$\mathbb{R}^2 = \mathbb{R}\times\mathbb{R}$ piano cartesiano

$$d_{\mathbb{R}^2}(P,Q) = \sqrt{(x_P-x_Q)^2+(y_P-y_Q)^2}$$

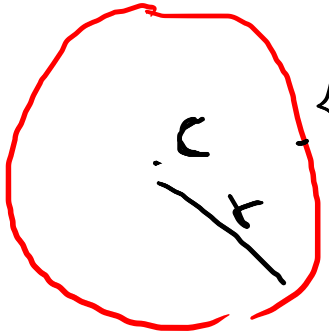
$P=(x_P,y_P)$ $Q=(x_Q,y_Q)$

$\mathbb{R}^3 = \mathbb{R}\times\mathbb{R}\times\mathbb{R}$ spazio tridimensionale

$$d_{\mathbb{R}^3}(P,Q) = \sqrt{(x_P-x_Q)^2+(y_P-y_Q)^2+(z_P-z_Q)^2}$$

$P=(x_P,y_P,z_P)$ $Q=(x_Q,y_Q,z_Q)$

Circonferenza di centro C e raggio r>0

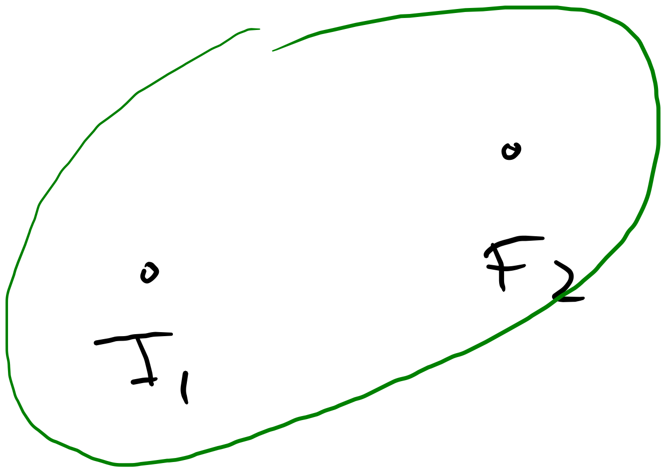


$\{P\in\mathbb{R}^2 : d_{\mathbb{R}^2}(P,C)=r\}$

Sfera di centro C e raggio r>0

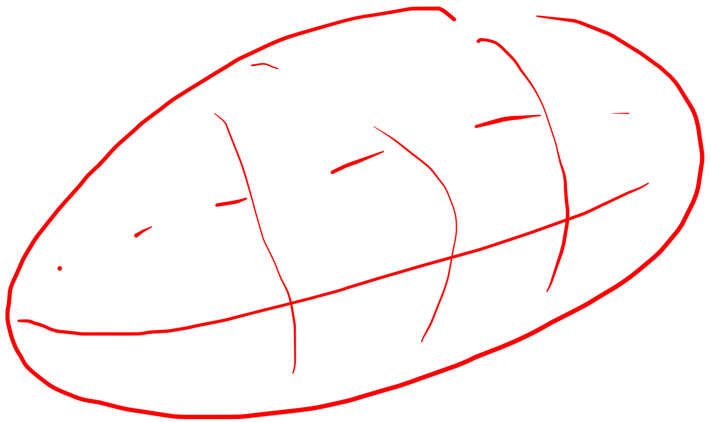
$=\{P\in\mathbb{R}^3 \; d_{\mathbb{R}^3}(P,C)=r\}$

Ellisse di fuochi F_1 e F_2



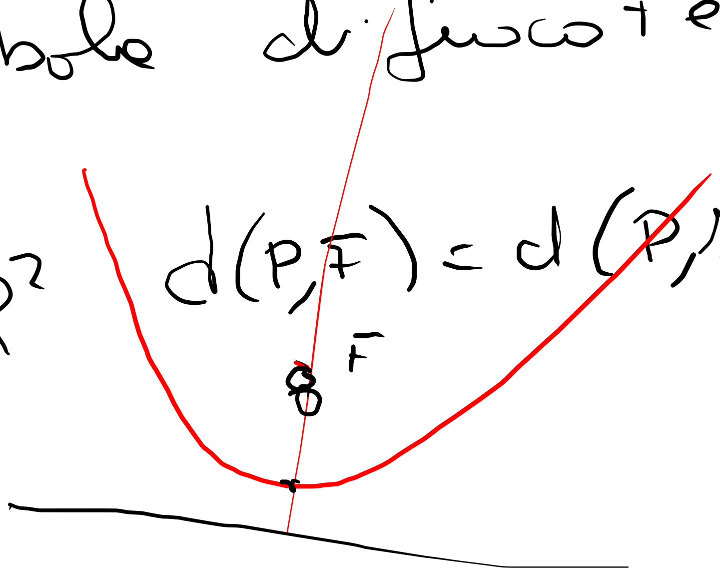
$$\{P \mid d_{\mathbb{R}^2}(P,F_1)+d_{\mathbb{R}^2}(P,F_2)=k>0\}$$

Ellissoide

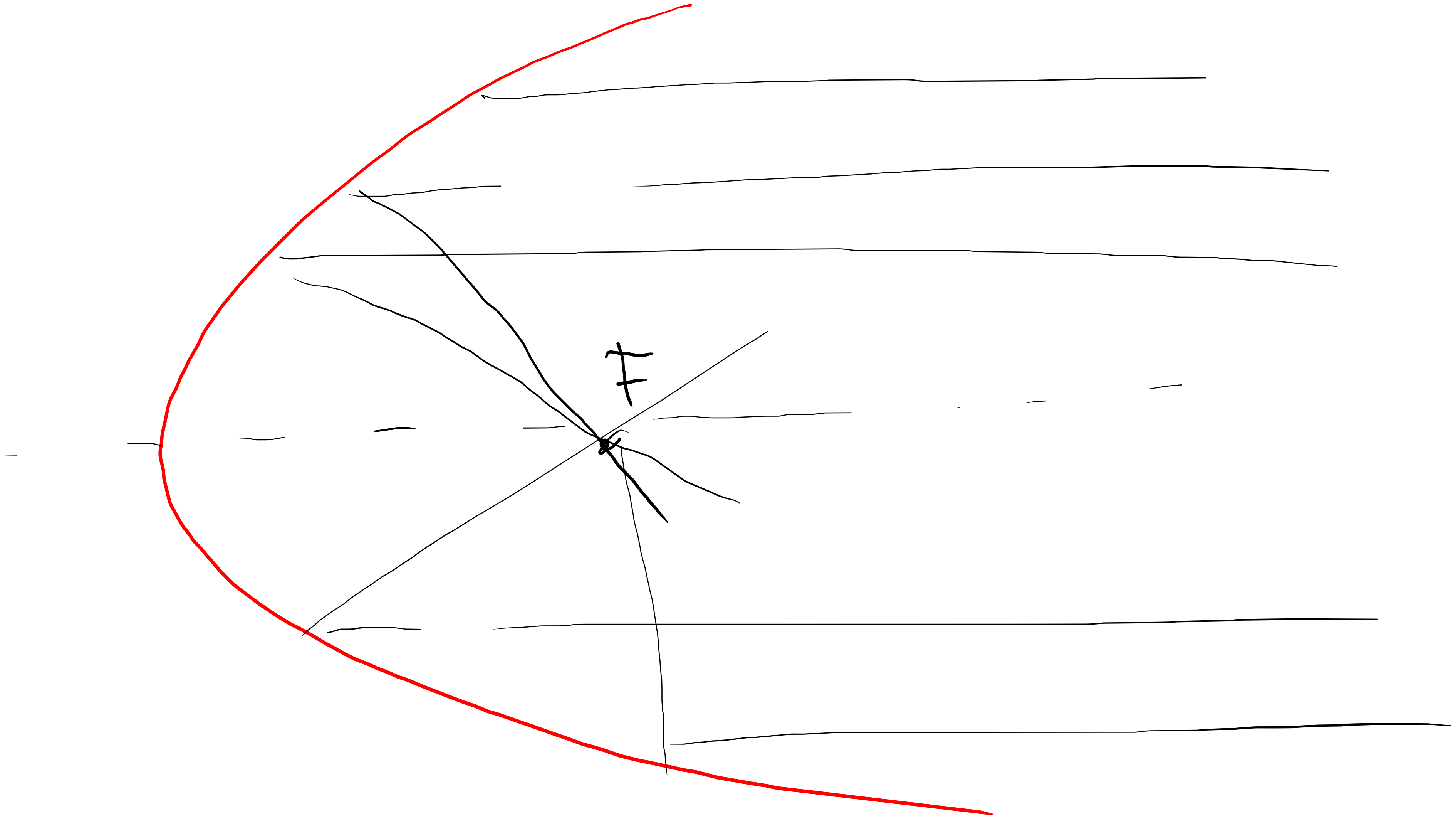


Parabola di fuoco F e direttrice s

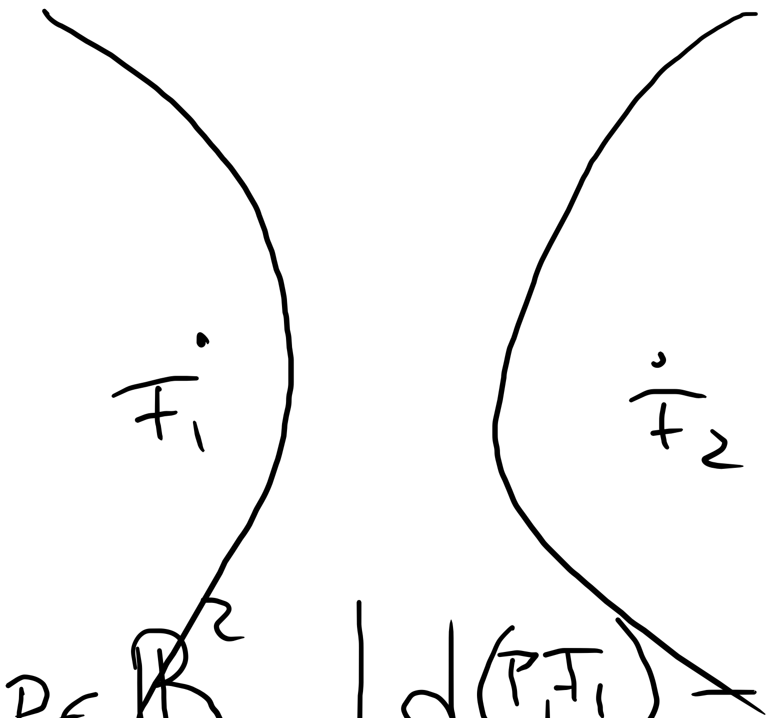
$$\{P\in\mathbb{R}^2 \mid d(P,F)=d(P,s)\}$$



Paraboloide

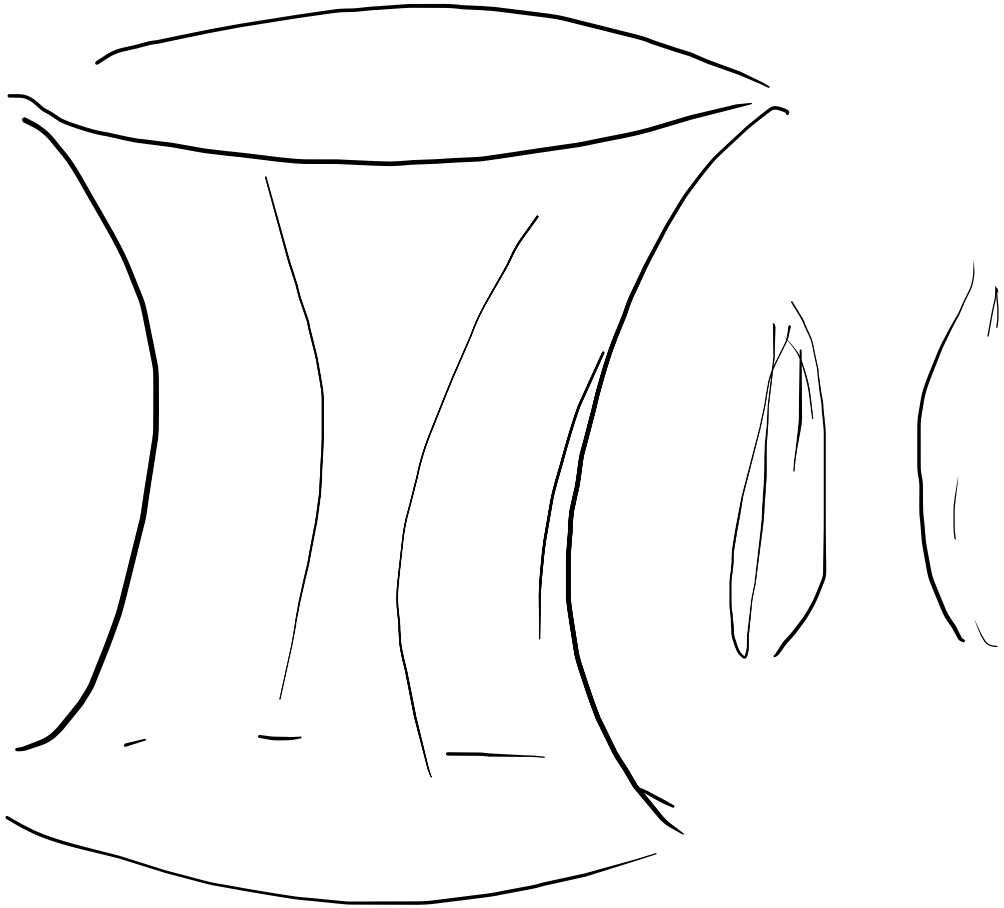

F

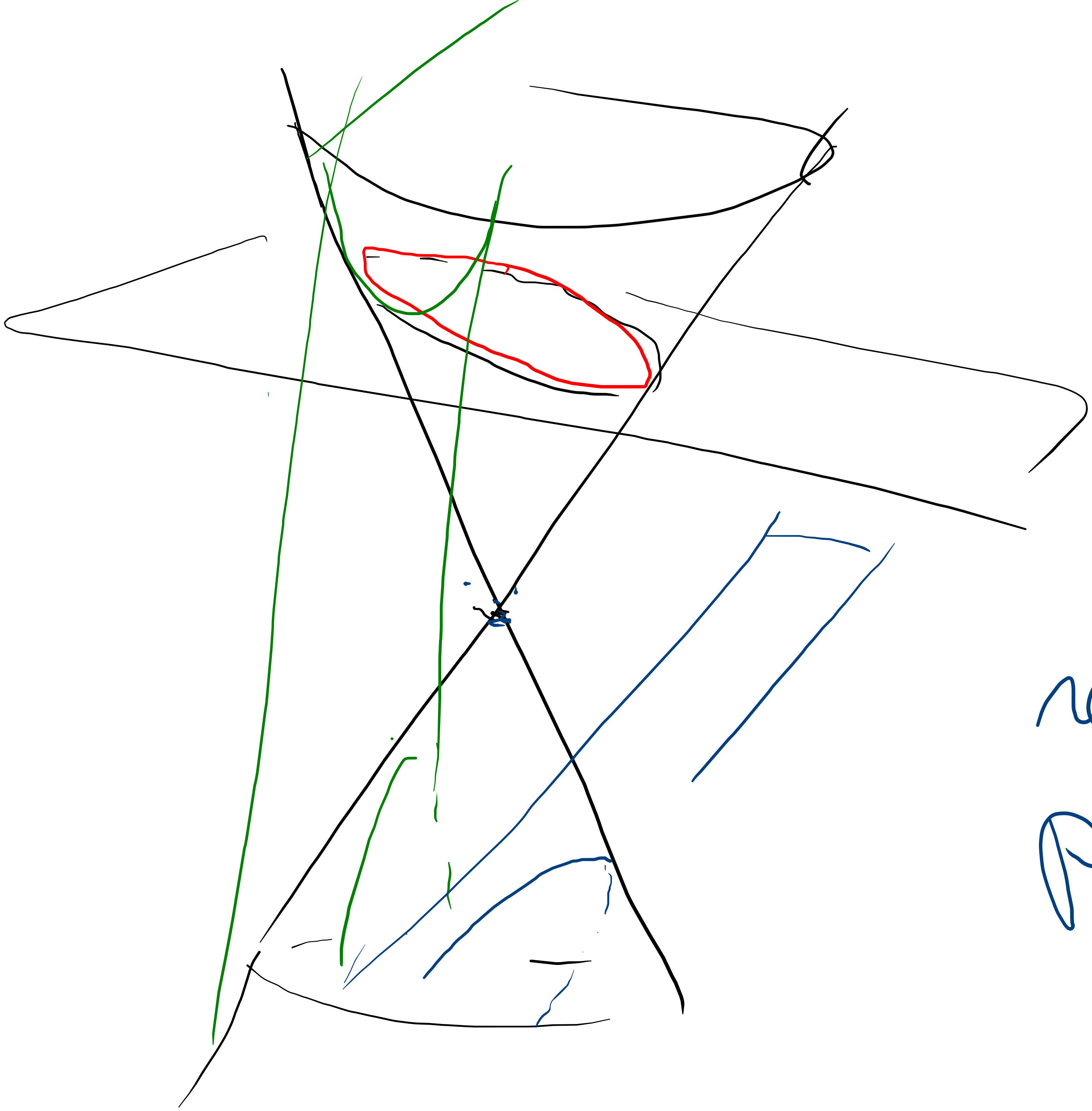
Iperbole



$$\{P \in \mathbb{R}^2 \mid |d(P,F_1) - d(P,F_2)| = k\}$$

Iperboloidi



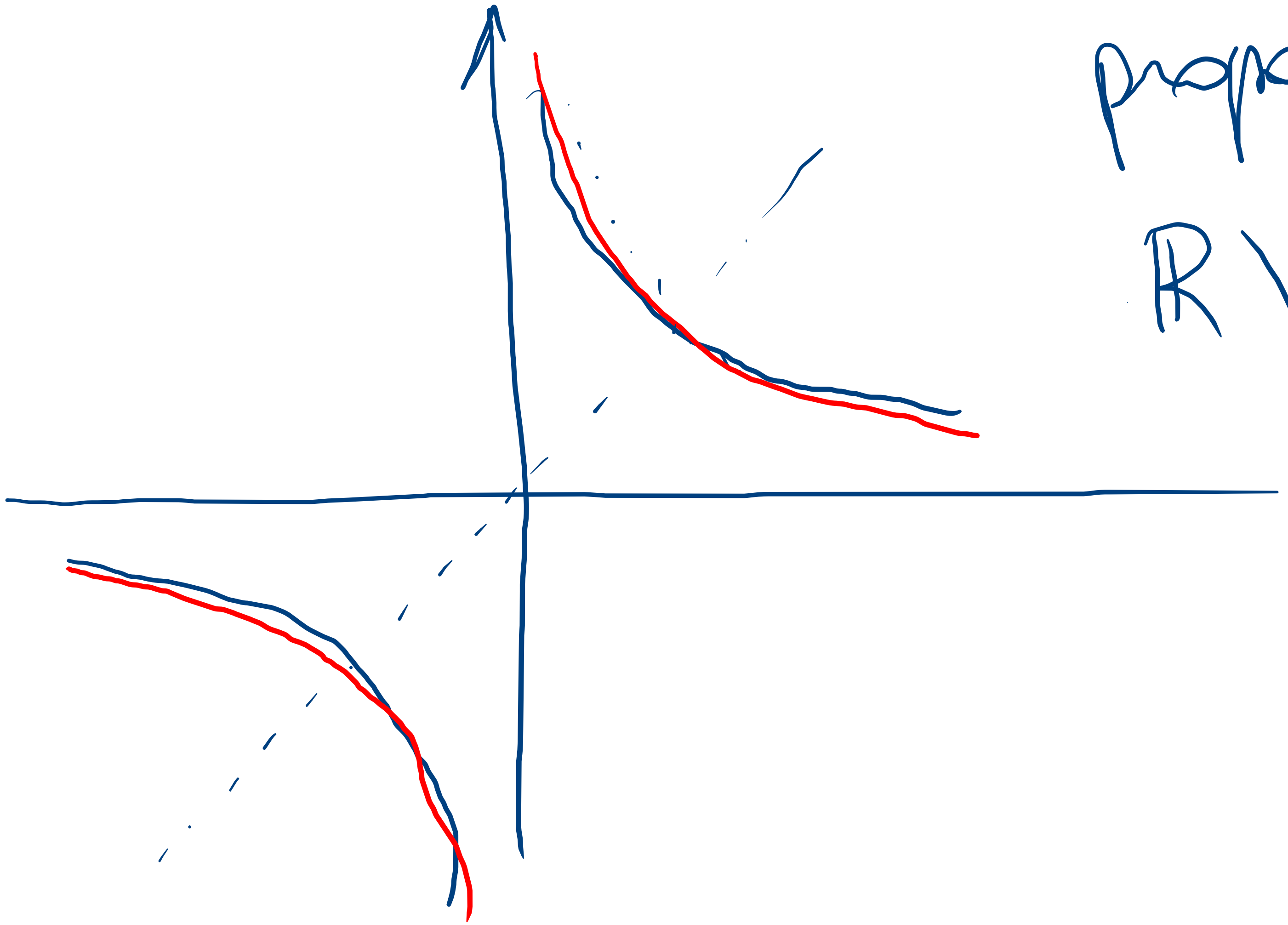


Coniche come intersezione del cono circolare retto con un piano.

$$y = \frac{1}{x}$$

dipendenza di y da x inversamente proporzionale

$\mathbb{R} \setminus \{0\}$



$$\boxed{x \cdot y = 1}$$

Retta nel piano come asse
di un segmento

$A, B \in \mathbb{R}^2 \qquad A \neq B$

$A = (x_A, y_A) \qquad B = (x_B, y_B)$

$P = (x, y) \qquad d_{\mathbb{R}^2}(P, A) = d_{\mathbb{R}^2}(P, B)$

$$\sqrt{(x - x_A)^2 + (y - y_A)^2} = \sqrt{(x - x_B)^2 + (y - y_B)^2}$$

$$\Updownarrow$$

$$(x - x_A)^2 + (y - y_A)^2 = (x - x_B)^2 + (y - y_B)^2$$

$$\cancel{x^2} - 2x x_A + x_A^2 + \cancel{y^2} - 2y y_A + y_A^2 =$$

$$= \cancel{x^2} - 2x x_B + x_B^2 + \cancel{y^2} - 2y y_B + y_B^2$$

$$2\underline{x}(x_B - x_A) + 2\underline{y}(y_B - y_A) + \underbrace{x_A^2 + y_A^2 - x_B^2 - y_B^2} = 0$$

$$\boxed{ax + by + c = 0}$$

$a = 0 \iff x_A = x_B \qquad b = 0 \iff y_A = y_B$

Ripetendo l'analogo calcolo in $\mathbb{R}^3$

si trova che

$$ax + by + cz + d = 0$$

è l'equazione di un piano in $\mathbb{R}^3$

se a, b, c non sono tutti nulli.

$$ax+by+c=0 \quad \dots\dots \quad r$$

$$a'x+b'y+c'=0 \quad \dots \quad r'$$

$$r=\{(x,y)\in\mathbb{R}^2 : ax+by+c=0 \quad a,b \text{ non entrambi nulli}\}$$

$$r'=\{(x,y)\in\mathbb{R}^2 \quad a'x+b'y+c'=0 \quad a',b' \text{ non entrambi nulli}\}$$

$r\cap r'\neq\emptyset \iff$ o r e r' coincidono o r e r' sono incidenti in un unico punto $\underline{P}_0$.

$r \cap r' = \emptyset \iff r, r'$ sono distinte
e parallele.

Caso 1 $\quad x = k \quad \ldots \quad r$

$\quad x = k' \quad\quad r'$

2 $\quad b \neq 0 \quad b' \neq 0$

$$y = -\frac{a}{b}x - \frac{c}{b} \quad \ldots \quad r$$

$$y = -\frac{a'}{b'}x - \frac{c'}{b'} \quad \ldots \quad r'$$

$$\begin{cases} y = -\frac{a}{b}x - \frac{c}{b} \\ y = -\frac{a'}{b'}x - \frac{c'}{b'} \end{cases} \longrightarrow -\frac{a'}{b'}x - \frac{c'}{b'} = -\frac{a}{b}x - \frac{c}{b}$$

sistema di equazioni

$$x\left(\frac{a'}{b'} - \frac{a}{b}\right) = \frac{c}{b} - \frac{c'}{b'}$$

$$\frac{a'}{b'} - \frac{a}{b} = T$$

$$x(m - m') = q' - q$$

$$x = \frac{q' - q}{m - m'} \quad \text{se } m \neq m'$$

$$T \neq 0 \iff \underbrace{-\frac{a'}{b'}}_{= m'} \neq -\frac{a}{b} = m$$

$$\begin{cases} y = -2x + 1 \\ y = x - 3 \end{cases}$$

$m = -2$

$m' = 1$ $\quad m \neq m'$

UNA SOLA SOLUZIONE COMUNE

$$P = \left(\frac{-3-1}{-2-1} = 4/3 \, , \; -5/3 \right)$$

$x = 4/3 \quad y = -2 \cdot 4/3 + 1$

$= 4/3 - 3$

Eq secondo grado $x^2 = k$

$x^2 \geq 0 \quad \forall x \in \mathbb{R}$

$\Longrightarrow$ Posso risolvere (ossia trovare un x tale che)

$x^2 = k$

$\Longleftrightarrow k \geq 0$.

Se $k \geq 0$

$x_1 = \sqrt{k} \qquad x_2 = -\sqrt{k}$

$$Ax^2+Bx+C=0 \qquad A\neq 0$$

$4A$

$$4A^2x^2+4ABx+4AC=0$$

$$(2Ax+B)^2-B^2 \qquad +4AC=0$$

$$(2Ax+B)^2 = B^2-4AC=\Delta$$

$$x=(2Ax+B)$$

$$x^2=\Delta$$

Δ è detto <u>discriminante</u>

Se $\Delta \geq 0$

$$(2Ax+B)^2 = \Delta$$

ho per adesso

$$x^2 = \Delta \qquad x = 2Ax+B$$

$A \neq 0$

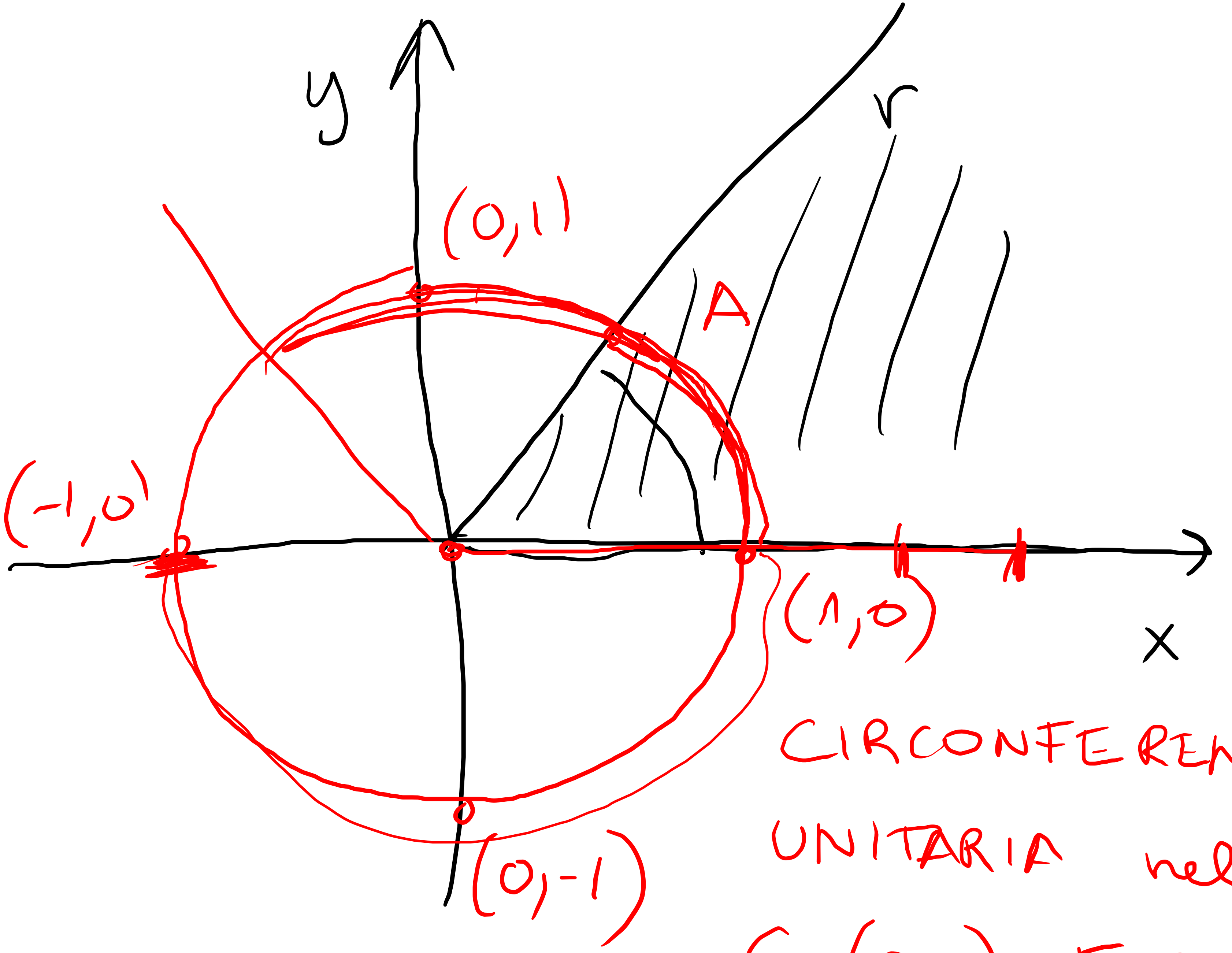
$$x_1 = \sqrt{\Delta} \qquad x_2 = -\sqrt{\Delta}$$

$$\| \qquad\qquad \|$$

$$2Ax_1 + B \qquad 2Ax_2 + B$$

$$\Longrightarrow \quad X_1 = \frac{-B+\sqrt{\Delta}}{2A}$$

$$X_2 = \frac{-B-\sqrt{\Delta}}{2A}$$



CIRCONFERENZA UNITARIA nell'ORIGINE

$C = (0,0) \quad r = 1$

r, r' sono parallele distinte $\Longleftrightarrow$ $m = m'$
$\quad\; \| \qquad \|$
$-\frac{a}{b} \quad -\frac{a'}{b'}$

parallele coincidenti $\quad$ $b \neq 0 \;\; b' \neq 0$

$\Longleftrightarrow$ $\boxed{\begin{array}{c} m = m' \\ q = q' \\ -\frac{a}{b} = -\frac{a'}{b'} \\ -\frac{c}{b} = -\frac{c'}{b'} \end{array}}$

$ax + by + c = 0$

$a'x + b'y + c' = 0$

Se $b = 0 \; b' = 0$

$\frac{c}{a} = \frac{c'}{a'} \Longleftrightarrow$

r ed r' sono incidenti in un solo punto $\quad (b \neq 0 \;\; b' \neq 0)$

$\Longleftrightarrow$ $m \neq m'$

$b = 0 \quad b' \neq 0$

Se $b=0 \Rightarrow a \neq 0$ $\quad x = -\dfrac{c}{a} = k$

eq retta verticale

Se $b \neq 0$

$$ax + by + c = 0$$

$$y = -\frac{a}{b}x - \frac{c}{b}$$

$$= mx + q$$

$$\boxed{m = -\frac{a}{b}}$$

$$\boxed{q = -\frac{c}{b}}$$