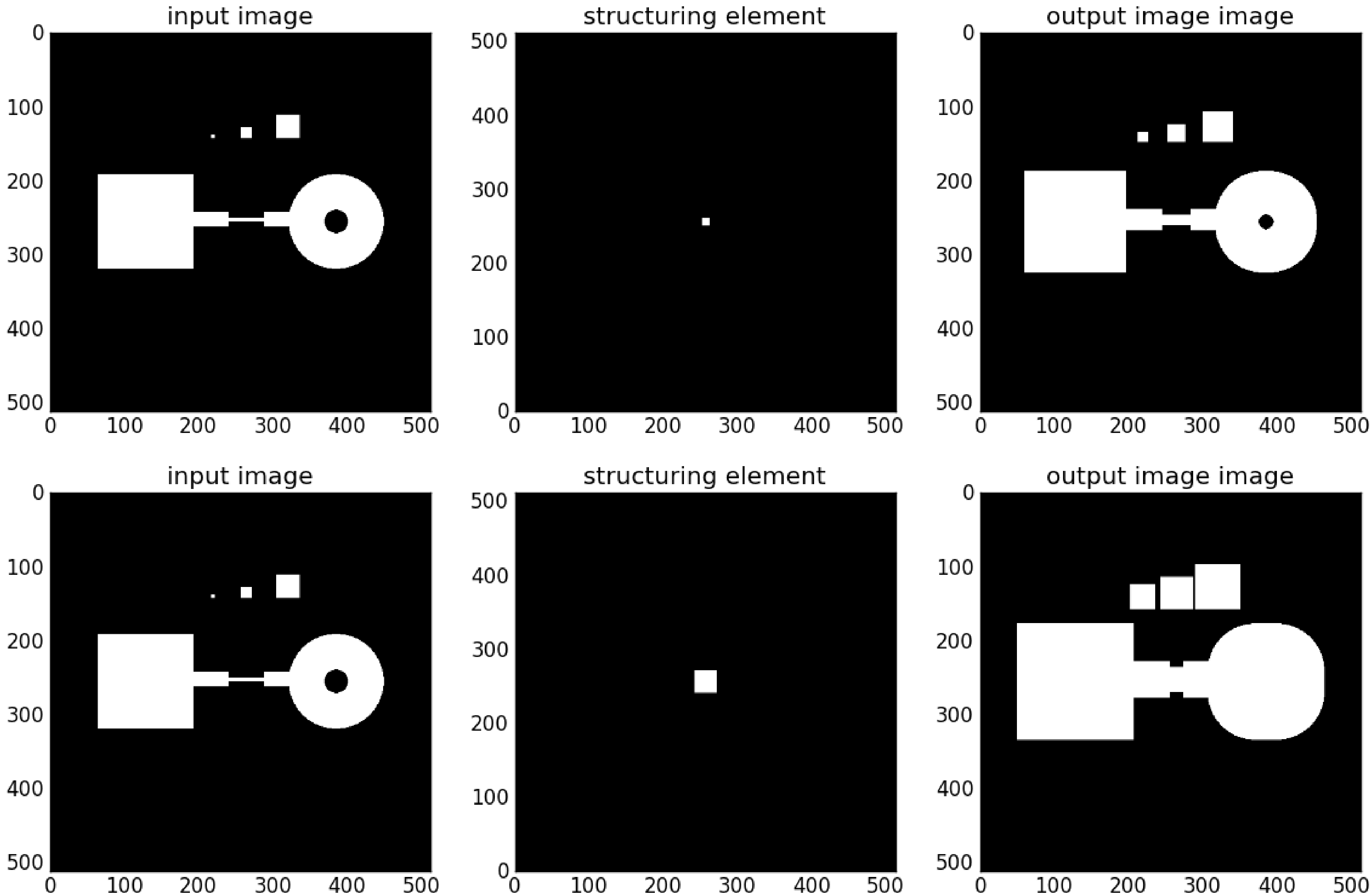


- Erosion: shrink region = dilation of complement



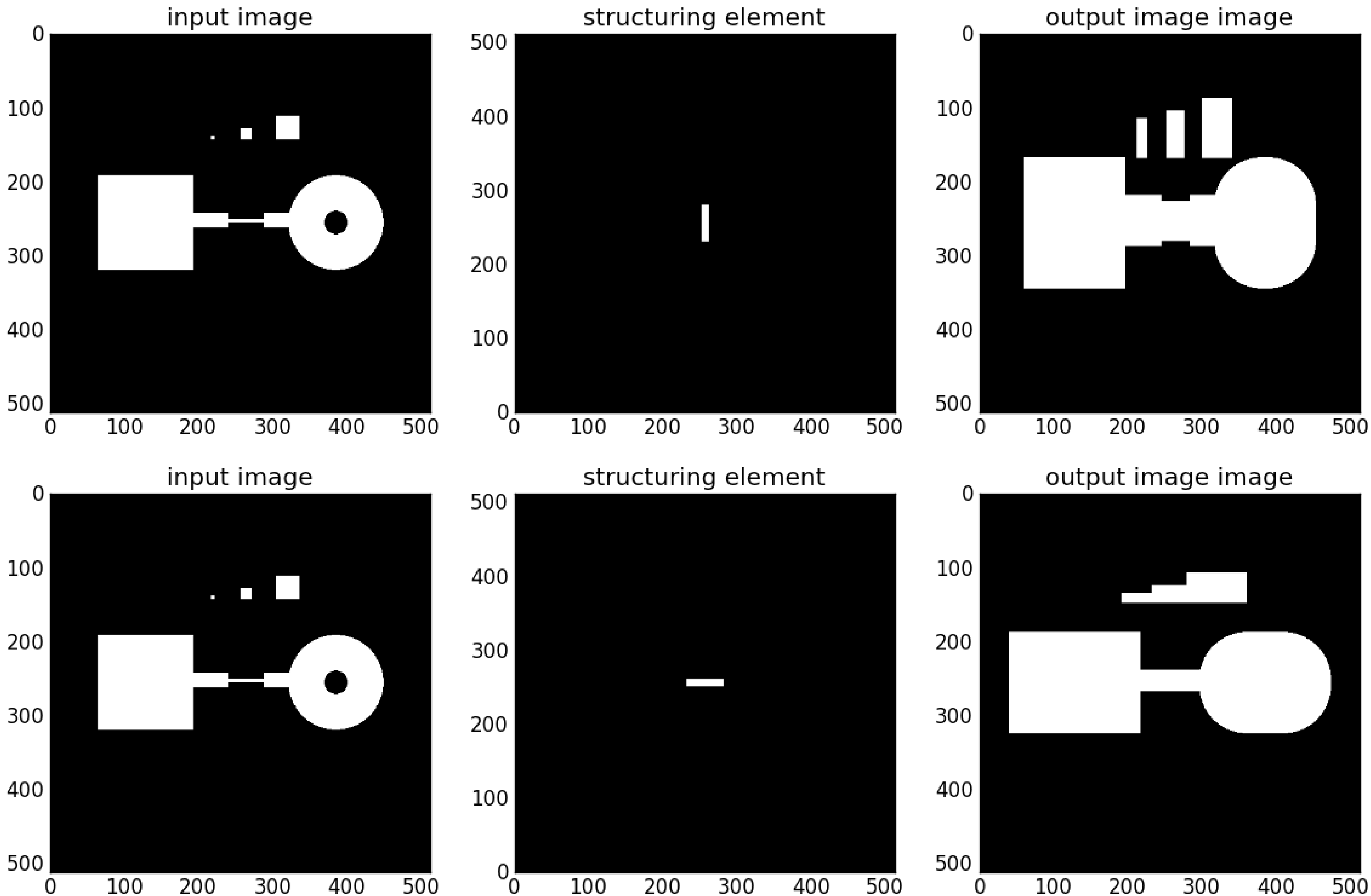
Morphological operations

- dilation



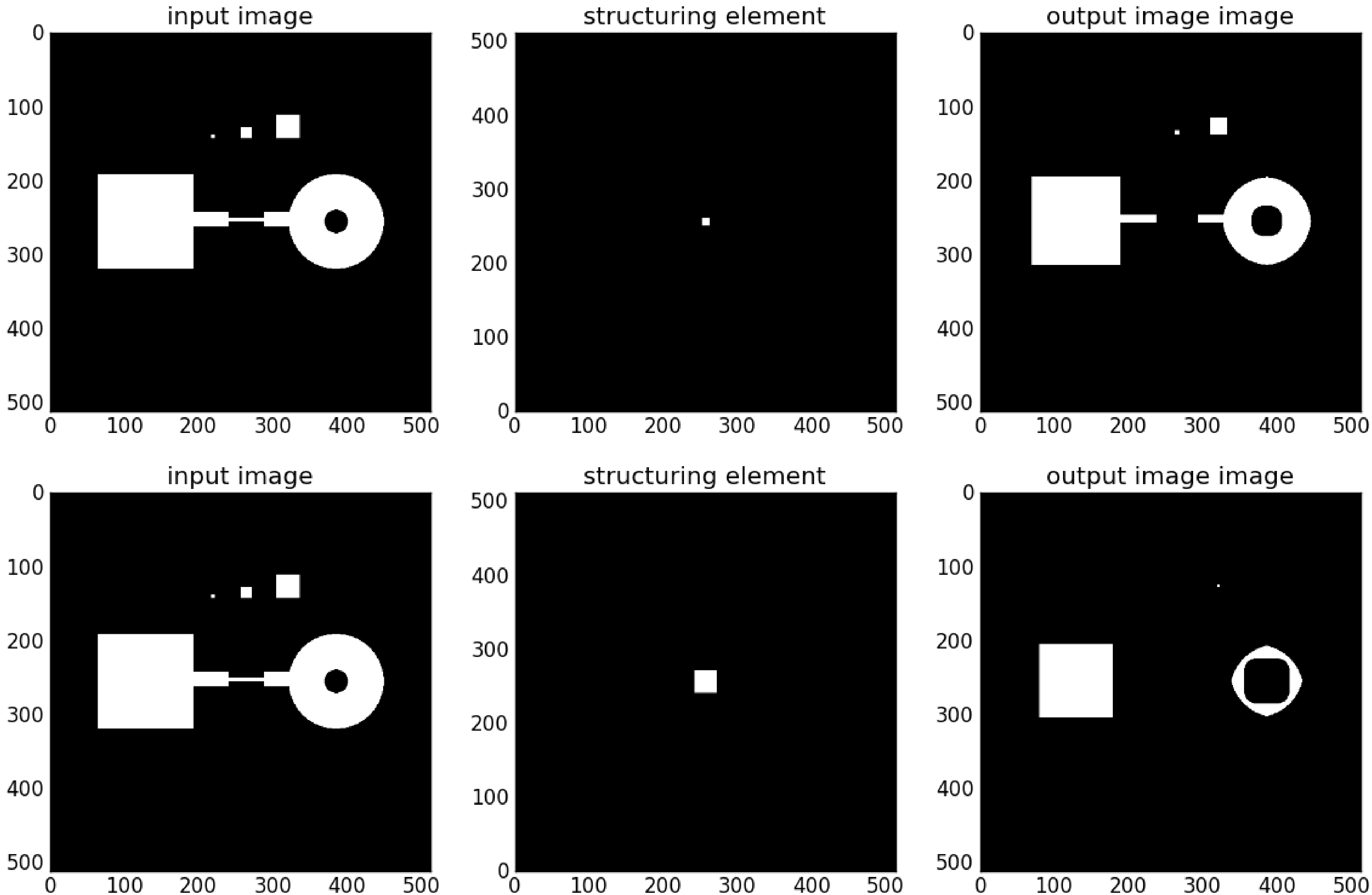
Morphological operations

- dilation



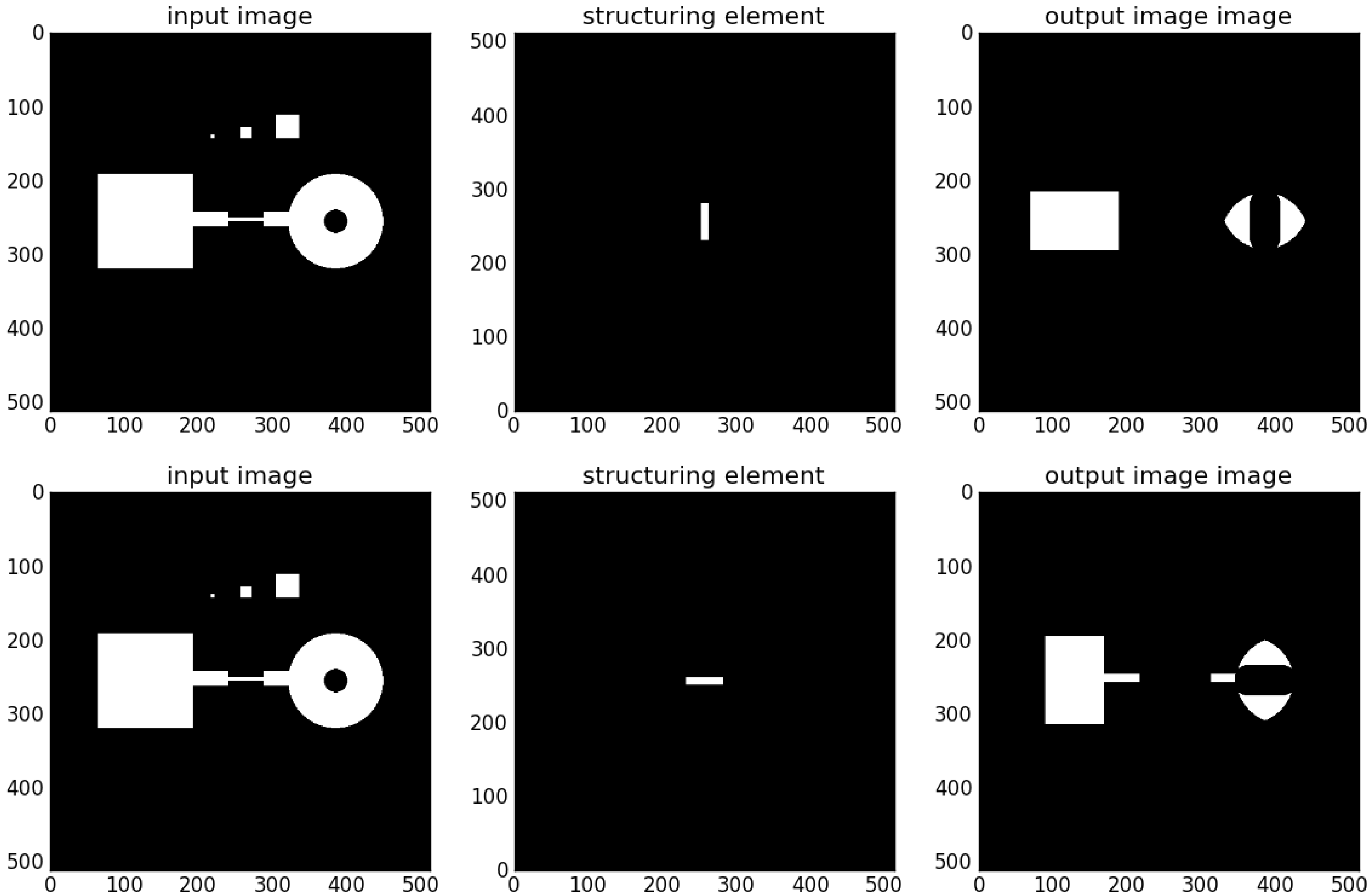
Morphological operations

- erosion



Morphological operations

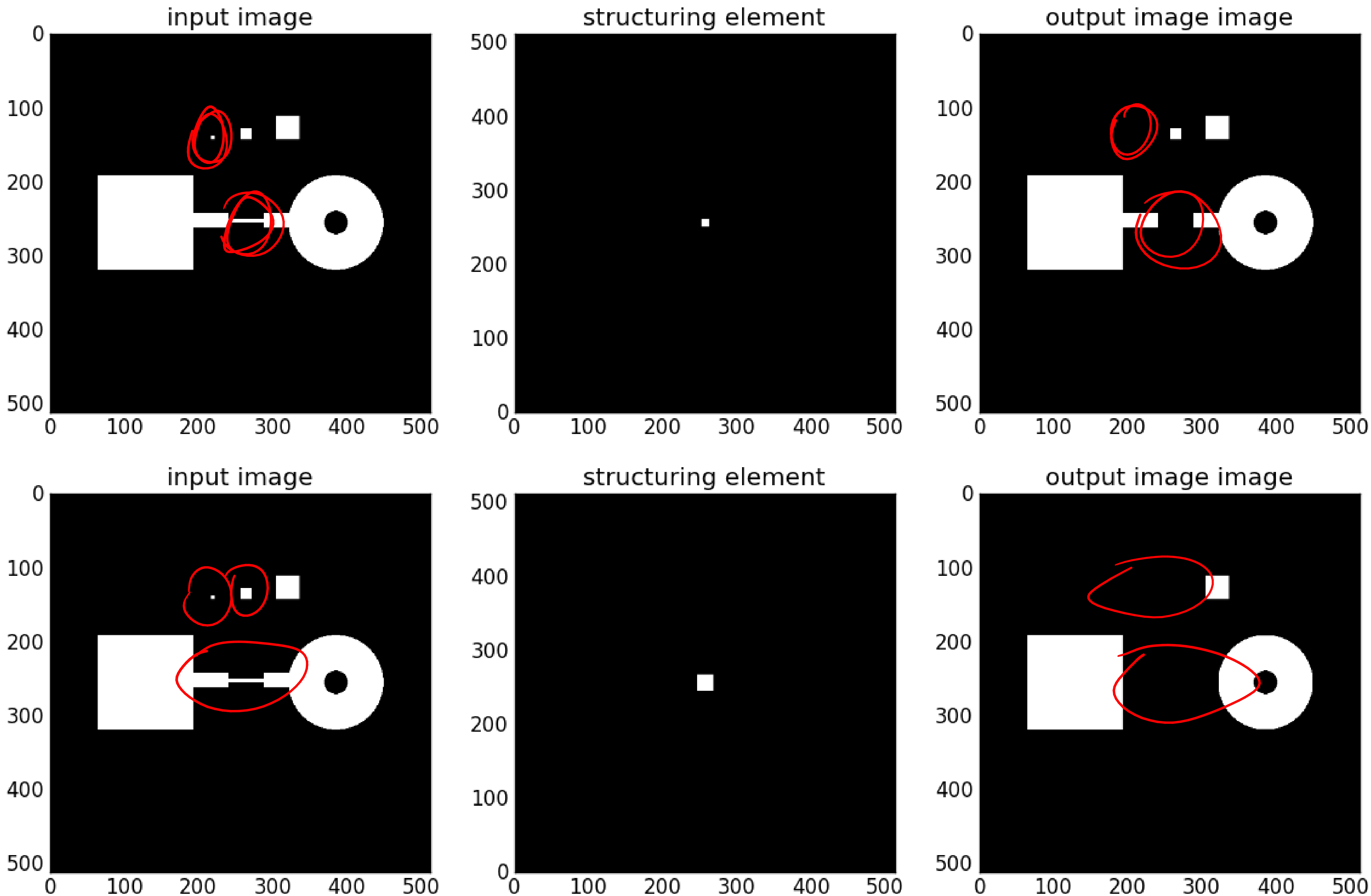
- erosion



Morphological operations

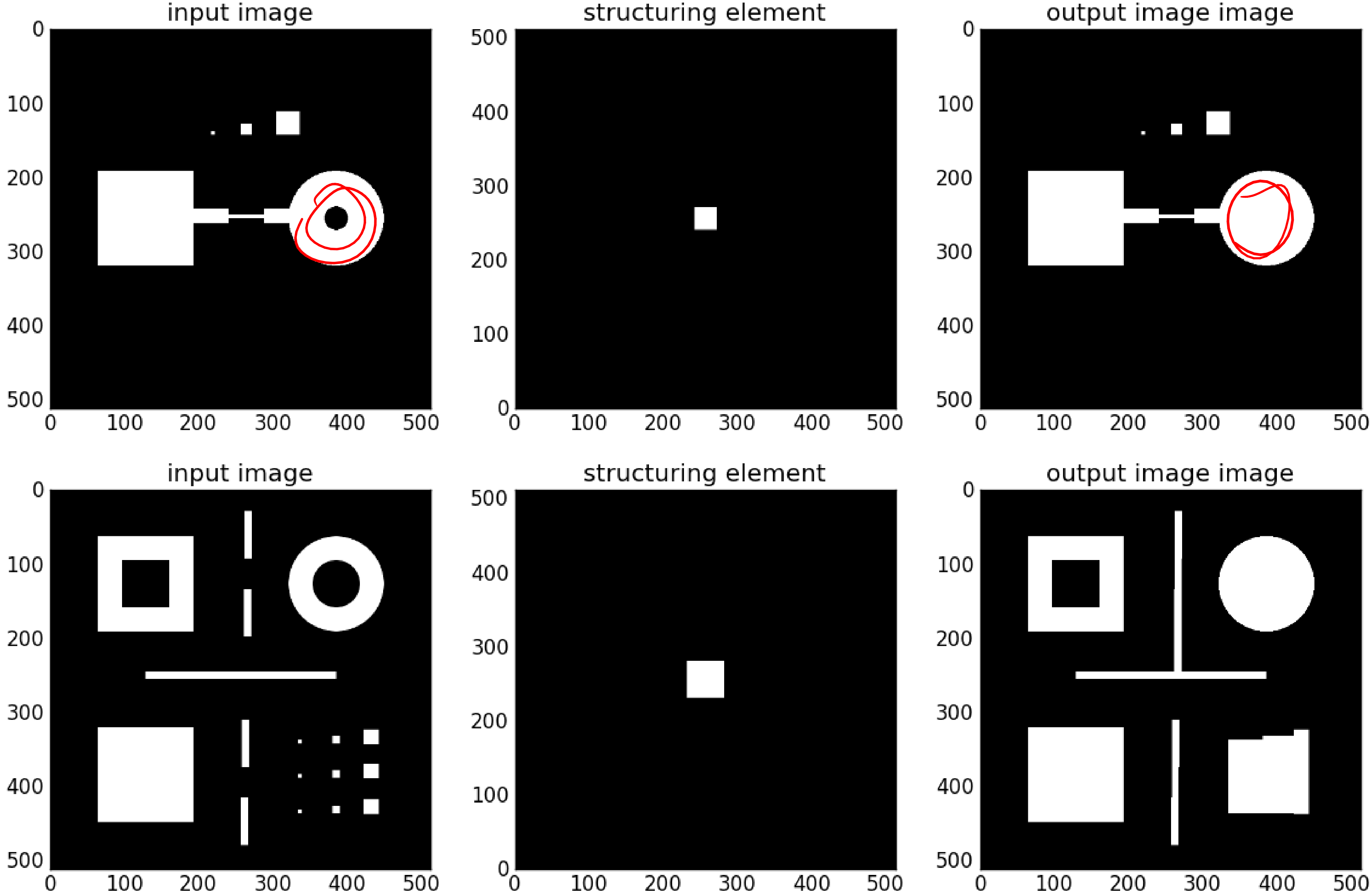
useful to remove noise artifacts

- opening: first erosion, then dilation



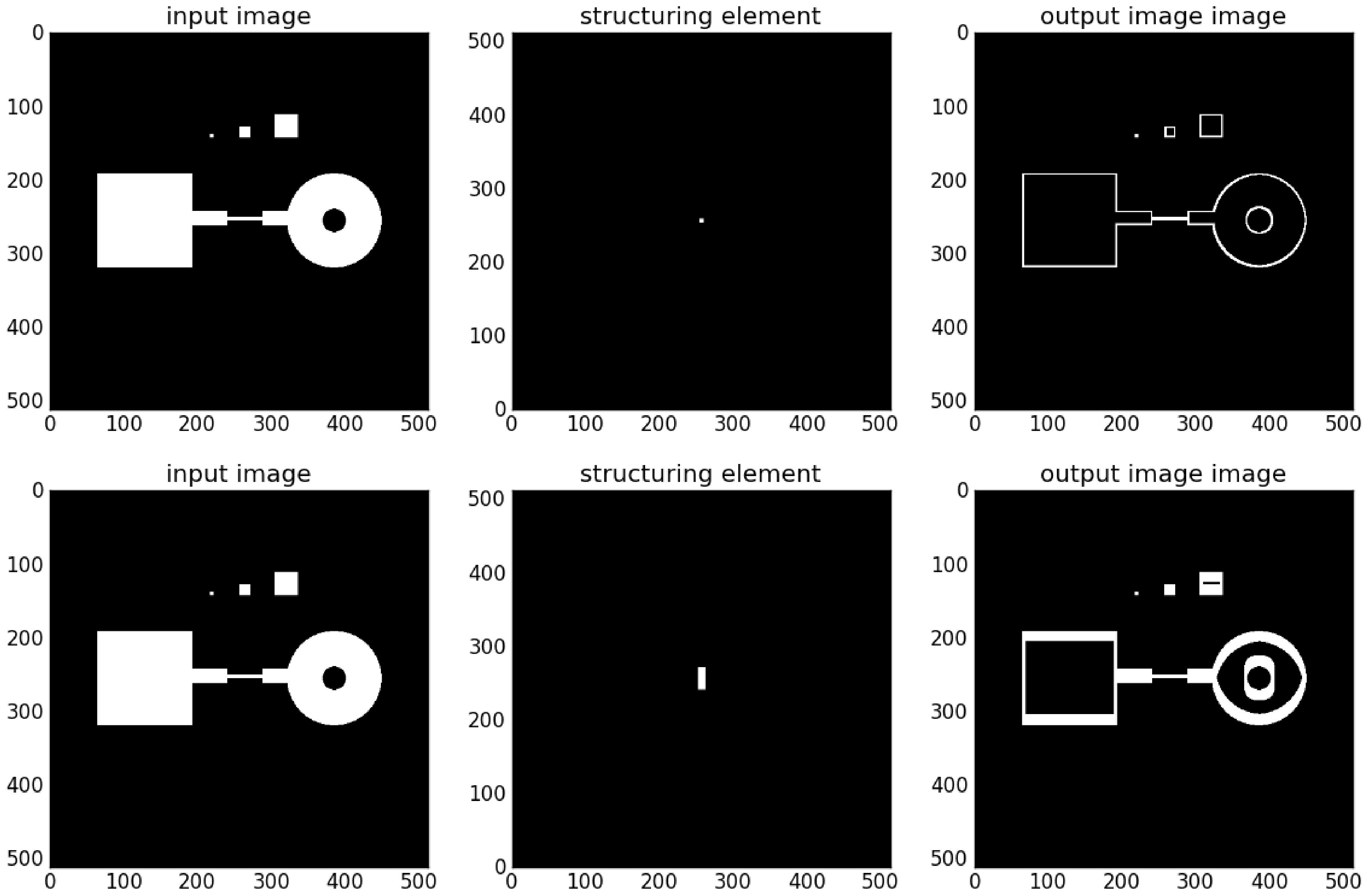
Morphological operations

- closing: first dilation, then erosion



Morphological operations

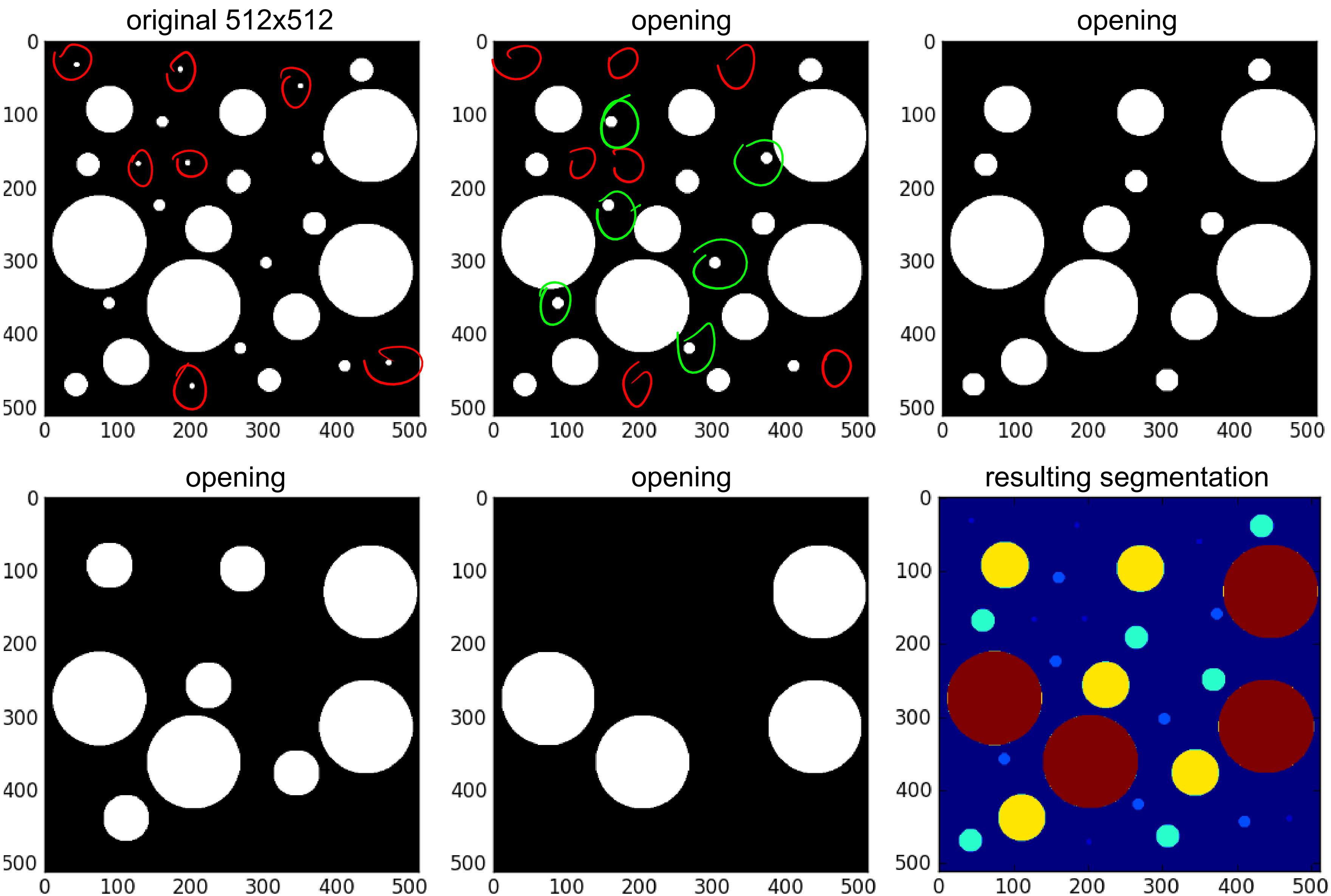
- boundary: original - erosion



Segmentation: Motivation

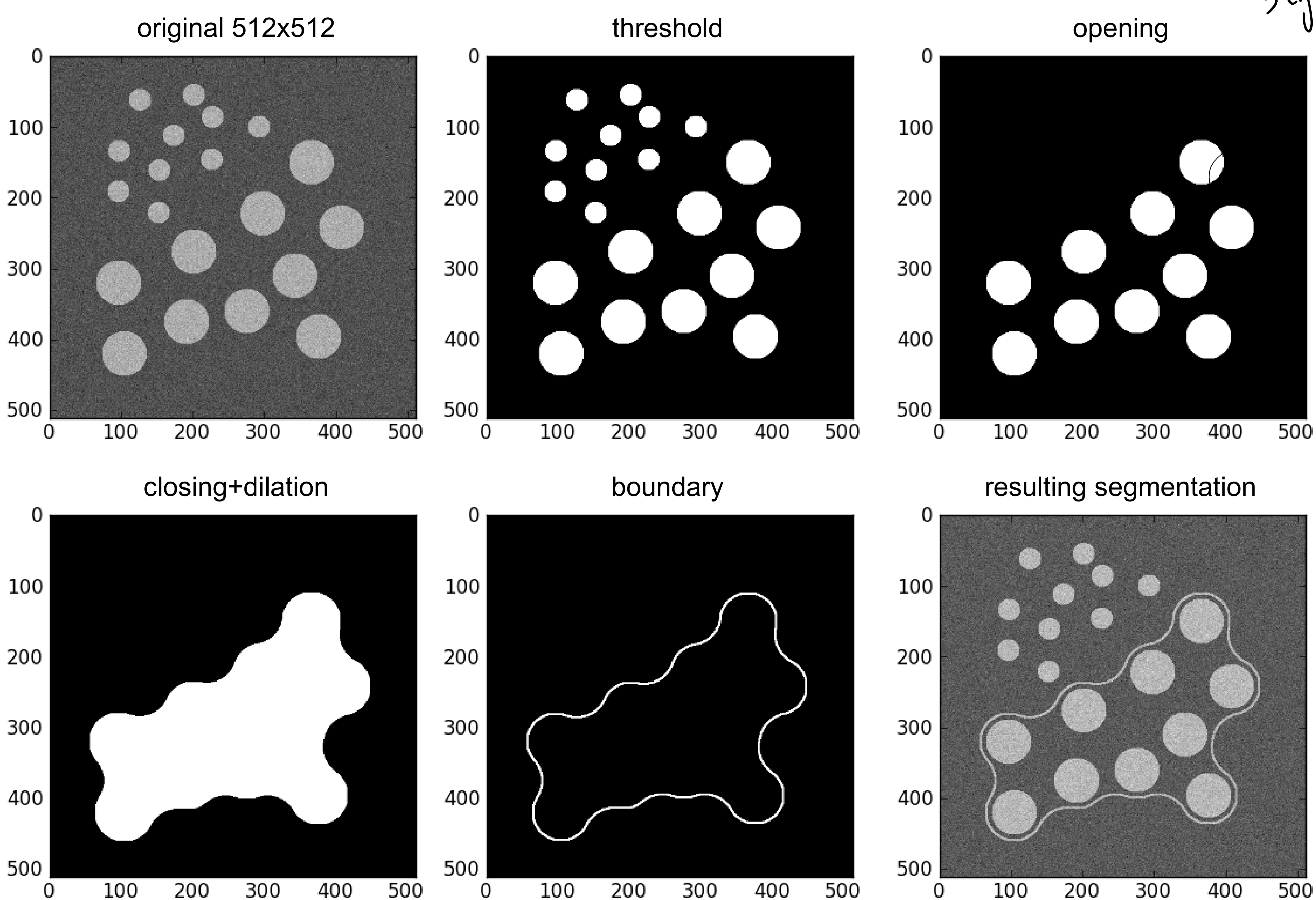
- Partitioning of image by regions-of-interest
- various methods available
  - by morphology
  - by intensity
  - by region
  - by boundary
  - ...

Segmentation by morphology



Segmentation by morphology

texture segmentation



Segmentation by intensity

- easy
- widely used

original

high window

mid window

low window

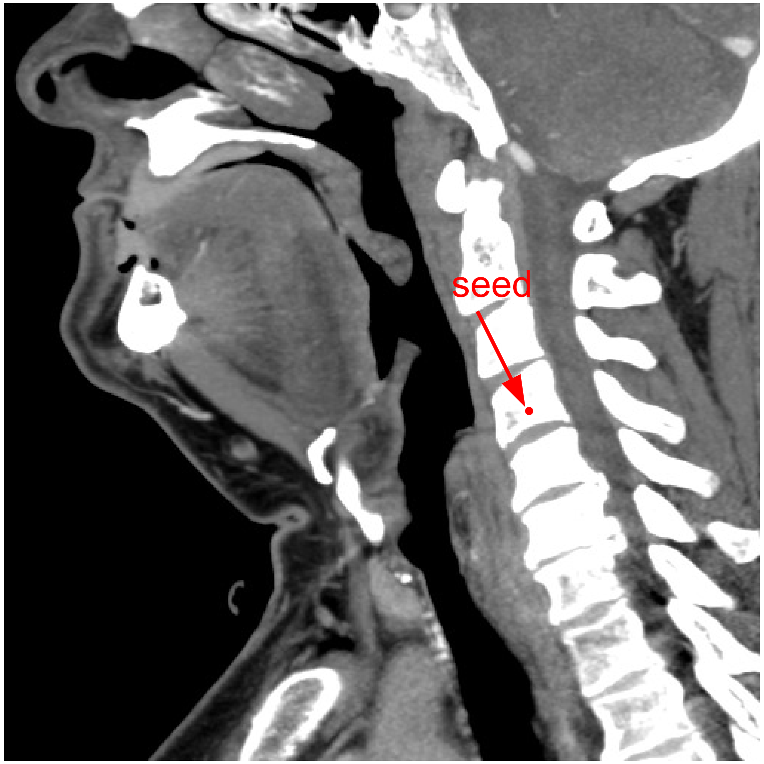
segmented

- noise prone
- no connectivity

Segmentation by region growth

- start with seed
- check intensity in neighborhood
- if intensity within window, set to 1
- iterate until no change

original



Segmentation by boundaries

- look for sharp changes in intensity
- more next week...

original

" laplace "

Linear transformation A

$$A(a_1 \vec{r}_1 + a_2 \vec{r}_2) = a_1 A \vec{r}_1 + a_2 A \vec{r}_2$$

Offset $\vec{r} \longrightarrow \vec{r} + \vec{r}_0$

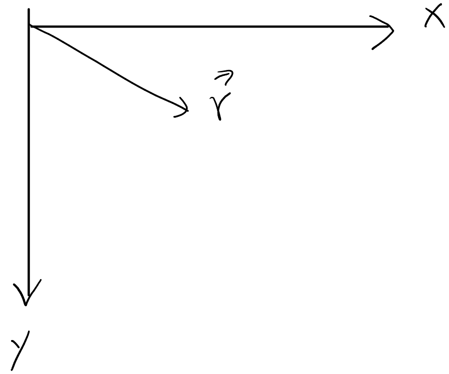
not linear $\vec{r}_1 + \vec{r}_2 \longrightarrow \vec{r}_1 + \vec{r}_2 + \vec{r}_0 \neq (\vec{r}_1 + \vec{r}_0) + (\vec{r}_2 + \vec{r}_0)$

Affine transformation:

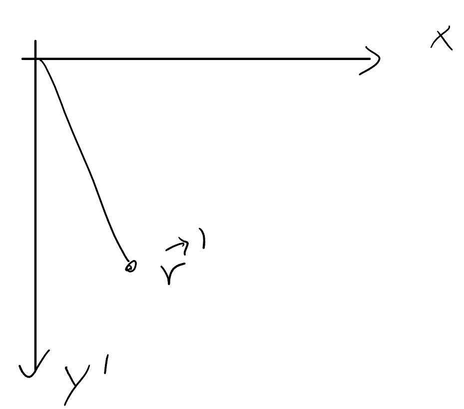
$$\boxed{\vec{r}\,' = A \vec{r} + \vec{r}_0}$$

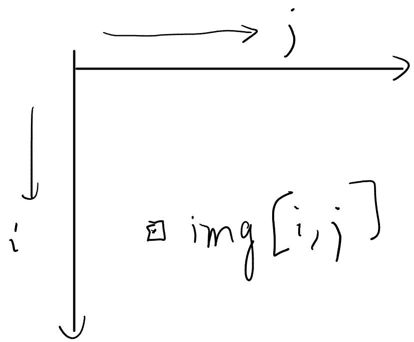
2D: total 6 parameters

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} A_{00} & A_{01} \\ A_{10} & A_{11} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \text{offset}_0 \\ \text{offset}_1 \end{pmatrix}$$

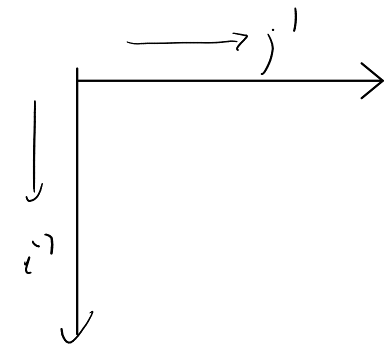


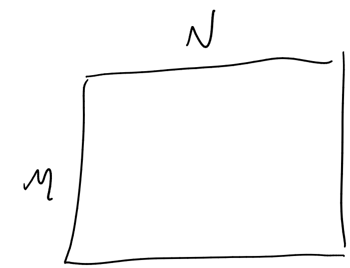
$$\vec{r}' = A\vec{r} + \vec{r}_0 \quad (1)$$





$$\begin{pmatrix} j \\ i \end{pmatrix} = M \begin{pmatrix} j' \\ i' \end{pmatrix} + D$$





$$x = j - N/2 \qquad x' = j' - N/2$$

$$y = i - M/2 \qquad y' = i' - M/2$$

$$(1) \quad \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} j' - N/2 \\ i' - M/2 \end{pmatrix} = A \begin{pmatrix} j - N/2 \\ i - M/2 \end{pmatrix} + \begin{pmatrix} r_{0x} \\ r_{0y} \end{pmatrix}$$

$$A^{-1}\begin{pmatrix} j' - N/2 - r_{0x} \\ i' - M/2 - r_{0y} \end{pmatrix} = \begin{pmatrix} j - N/2 \\ i - M/2 \end{pmatrix}$$

$$\begin{pmatrix} j \\ i \end{pmatrix} = \underbrace{A^{-1}}_{M}\begin{pmatrix} j' \\ i' \end{pmatrix} + \underbrace{\begin{pmatrix} N/2 \\ M/2 \end{pmatrix} - A^{-1}\begin{pmatrix} N/2 + r_{0x} \\ M/2 + r_{0y} \end{pmatrix}}_{\Delta}$$

$$\begin{pmatrix} j \\ i \end{pmatrix} = M\begin{pmatrix} j' \\ i' \end{pmatrix} + \Delta$$