Circonferenza di centro C e raggio $r>0$

$C = (x_c, y_c)$

$r > 0$

$$\mathcal{C}_{C,r} = \{ P = (x,y) \quad d(P,C) = r \}$$

$$d(P, C) = r \qquad P = (x, y) \qquad C = (x_c, y_c)$$

$$\Updownarrow$$

$$\sqrt{(x - x_c)^2 + (y - y_c)^2} = r$$

$$\Updownarrow$$

$$(x - x_c)^2 + (y - y_c)^2 = r^2 \iff \begin{aligned} &x^2 - 2x_c x + x_c^2 \\ &+ y^2 - 2y_c y + y_c^2 = r^2 \end{aligned}$$

$$\mathcal{C}_{c,r} = \{ (x,y) = P \quad : \quad d(P,C) = r \}$$

$$C = (x_c, y_c)$$

$$\iff$$

$$\mathcal{C}_{c,r} = \{ (x,y) \quad : \quad \underbrace{x^2 - 2x_c x + x_c^2 + y^2 - 2y_c y + y_c^2 = r^2}_{\text{Equazione di una circonferenza}} \}$$

$$A = -2x_c$$

$$B = -2y_c$$

$$C = r^2 - x_c^2 - y_c^2$$

$$x^2 + y^2 + Ax + By + C = 0$$

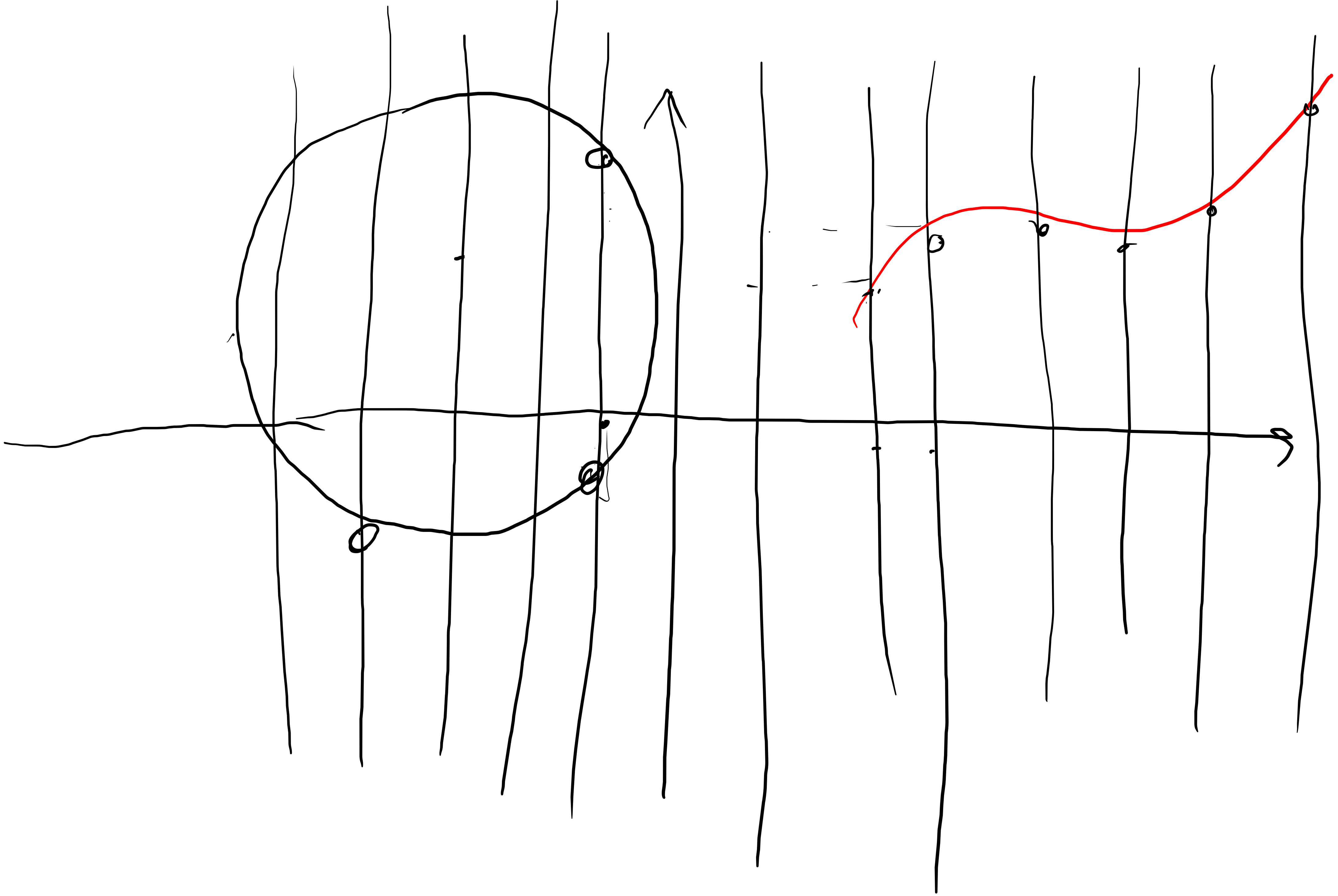
$$x^2 + y^2 + 1 = 0$$

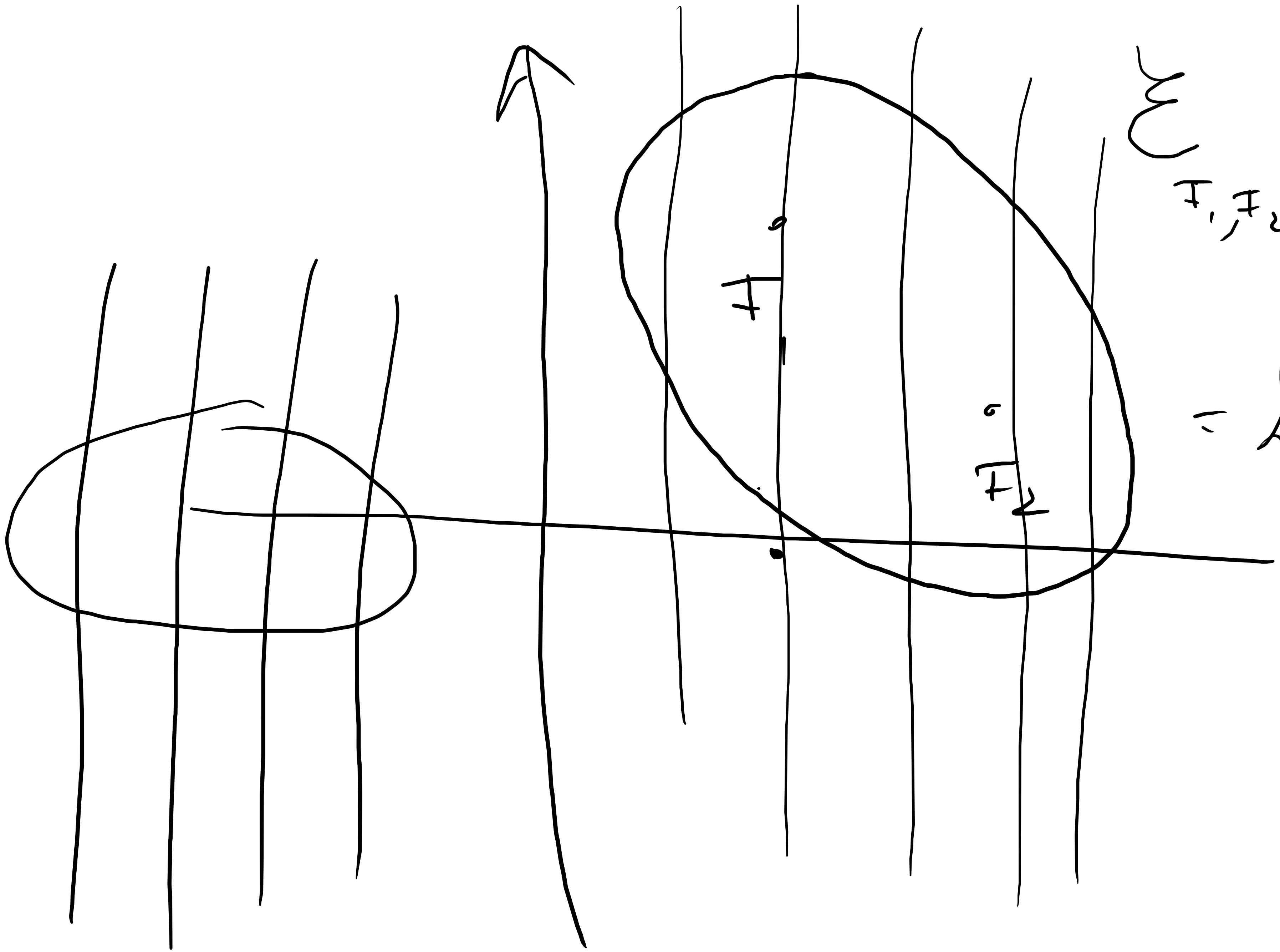
$A = 0$

$B = 0$

$C = 1$

$$C + x_c^2 + y_c^2 = r^2$$



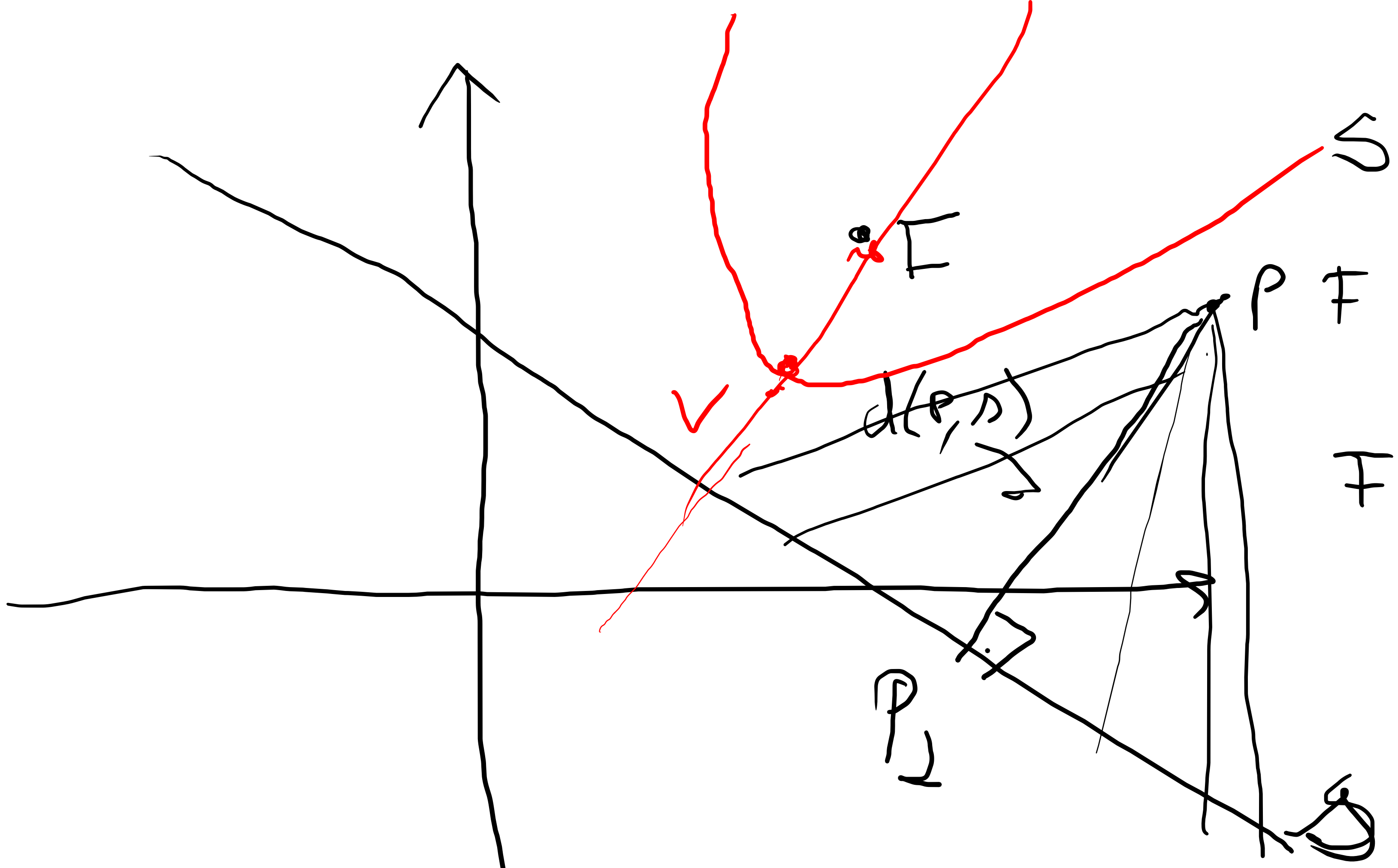


$$\mathcal{E}_{F_1,F_2,d} =$$

$$= \{ (x,y) = P$$

$$d(P,F_1) + d(P,F_2) = d \}$$

$$d > 0$$



S retta nel piano

F punto dello fuoco

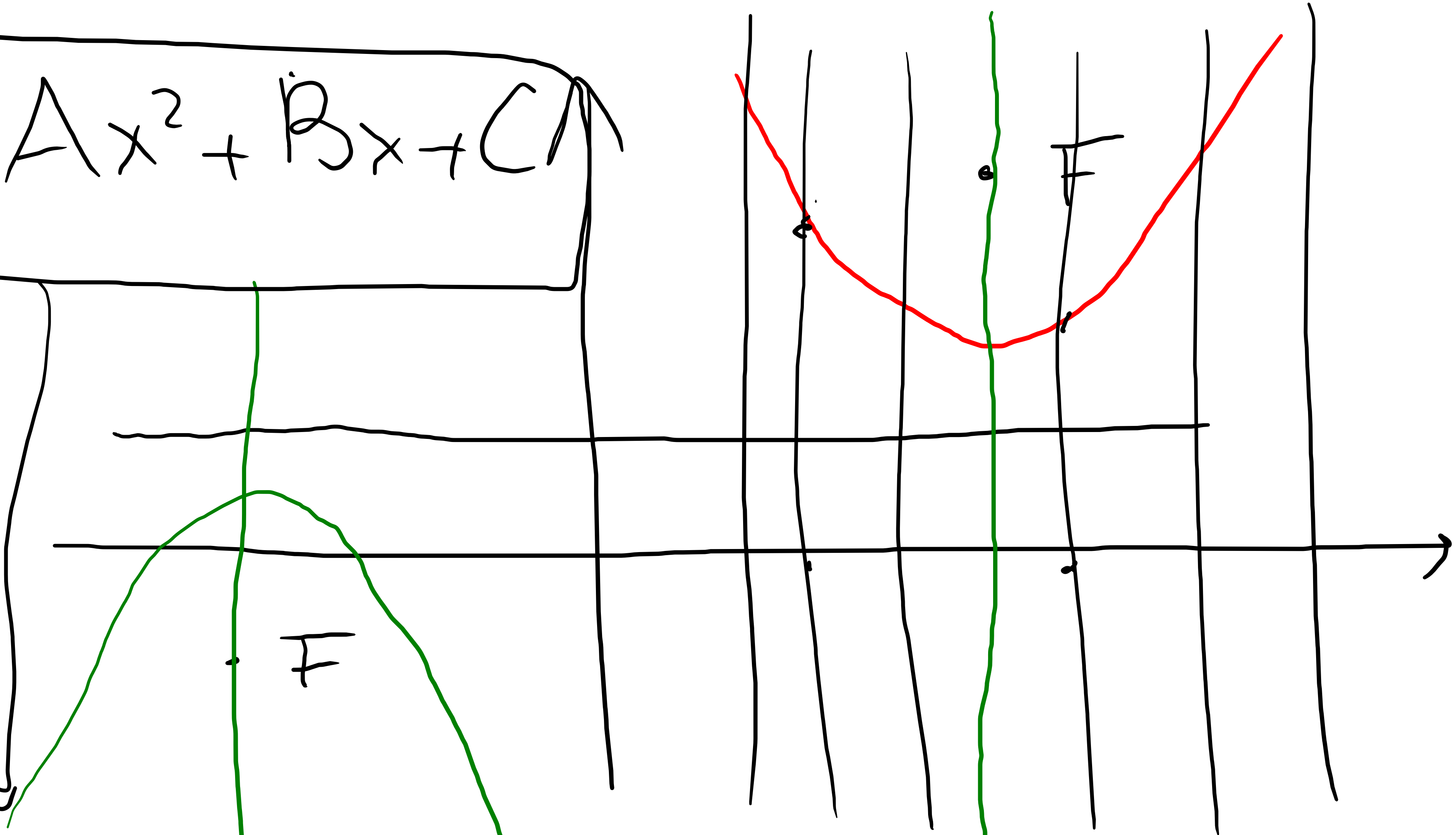
$F \notin S$

$d(P, P_\perp)$

$$\mathcal{P}_{F,S} = \{ (x,y)=P \mid d(P,F) = d(P,S) \}$$

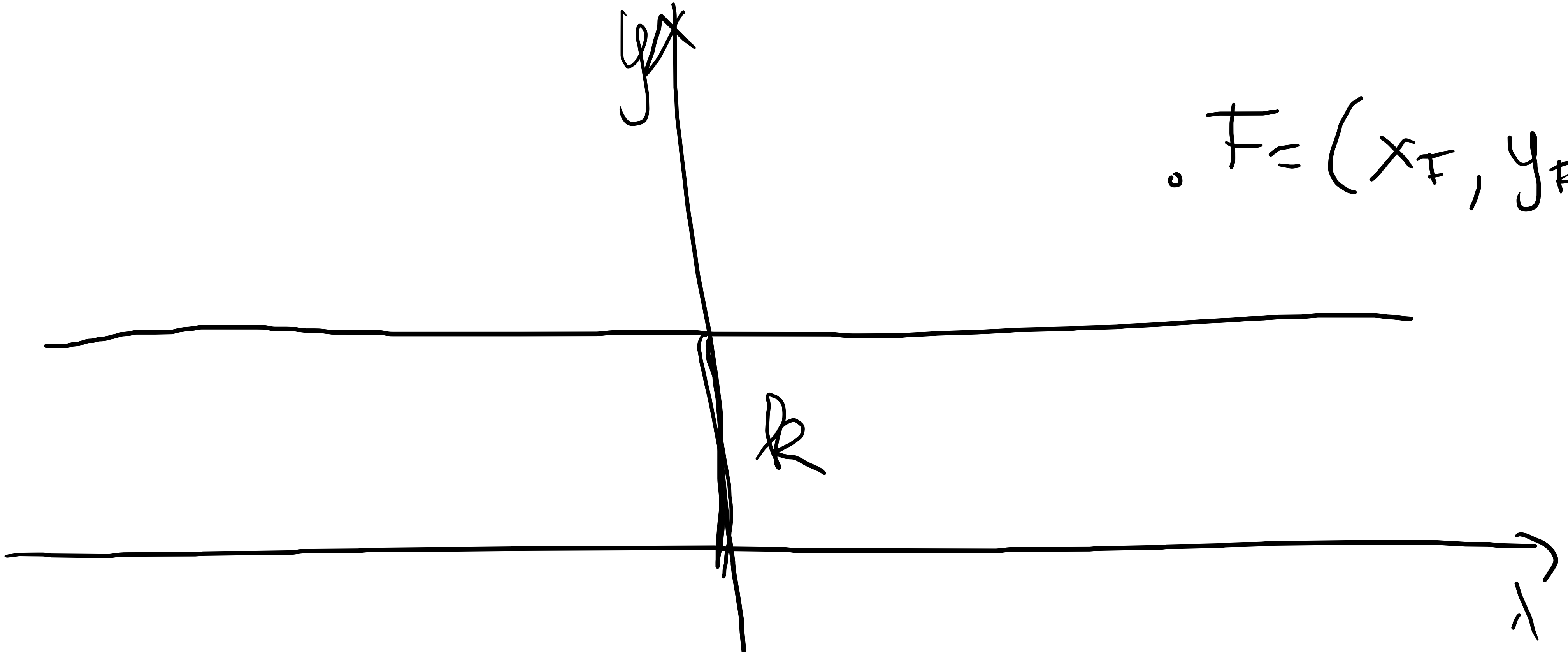
$$\boxed{y = Ax^2 + Bx + C}$$

$A \neq 0$



generale equazione di PARABOLA

con asse verticale ossia parallelo all'asse delle y.



$\cdot\, F = (x_F, y_F)$

r

$F \notin r$

$\Updownarrow$

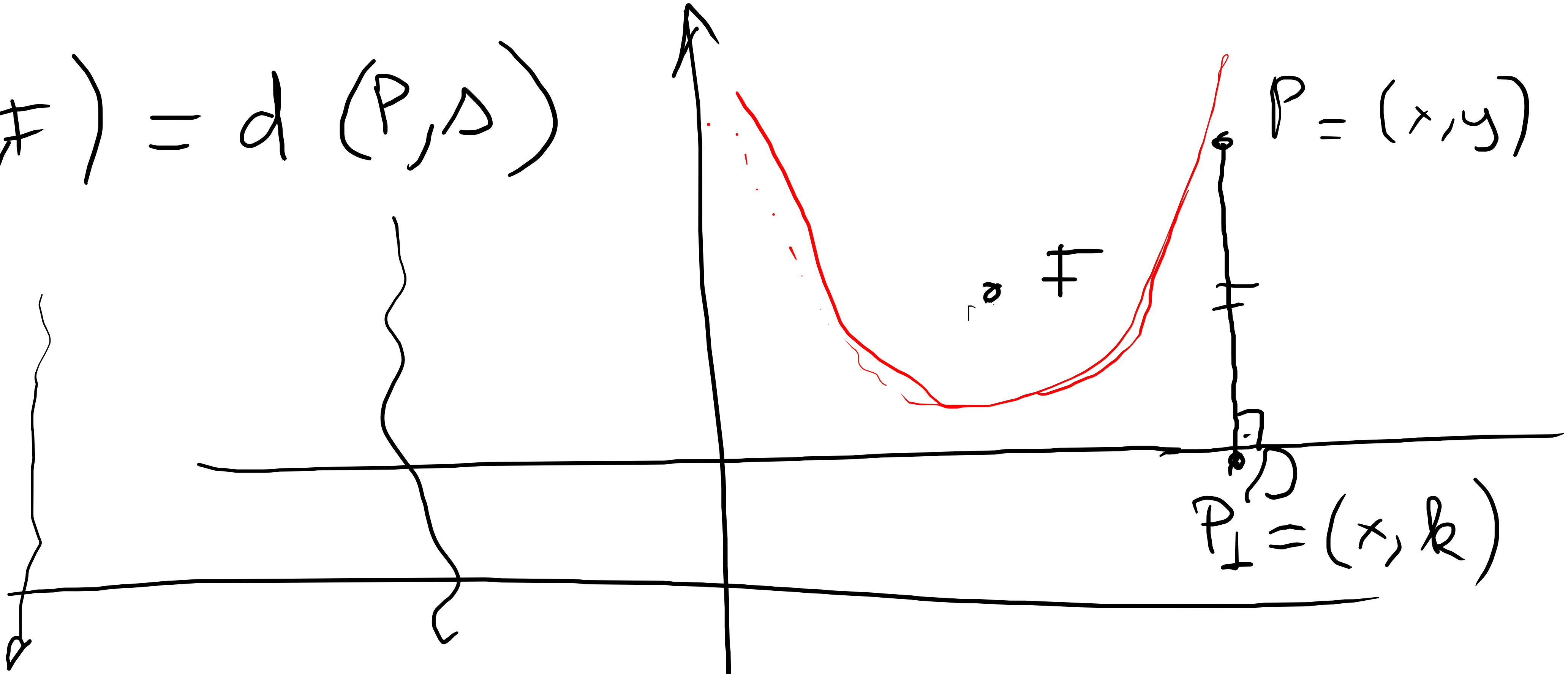
$y_F \neq k$

$$r = \{ (x,y) \in \mathbb{R}^2 \quad \underline{y = k} \}$$

$$d(P,F) = d(P,\Delta)$$

$F \notin \Delta$

$\Delta = \{ y = k \}$



$$\sqrt{(x-x_F)^2+(y-y_F)^2} = \sqrt{\cancel{(x-x)^2}+(y-k)^2} = |y-k|$$

Introduciamo questo nuovo simbolo

$$|y-k| = \sqrt{(y-k)^2} = \begin{cases} y-k & \text{se } y \geq k \\ k-y & \text{se } y < k \end{cases}$$

valore assoluto di $y-k$

$$\sqrt{(x-x_F)^2+(y-y_F)^2} = |y-k|$$

elevando al quadrato entrambi ottengo

$$(x-x_F)^2+(y-y_F)^2=(y-k)^2 \geq 0$$

$$x^2-2xx_F+x_F^2+\cancel{y^2}-2y_Fy+y_F^2=\cancel{y^2}-2ky+k^2$$

$$x^2-2xx_F+x_F^2+y_F^2-k^2=2y_Fy-2ky$$

$$=2y(y_F-k)$$

$y_F \neq k$ poiché $F \notin D$

$$y_F - k \neq 0 \Rightarrow \exists \ (y_F - k)^{-1} = \frac{1}{y_F - k}$$

inverso o reciproco di $y_F - k$

$$x^2 - 2x\, x_F + x_F^2 - k^2 + y_F^2 = 2y\,(y_F - k)$$

divido per $2 \cdot (y_F - k)$ cioè moltiplico ambo i membri dell'equazione per il reciproco di $2(y_F - k)$

$$\overbrace{\frac{x^2}{2(y_F-k)}}^{A} - \overbrace{\frac{\cancel{2}x x_F}{\cancel{2}(y_F-k)}}^{B} + \overbrace{\frac{x_F^2+y_F^2-k^2}{2(y_F-k)}}^{C} = y$$

$$\boxed{y = Ax^2 + Bx + C}$$

L'asse della parabola è verticale per costruzione

se $y_F - k > 0 \iff y_F > k \iff A > 0$

$y_F - \; < 0 \iff y_F < k \iff A < 0$

$A \neq 0$

$$A = \frac{1}{2(y_F - k)} \qquad B = \frac{-2x_F}{2(y_F - k)} = \frac{-x_F}{y_F - k}$$

$$\boxed{x_F = \frac{-B}{2A}}$$

$$B = \frac{-x_F}{y_F - k}$$

$$x_F = -B \cdot (y_F - k)$$

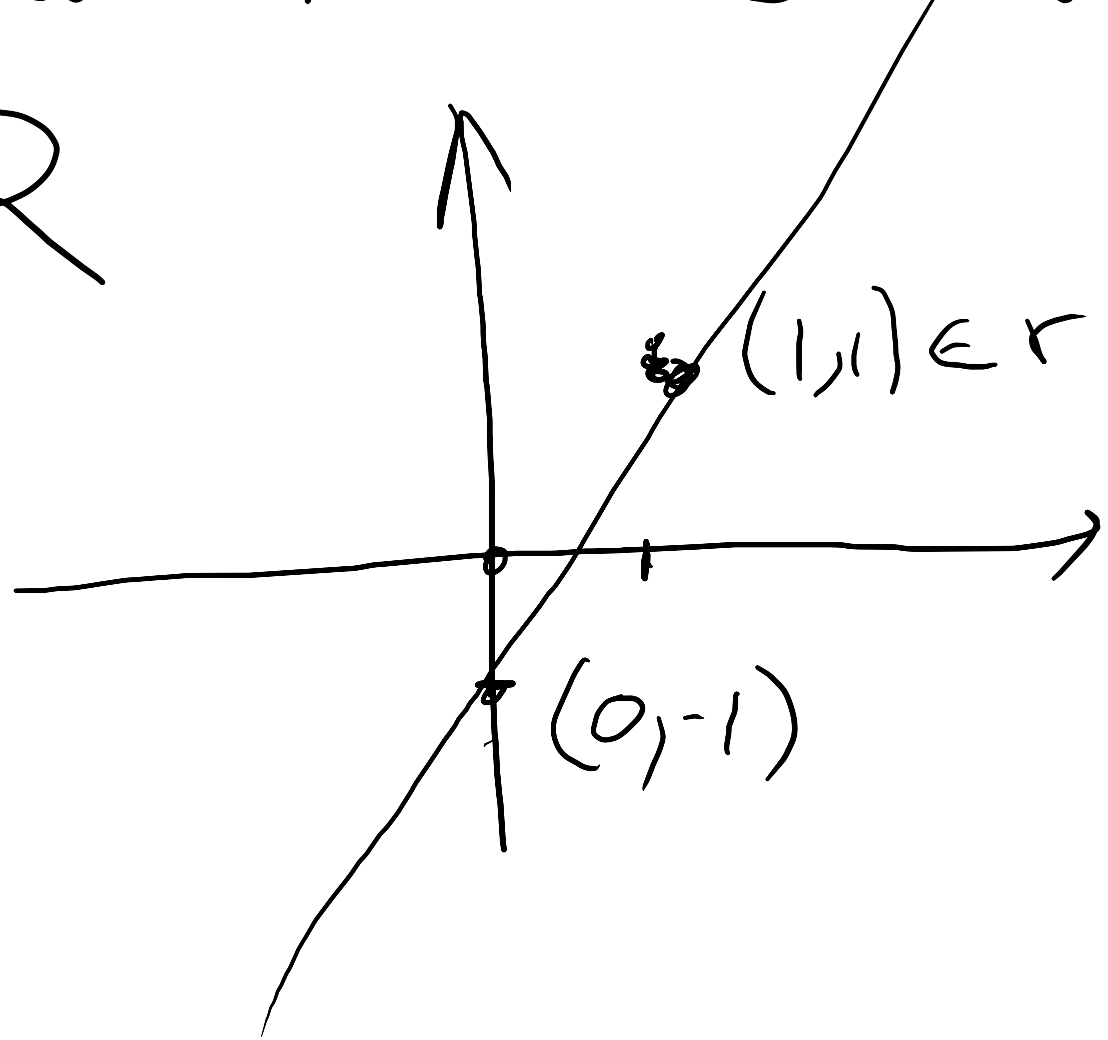
$$= -\frac{1}{2A} \cdot B$$

$y = Ax^2 + Bx + C$

$(0, C)$ appartiene alla parabola

$r: \; y = 2x - 1$ è un esempio di funzione CRESCENTE

$A = \mathbb{R}$ $\quad B = \mathbb{R}$

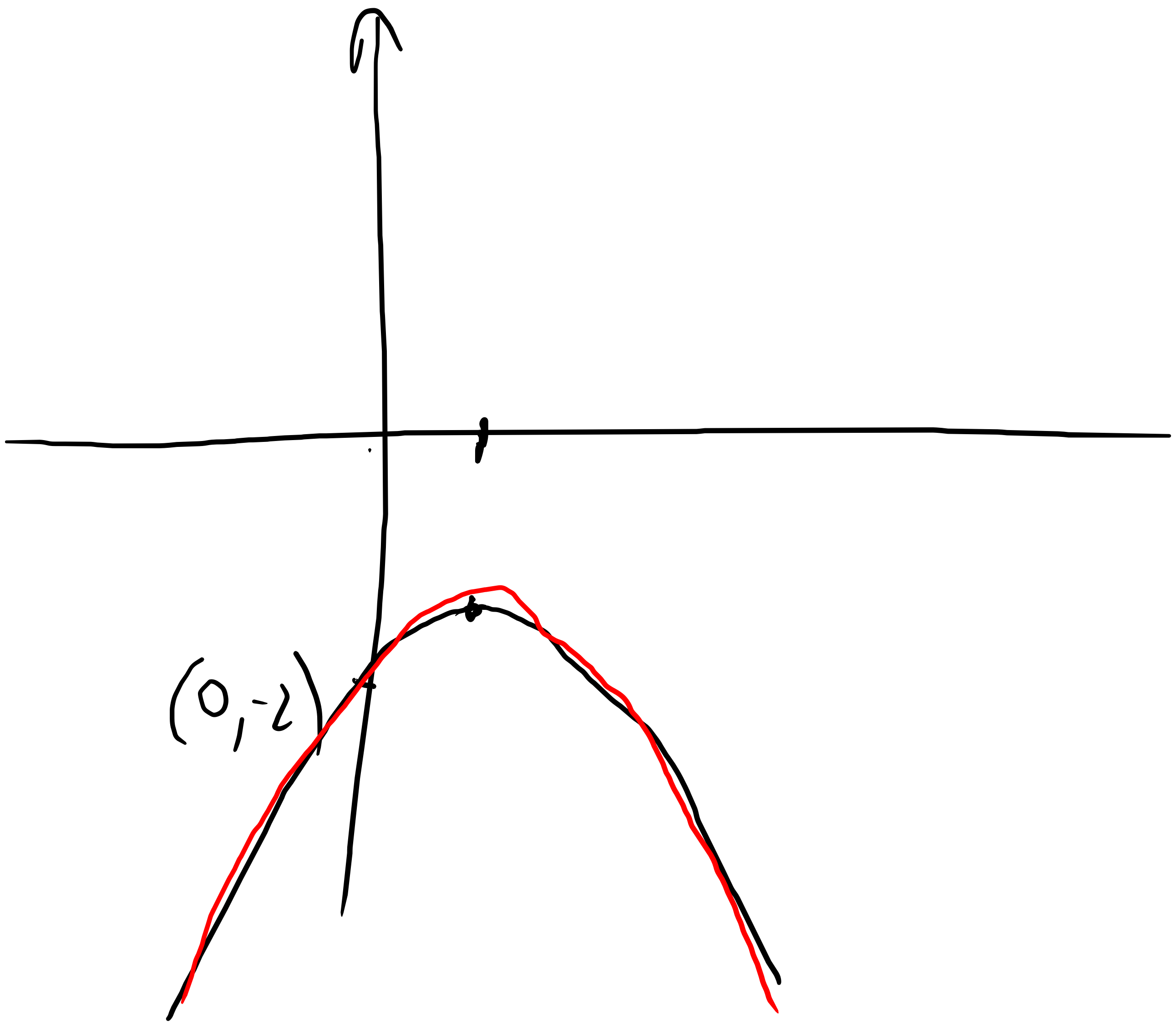


$$y = -x^2 + x - 2$$

$A = -1$

$B = 1$

$C = -2$



Def Siano A e B due insiemi ordinati (ad esempio $\mathbb{N}$, $\mathbb{R}$ ...)

Diremo che $f: A \to B$ è crescente (decrescente) se comunque presi a_1 e a_2 in A con $a_1 < a_2$, risulta

$$f(a_1) < f(a_2) \qquad \big(f(a_1) > f(a_2)\big)$$