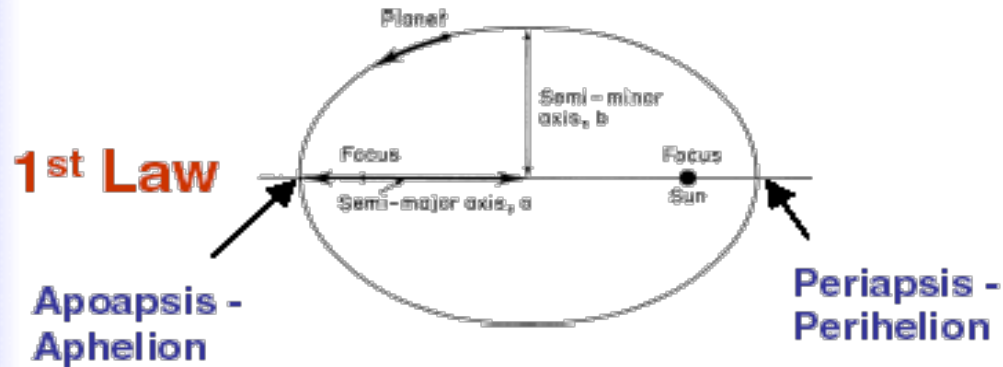


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# Le orbite di un satellite

- Leggi di Keplero
- Leggi di Newton
- Equazioni del moto
- Anomalie
- Elementi Orbitali
- Perturbazioni
- Orbite varie

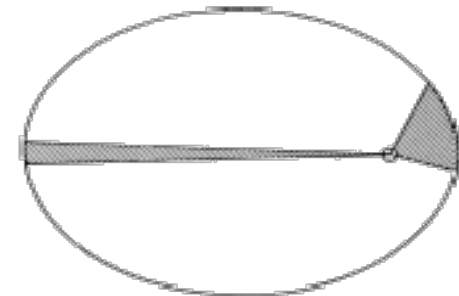
# Leggi di Keplero



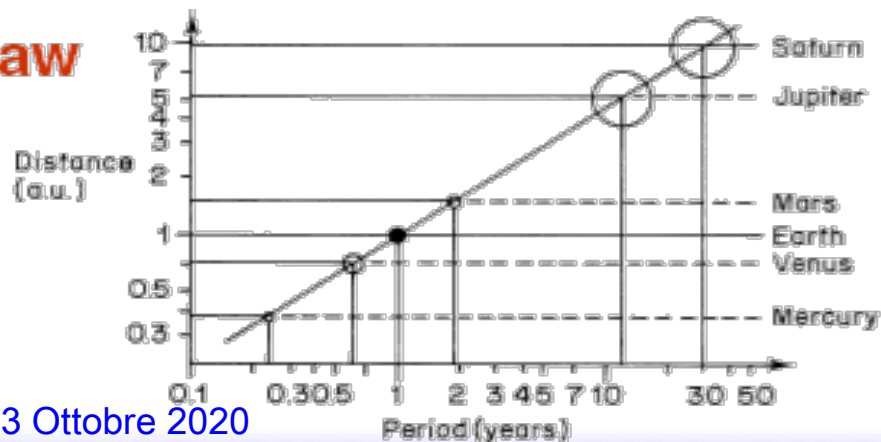
L'orbita di un pianeta è un'ellisse con il Sole in uno dei fuochi

Una linea congiungente il Sole ad un pianeta spazza aree uguali in intervalli di tempo uguali

**2nd Law**



**3rd Law**



Il quadrato del periodo dell'orbita di un pianeta è proporzionale al cubo della sua distanza media dal Sole

# Leggi di Newton 1/2

- 1<sup>a</sup> Legge: La legge di inerzia
- 2<sup>a</sup> Legge: Forza = massa × accelerazione
- 3<sup>a</sup> Legge: Azione e reazione

In assenza di forze, un "corpo" in quiete resta in quiete, e un corpo che si muova a velocità rettilinea e uniforme continua così indefinitamente

LA LEGGE DI GRAVITA':  $F = \frac{GMm}{r^2}$

F Forza gravitazionale tra due corpi

G Costante di gravitazione universale:  $G = 6.670 \times 10^{-11} \text{ N.m}^2\text{kg}^{-2}$

M Massa di un corpo (Terra o Sole)

m Massa di un altro corpo (il satellite)

r Separazione tra i corpi

$$GM_{\text{Earth}} = \mu = 3.986004418 \times 10^{14} \text{ m}^3.\text{s}^{-2}$$
$$\sim 400'000 \text{ km}^3.\text{s}^{-2}$$

# Leggi di Newton 1/2

- 1<sup>a</sup> Legge: La legge di inerzia
- 2<sup>a</sup> Legge: Forza = massa × accelerazione
- 3<sup>a</sup> Legge: Azione e reazione

Quando una forza è applicata a un oggetto, esso accelera.  
L'accelerazione  $a$  è nella direzione della forza ed è proporzionale alla sua grandezza, ed inversamente alla massa dell'oggetto.

$$\text{LA LEGGE DI GRAVITA': } F = \frac{GMm}{r^2}$$

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# Leggi di Newton 1/2

- 1<sup>a</sup> Legge: La legge di inerzia
- 2<sup>a</sup> Legge: Forza = massa × accelerazione
- 3<sup>a</sup> Legge: Azione e reazione

Per ogni azione esiste una reazione uguale e contraria: se il corpo n.1 esercita una forza F sul corpo n.2, allora il corpo n.2 eserciterà sul corpo n.1 una forza di uguale grandezza e di verso opposto

$$\text{LA LEGGE DI GRAVITA': } F = \frac{GMm}{r^2}$$

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- 2<sup>a</sup> Legge: Forza = massa × accelerazione
- 3<sup>a</sup> Legge: Azione e reazione

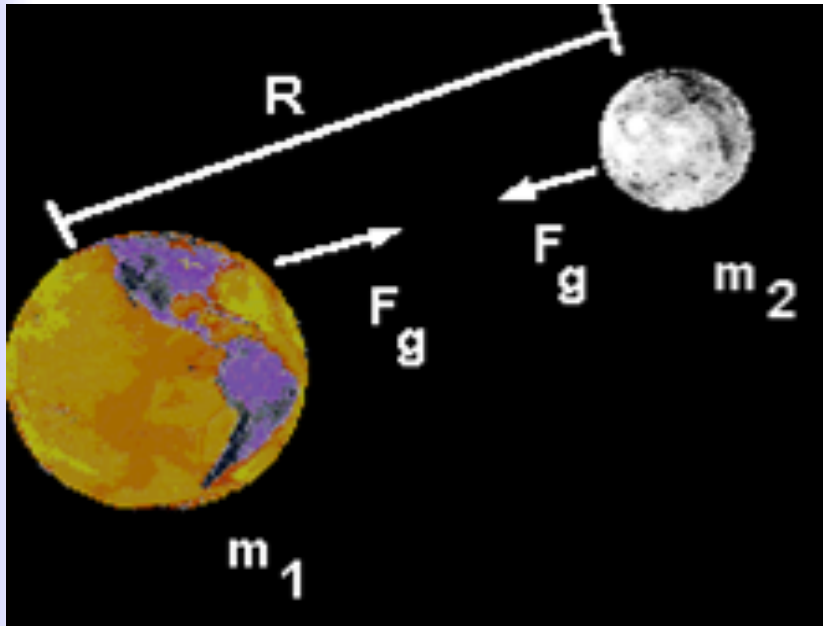
Quando una forza è assistita da un'altra, il risultato è la somma delle due forze. L'azione e la reazione sono forze di uguale intensità e di uguale direzione, ma di verso opposto.

$$\text{LA LEGGE DI GRAVITA': } F = \frac{GMm}{r^2}$$

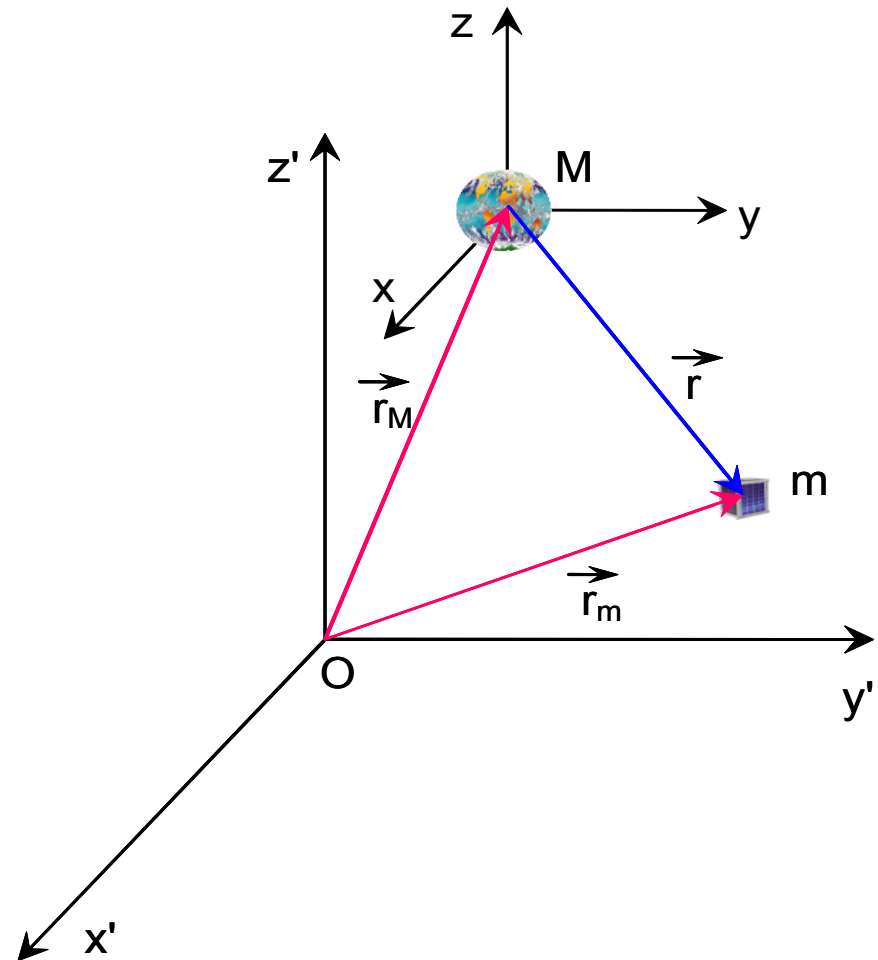
F	Forza gravitazionale tra due corpi
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M	Massa di un corpo (Terra o Sole)
m	Massa di un altro corpo (il satellite)
r	Separazione tra i corpi

$$GM_{\text{Earth}} = \mu = 3.986004418 \times 10^{14} \text{ m}^3.\text{s}^{-2}$$
$$\sim 400'000 \text{ km}^3.\text{s}^{-2}$$

# Leggi di Newton 2/2



Sistema a due corpi  
di massa  $M$  e  $m$   
( $M \gg m$ )



# Equazioni del moto 1/2

---

$$\ddot{\vec{r}} = -\mu \vec{r} / r^3$$



$$a_r = -\mu / r^2$$

$$a_\theta = 0$$

MOTO CENTRALE

$$v_r = dr/dt$$

velocità radiale

$$v_\theta = r d\theta/dt$$

velocità tangenziale

$$a_r = d^2r/dt^2 - r (d\theta/dt)^2$$

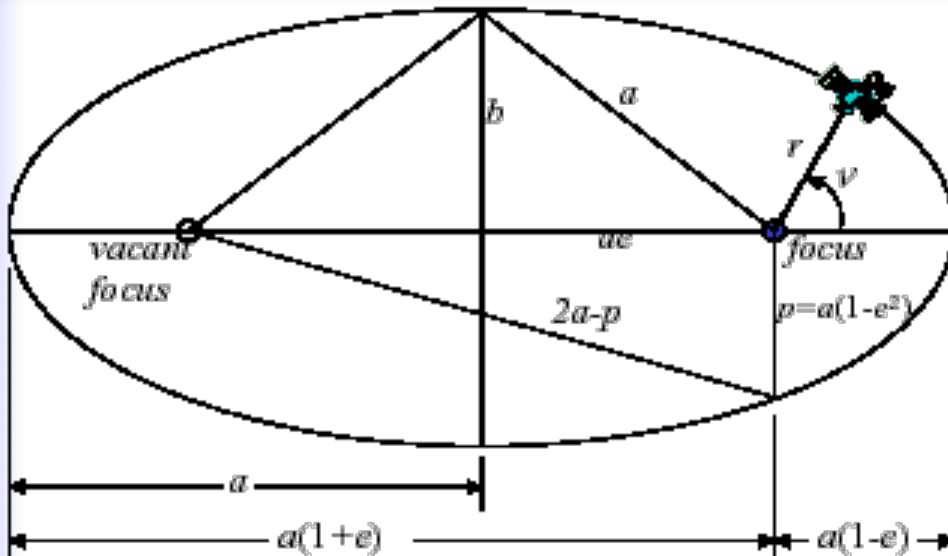
accelerazione radiale

$$a_\theta = r d^2\theta/dt^2 + 2 dr/dt d\theta/dt$$

accelerazione tangenziale

$$= 1/r d(r^2 d\theta/dt)/dt$$

# Equazioni del moto 2/2



S = satellite

T = terra (fuoco)

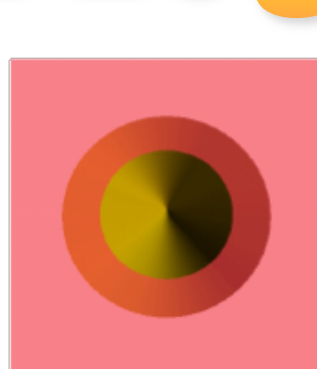
$\vec{r}$  = vettore posizione S rispetto al centro di T

$\vec{v}$  = vettore velocità S rispetto a T

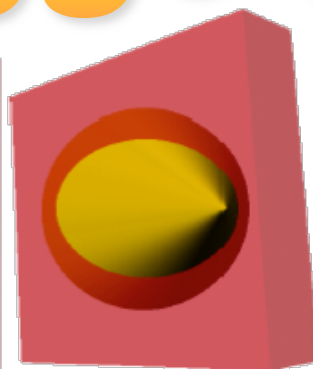
- a      semiasse maggiore
- b      semiasse minore:  $b = a\sqrt{1-e^2}$
- c      semidistanza fra i fuochi
- e      eccentricità:  $e = c/a = \sqrt{a^2-b^2}/a$
- p      semi-latus rectus:  $p = a(1-e^2)$
- $r_p$     raggio del perigeo
- $r_a$     raggio dell'apogeo
- $\theta$     angolo polare (anomalia vera)
- $\varphi$     anomalia eccentrica

# Coniche 1/3

$$r = p / (1 + e \cos \theta)$$



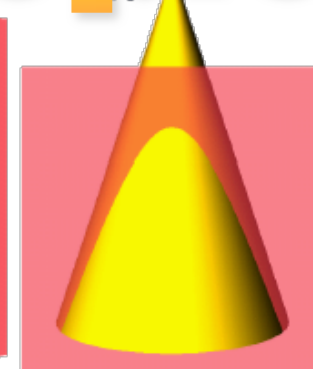
Circle  
 $e = 0$



Ellipse  
 $0 < e < 1$



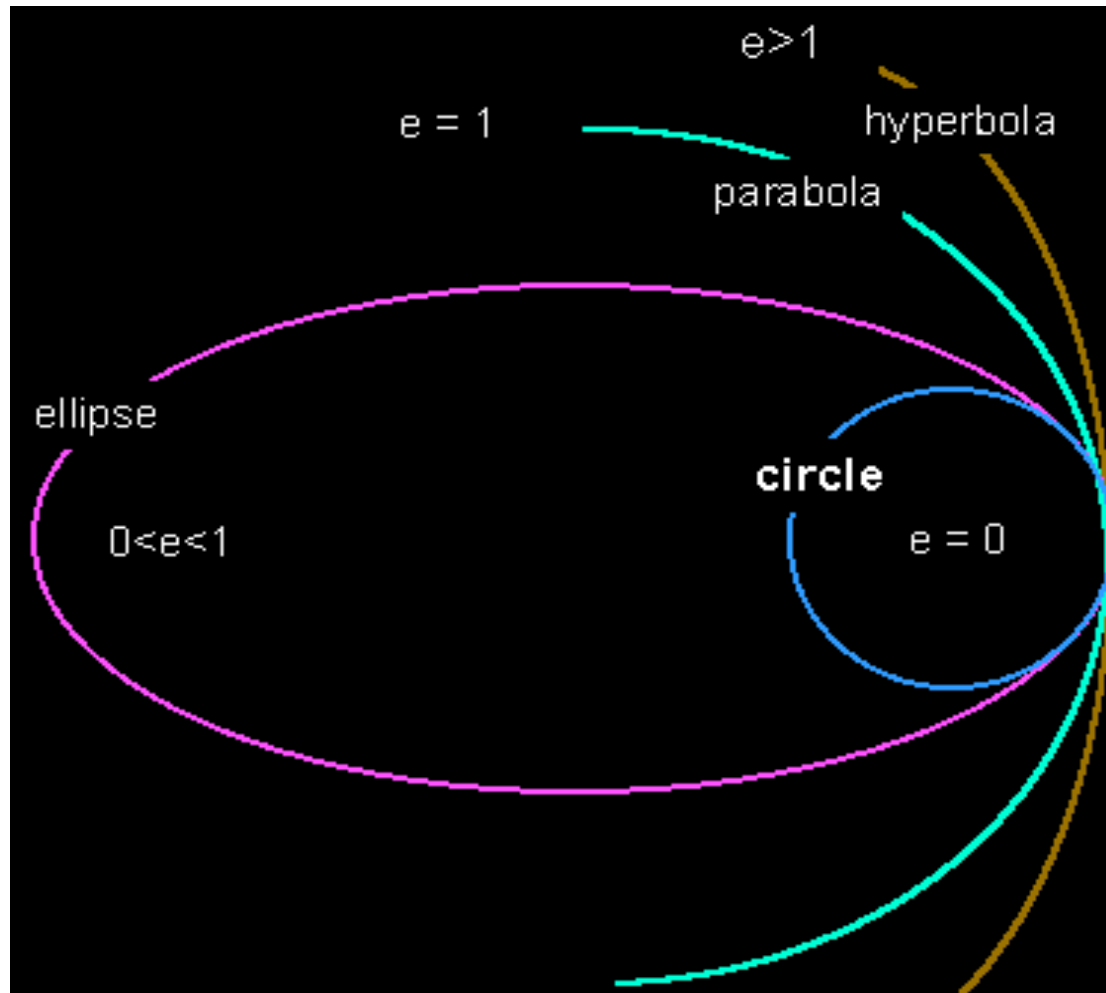
Parabola  
 $e = 1$



Hyperbola  
 $e > 1$

# Coniche 2/3

$$r = p / (1 + e \cos \theta)$$

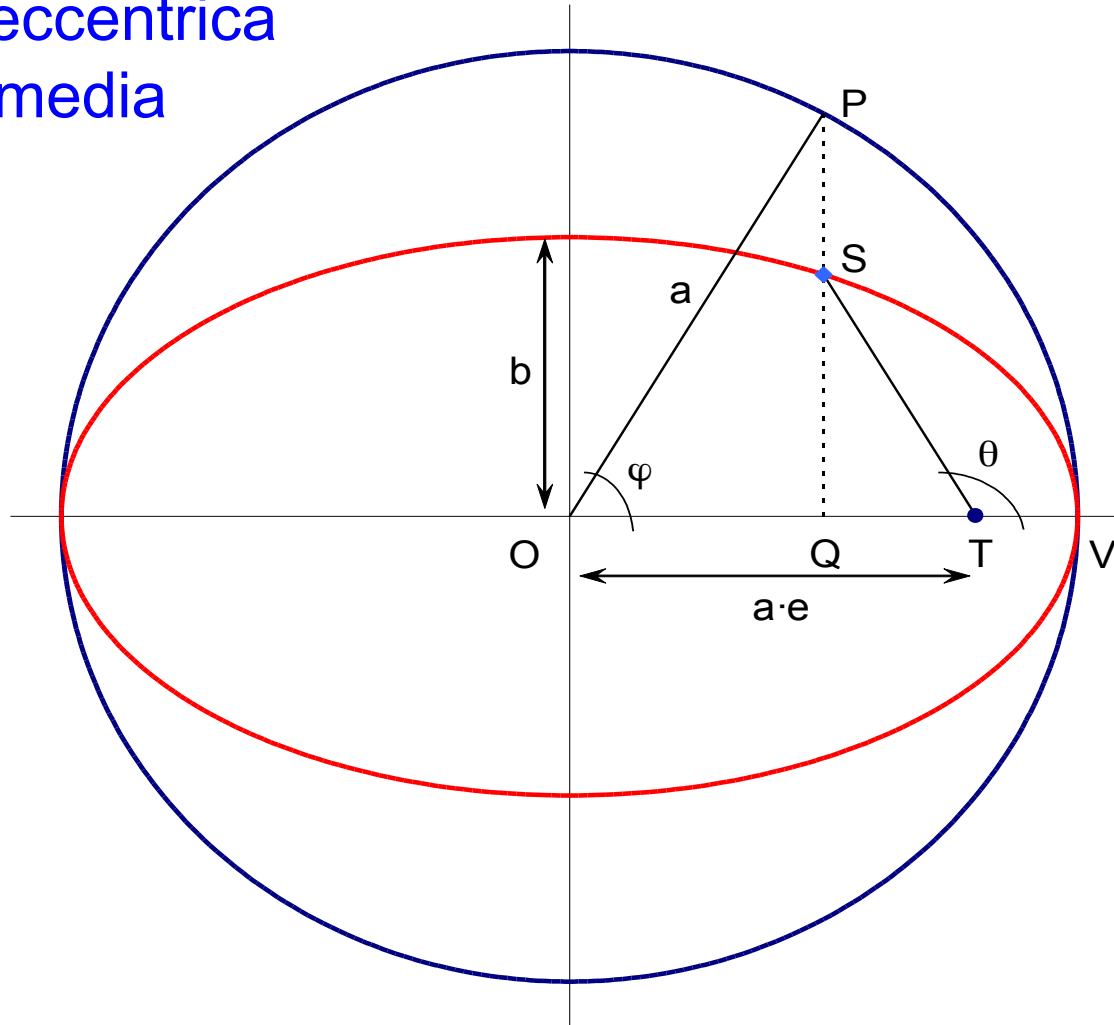


# Coniche 3/3

$\theta$  = anomalia vera

$\varphi$  = anomalia eccentrica

$M$  = anomalia media





# Anomalia

---

anomalia vera:  $r = \frac{a(1-e^2)}{1+e \cdot \cos \theta}$

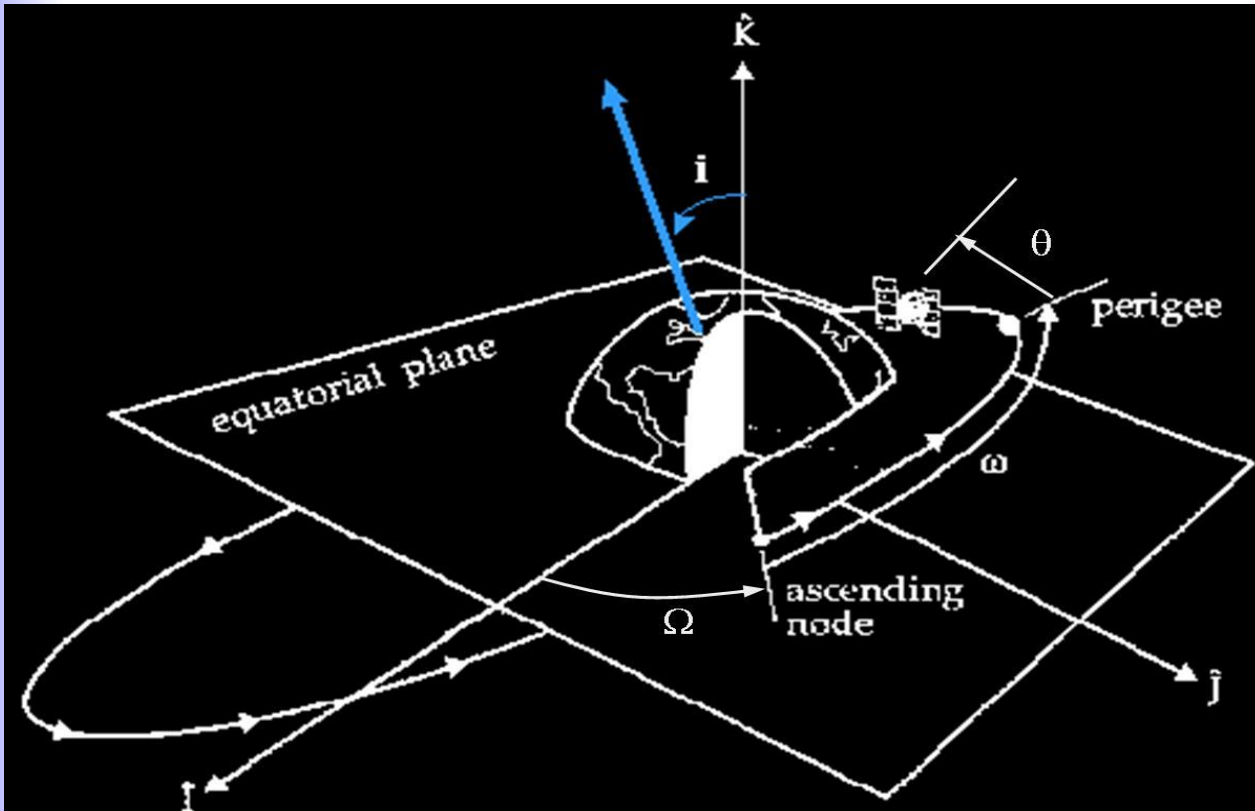
anomalia eccentrica:  $r = a(1-e \cdot \cos \varphi)$

anomalia media:  $M = \varphi - e \cdot \sin \varphi = \sqrt{\frac{\mu}{a^3}}(t - t_p)$

$$\cos \theta = \frac{e - \cos \varphi}{e \cdot \cos \varphi - 1}$$

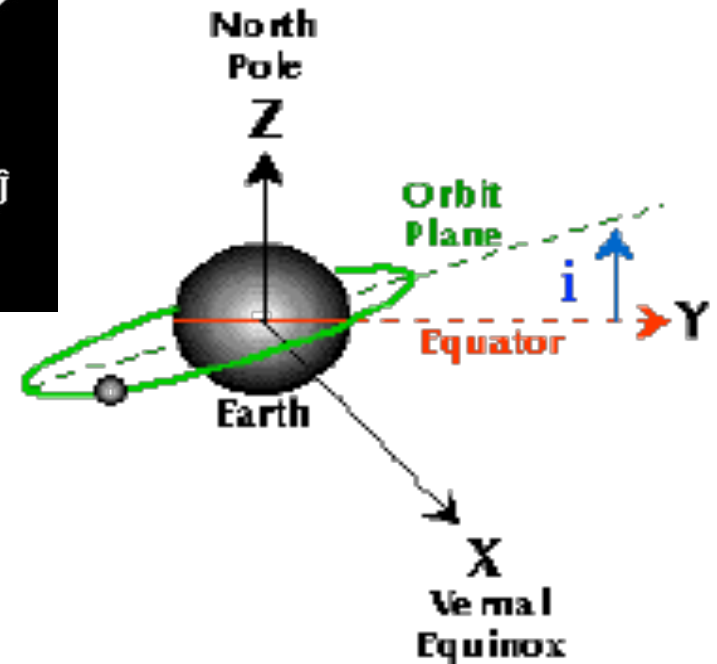


# Elementi Orbitali 2/2



esempi:

- orbita circolare,  $i=0$ ,  $\Omega, \omega$ ?
- orbita circolare,  $i=10^\circ$ ,  $\Omega, \omega$ ?
- $\Omega=0$ ,  $\omega=0$ ,  $90^\circ$ ,  $180^\circ$
- $\Omega=90^\circ$ ,  $\omega=0$ ,  $90^\circ$ ,  $180^\circ$



# Perturbazioni 1/3

$$d^2\mathbf{r}/dt^2 + \mu/r^3 \mathbf{r} = \mathbf{f}$$

**Energia:**

$$da/dt = 2a^2 / \sqrt{\mu p} ( f_r e \sin\theta + f_\theta (1+e \cos\theta) )$$

**Momento Angolare Specifico:**

$$d\Omega/dt \sin i = 1/\sqrt{\mu p} r f_n \sin \nu \quad (\nu = \omega + \theta)$$

$$di/dt = 1/\sqrt{\mu p} r f_n \cos \nu$$

$$de/dt = \sqrt{p/\mu} ( f_r \sin \theta + f_\theta (\cos\theta + \cos\varphi) )$$

$$d\omega/dt + d\Omega/dt \cos i = 1/e \sqrt{p/\mu} ( -f_r \cos\theta + f_\theta (1+r/p) \sin\theta )$$

# Perturbazioni 2/3

## Triassialità della Terra

$$\Phi = \frac{\mu}{r} \left( 1 - \sum_{n=2}^{\infty} J_n \left( \frac{R_{eq}}{r} \right)^n P_n(\sin(\lambda)) \right) \quad (f = -\nabla\Phi^*)$$

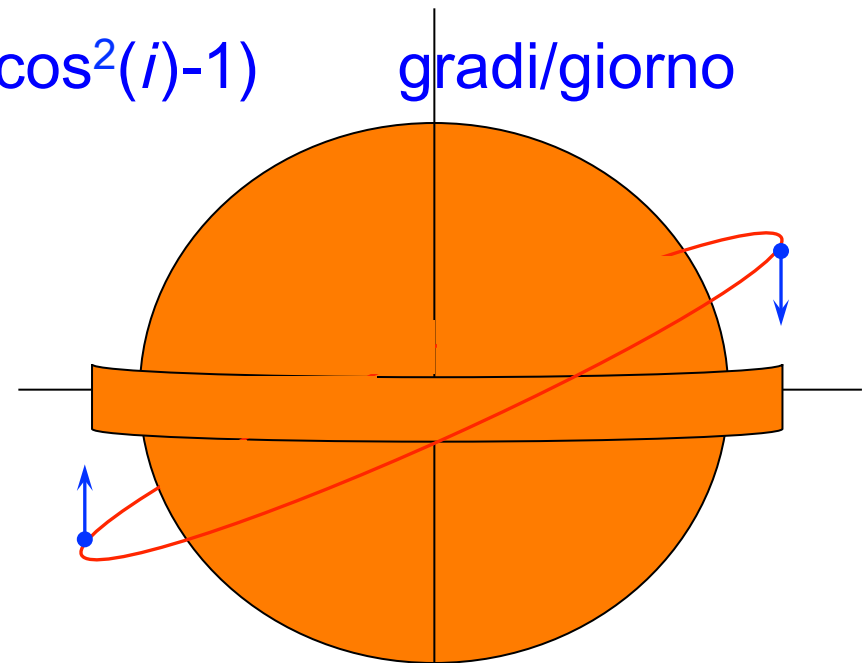
$$d\Omega/dt \sim -9.97 (R_{eq}/a)^{3.5} (1-e^2)^{-2} \cos(i) \quad \text{gradi/giorno}$$

$$d\omega/dt \sim 4.98 (R_{eq}/a)^{3.5} (1-e^2)^{-2} (5 \cdot \cos^2(i) - 1) \quad \text{gradi/giorno}$$

$$r_p - r_{pe} \sim -6.8 \sin(i) \sin(\omega)$$

$$J_2 = 1.08263 \times 10^{-3}$$

$$J_3 = 2.54 \times 10^{-6}$$



# Perturbazioni 2/3

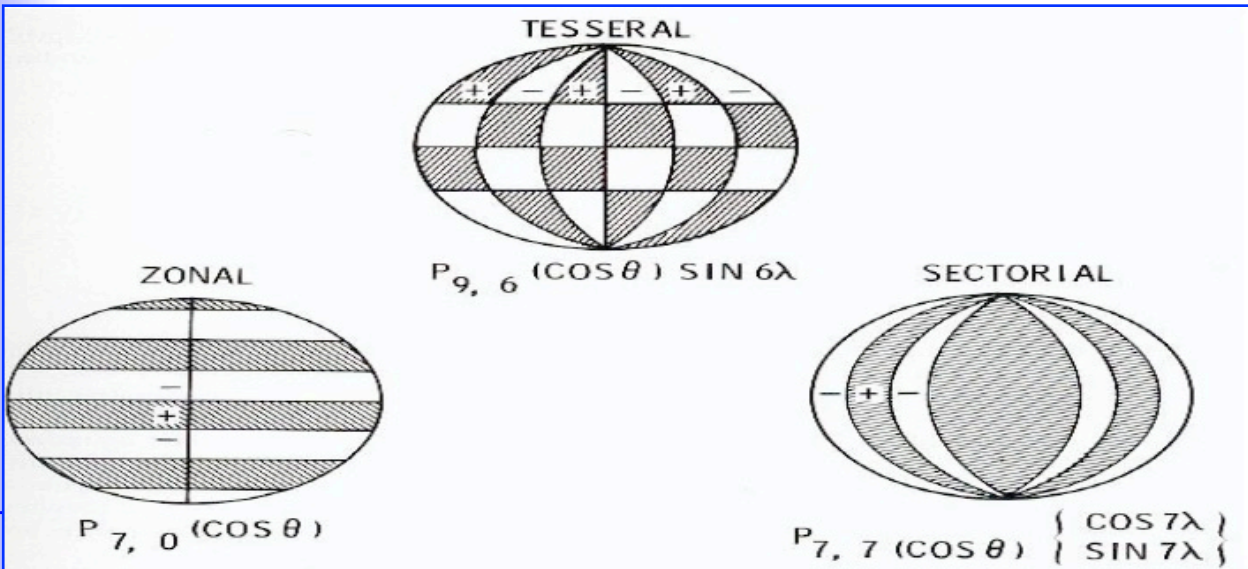
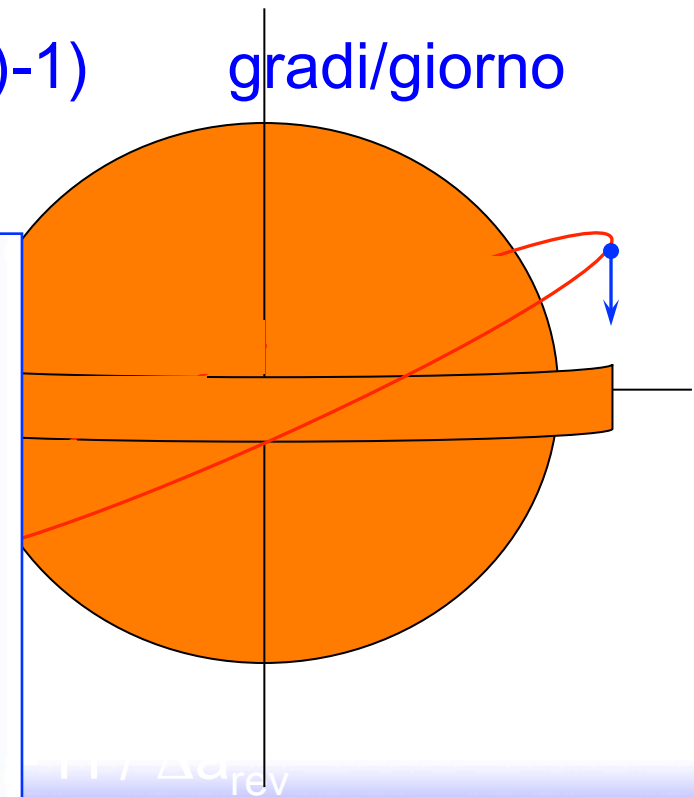
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grad

grad

km

Shape	Drag Coefficient
Sphere	0.47
Half-sphere	0.42
Cone	0.50
Cube	1.05
Angled Cube	0.80
Long Cylinder	0.82
Short Cylinder	1.15
Streamlined Body	0.04
Streamlined Half-body	0.09

Measured Drag Coefficients

## Attrito Atmosferico

$$F = ma = -1/2 \rho v^2 A C_D \quad \Rightarrow \quad \Delta a_{rev} = -2\pi (C_D A / m) \rho a^2$$

$$\Delta v_{rev} = \pi (C_D A / m) \rho a v$$

$$\tau_{life} = -H / \Delta a_{rev}$$

# Perturbazioni 2/3

## Triassialità della Terra

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$$J_2 = 1.08263 \times 10^{-3}$$

$$J_3 = 2.54 \times 10^{-6}$$



Rotazione terrestre

ZONAL

Forma a pera

Variazione raggio equatoriale

$P_{7,0}(\cos \theta)$

Rigonfiamento equatoriale e appiattimento

poli:  $r_{polo} = 21 \text{ km}$  più corto  $r_{eq}$

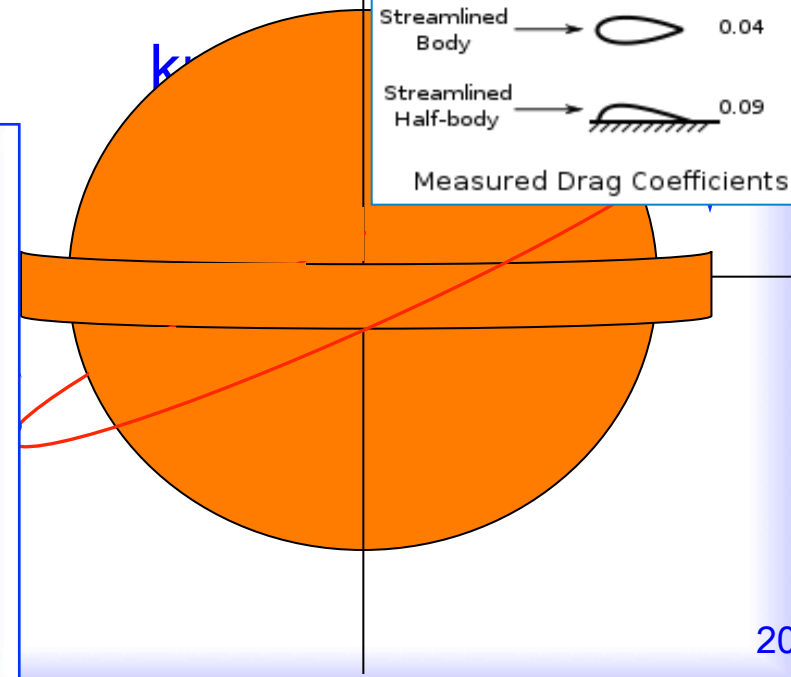
Centro di massa sud equatore

Assimmetria gravitazionale nel piano equatoriale  $\Delta r \sim 70 \text{ m}$

$P_{7,7}(\cos \theta) \begin{cases} \cos 7\lambda \\ \sin 7\lambda \end{cases}$

Shape	Drag Coefficient
Sphere	0.47
Half-sphere	0.42
Cone	0.50
Cube	1.05
Angled Cube	0.80
Long Cylinder	0.82
Short Cylinder	1.15
Streamlined Body	0.04
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Measured Drag Coefficients





# Perturbazioni 2/3

	a (km)	e	i (deg)	$\Delta\Omega$ (deg/day)	$\Delta\omega$ (deg/day)
Shuttle	6700	0	28	-7.35	12.05
GPS	26600	0	60	0.033	0.008
GEO	42160	0	0	-0.013	0.025
Molniya	26600	0.75	63.4	-0.30	0.00

## Attrito Atmosferico

$$F = ma = -1/2 \rho v^2 A C_D \quad \Rightarrow \quad \Delta a_{\text{rev}} = -2\pi (C_D A / m) \rho a^2$$

$$\Delta v_{\text{rev}} = \pi (C_D A / m) \rho a v$$

$$\tau_{\text{life}} = -H / \Delta a_{\text{rev}}$$

# Perturbazioni 3/3

## Forze Gravitazionali del Sole e della Luna

$$d\Omega/dt_L = -3.38 \cdot 10^{-3} \cos(i) / n \quad \text{gradi/giorno}$$

$$d\Omega/dt_S = -1.54 \cdot 10^{-3} \cos(i) / n \quad \text{gradi/giorno}$$

$$d\omega/dt_L = 1.69 \cdot 10^{-3} ( 5 \cdot \cos^2(i) - 1 ) / n \quad \text{gradi/giorno}$$

$$d\omega/dt_S = 0.77 \cdot 10^{-3} ( 5 \cdot \cos^2(i) - 1 ) / n \quad \text{gradi/giorno}$$

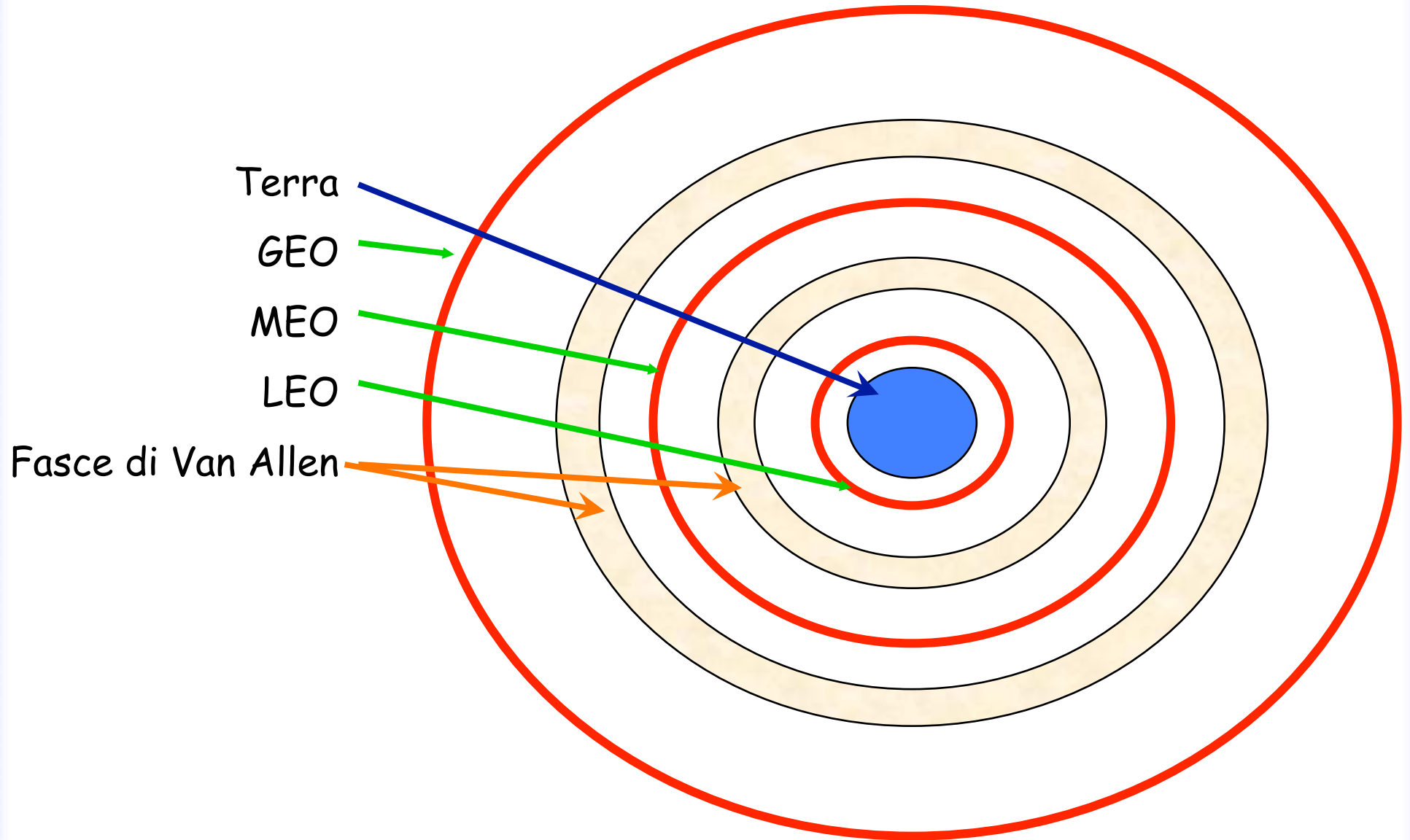
$n$  = numero di rivoluzioni / giorno

## Pressione di Radiazione

$$f = - 4.5 \cdot 10^{-6} (1+r) A/m \text{ m/s}^2$$

$r$  = coefficiente riflessione

# ORBITE



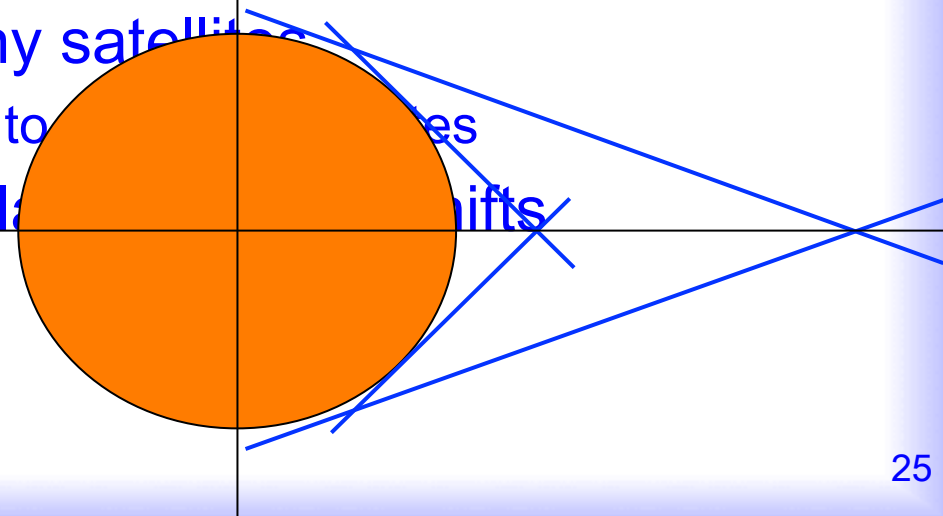
# Low Earth Orbit (LEO)

---

- ✦ LEOs are either circular (or elliptical) orbits less than 2 000 km above the surface of the earth
  - Satellites generally some 700 to 1400 km up
  - Orbit periods between 90 to 120 minutes
    - Maximum time during which a satellite is above the horizon for an observer on the earth is 20 minutes.
  - Footprint radius is generally 3 000 to 4 000 km
  - A global system requires many satellites
    - Needs to hand over the service to different satellites
  - Need to be able to cope with large Doppler shifts

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  - A global system requires many satellites
    - Needs to hand over the service to other satellites
  - Need to be able to cope with large orbital drifts



# Medium Earth Orbits (MEO)

---

- ✦ MEOs are circular orbits at an altitude of around 10000 km, with an orbit period of around 6 hours
  - The time during which a MEO satellite is in view for an observer on the earth is in the order of a few hours
  - A global communications system using this type of orbit, requires a modest number of satellites (around 10 to 20) in 2 to 3 orbital planes to achieve global coverage
  - Compared to a LEO system, hand-over is less frequent, and propagation delay and free space loss are greater

# Geosynchronous Orbit (GEO)

---

- ✦ A circular orbit with an orbital period equal to that of the Earth
  - When in the equatorial plane (geostationary: inclination =  $0^\circ$ ), it appears fixed from an observer on Earth
    - This is achieved with an orbital height of 35 786 km (or an orbital radius of 6.6107 Equatorial Earth Radii)
  - A GEO orbit has small non-zero values for inclination and eccentricity
    - causing the satellite to trace out a small figure of eight in the sky
  - The round-trip delay is approximately 250 ms

# Orbite particolari

---

☺ Sun-Synchronous:

➤  $d\Omega/dt_{J_2} = \text{vel.ang. Terra}$

☺ Molniya:

➤  $\tau = 12 \text{ hr}$ ,  $d\omega/dt_{J_2} = 0$ ,  $e=0.75$ ,  $i=63.5^\circ$

☺ Geo-Synchronous:

➤  $\tau = \text{vel.rot. Terra}$

☺ Geo-Stazionarie:

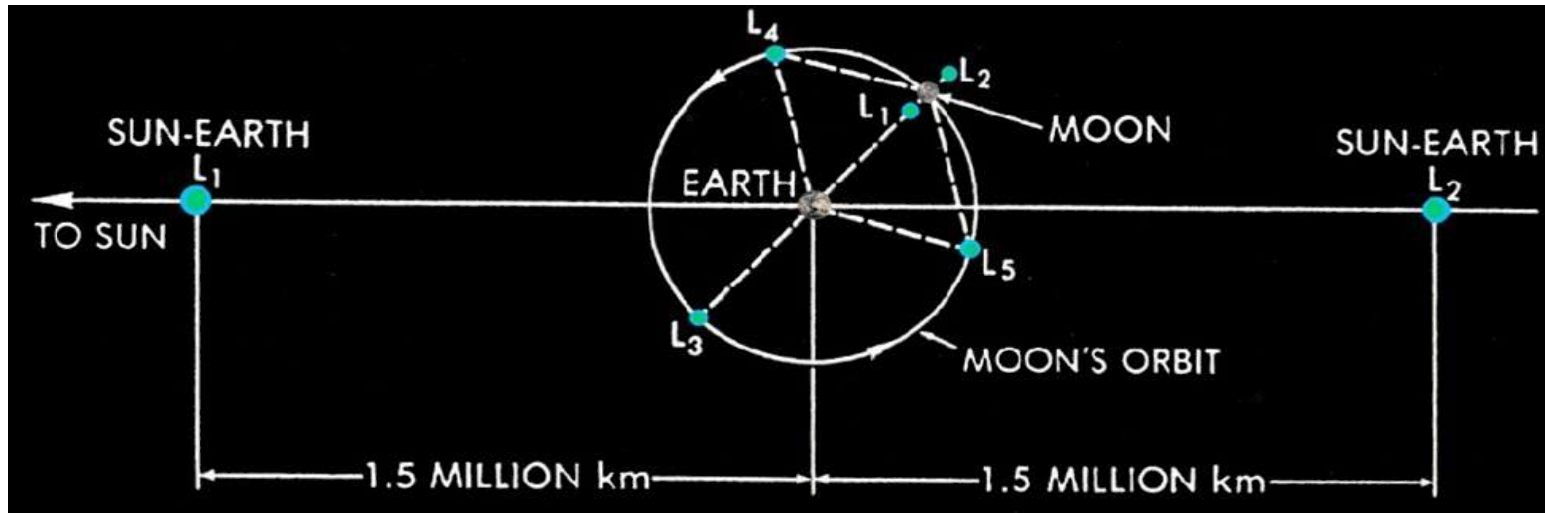
➤  $\tau = \text{vel.rot. Terra}$ ,  $i=0^\circ$

☺ Lagrangiane:

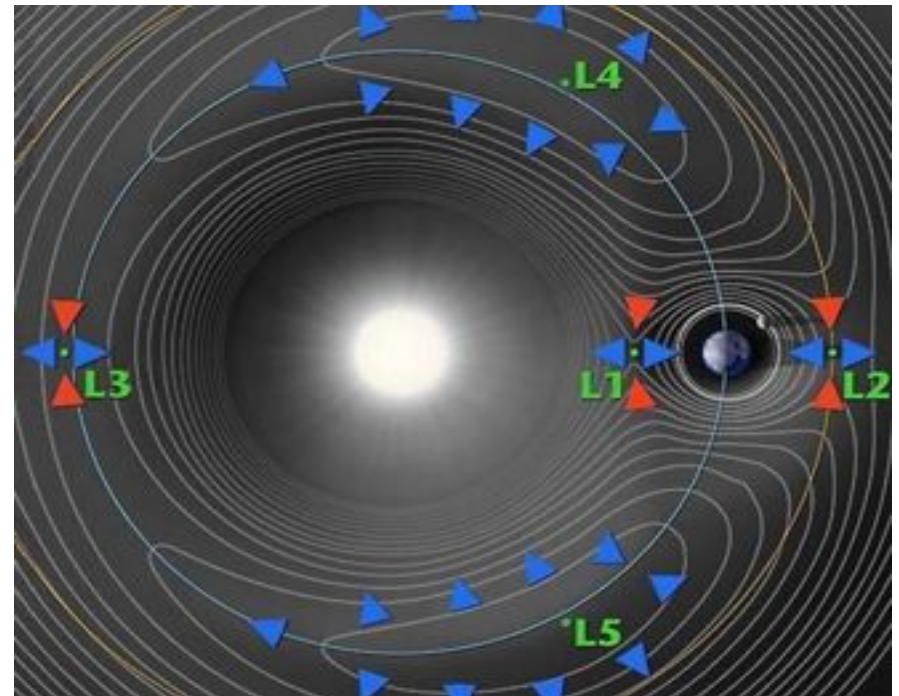
➤ equilibrio Luna/Terra/sat



# Lagrangiane

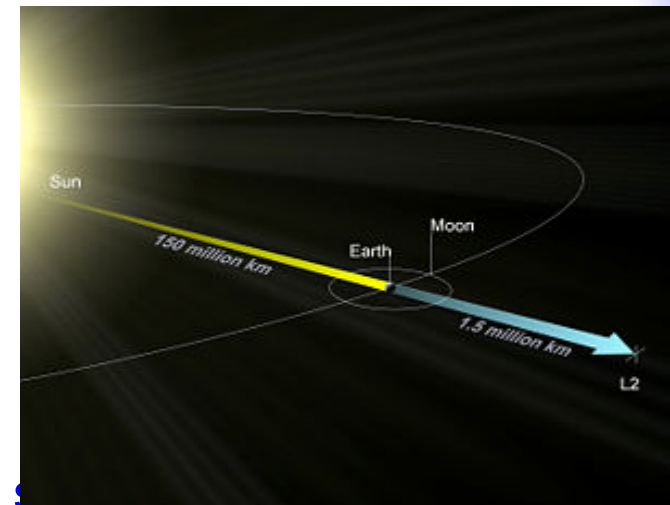
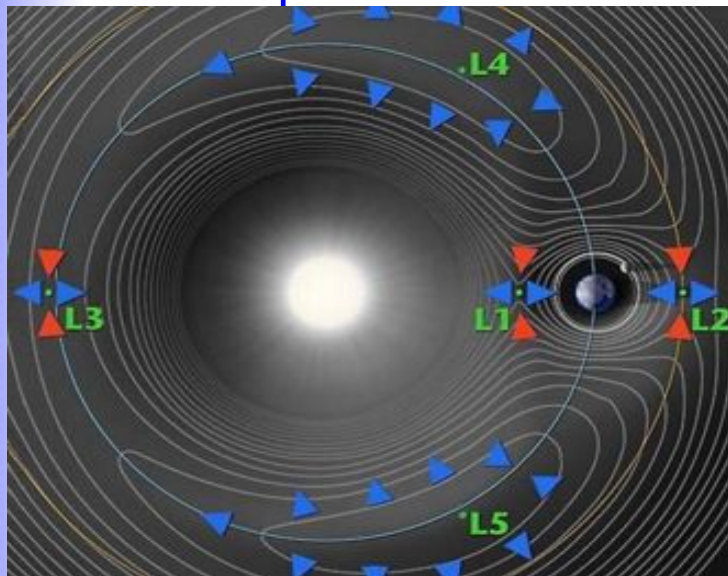


$TL_1$	=	322127 km
$TL_2$	=	442060 km
$TL_3$	=	386322 km
$TL_{4,5}$	=	384400 km
$TS_2$	=	$1.5E+06$ km
$TS$	=	$1.5E+08$ km



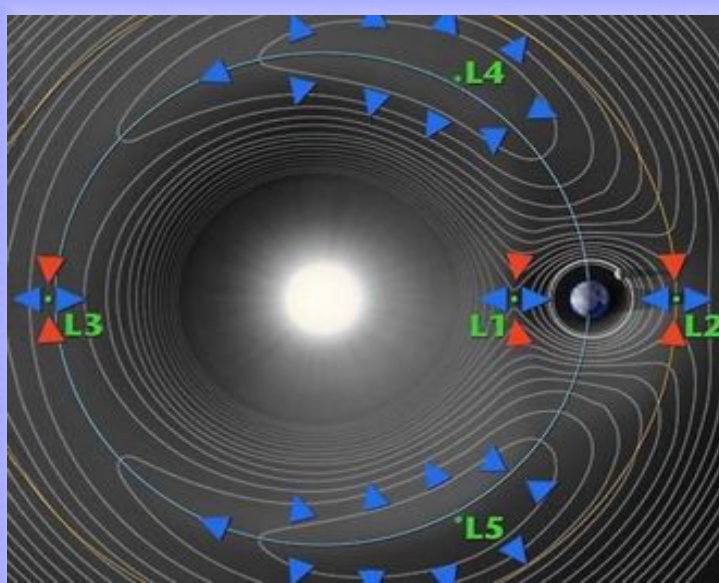
# Lagrangian points

- ✦ L1: lies on the line defined by and between the 2 large masses  $M1$  and  $M2$ . It is the most intuitively understood of the Lagrangian points: where the gravitational attractions of the 2 objects effectively cancel each other out
- ✦ L2: lies on the line defined by the 2 large masses, beyond the smaller of the 2. The gravitational forces of the 2 large masses balance the centrifugal force on the smaller mass
- ✦ L3: lies on the line defined by the 2 large masses, beyond the larger of the 2: the combined pull of Earth and Sun again causes the object to orbit with the same period as the Earth

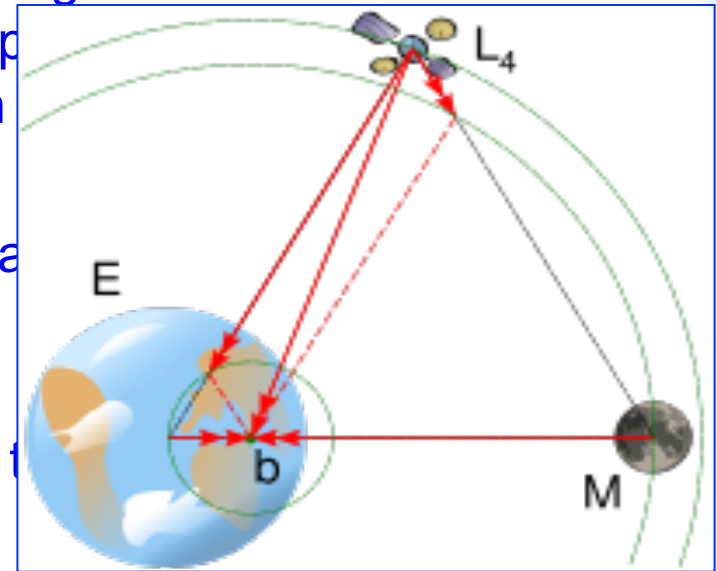


rest of the system

# Lagrangian points



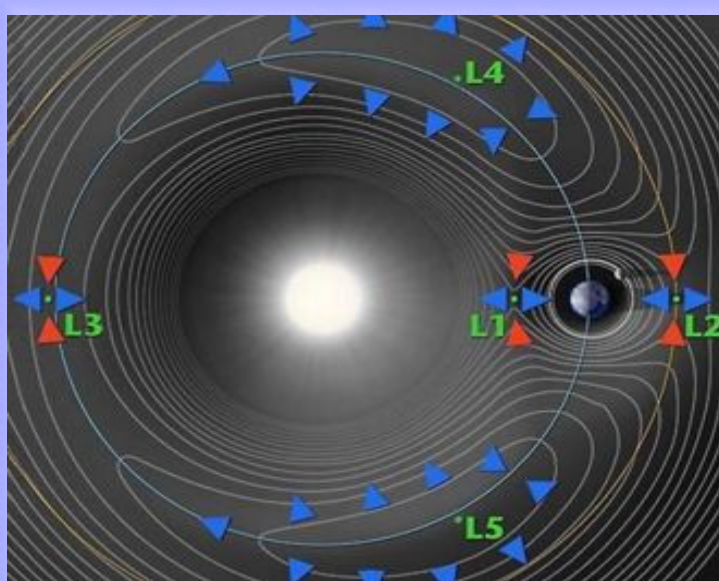
defined by and between the 2 large masses  $M1$  and  $M2$ . It is understood of the Lagrangian points where the gravitational forces effectively cancel each other out. L1, L2, and L3 are defined by the 2 large masses, while L4 and L5 are defined by the 2 large masses balanced by the smaller mass.



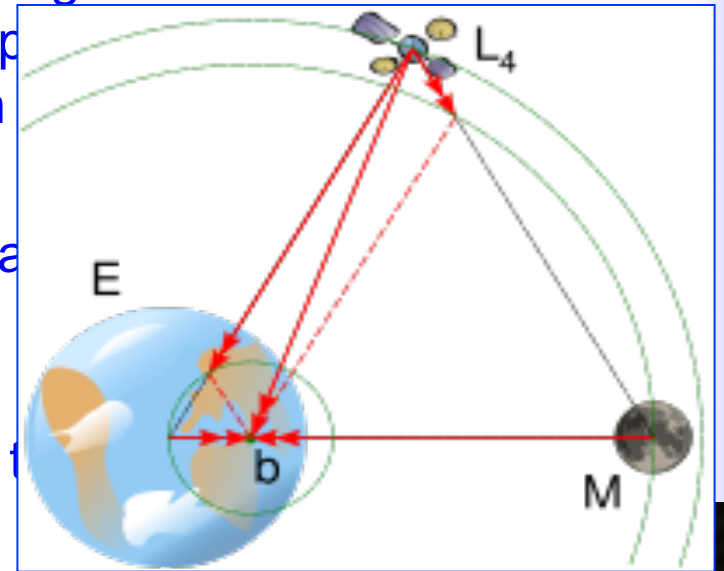
- ✦ L3: lies on the line defined by the 2 large masses, the combined pull of Earth and Sun again causes the same period as the Earth
- ✦ L4/L5: lie at the 3rd corners of the 2 equilateral triangles in the plane of orbit, behind (L5) or ahead of (L4) the smaller mass wrt its orbit around the larger mass. The distances to the 2 masses are equal  $\Rightarrow$  gravitational forces from the 2 massive bodies are in the same ratio as their masses and the resultant force acts through the barycentre of the system: being both the centre of mass and centre of rotation of the system, the resultant force is exactly that required to keep a body in orbital equilibrium with the rest of the system



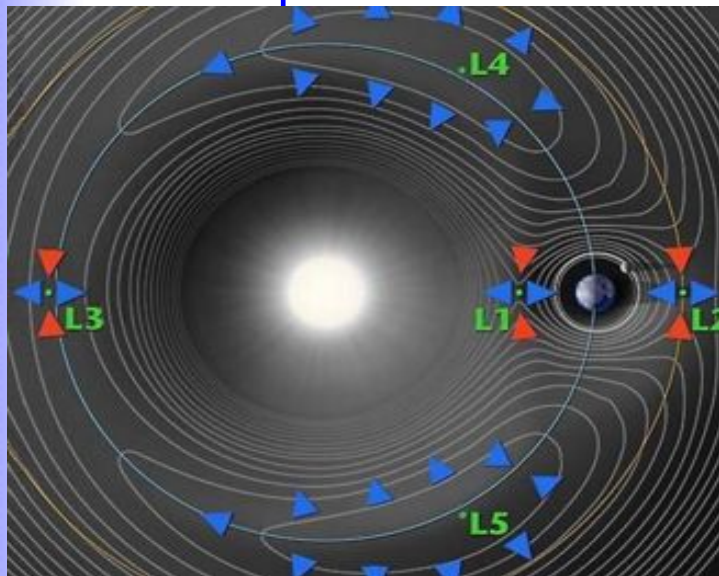
# Lagrangian points



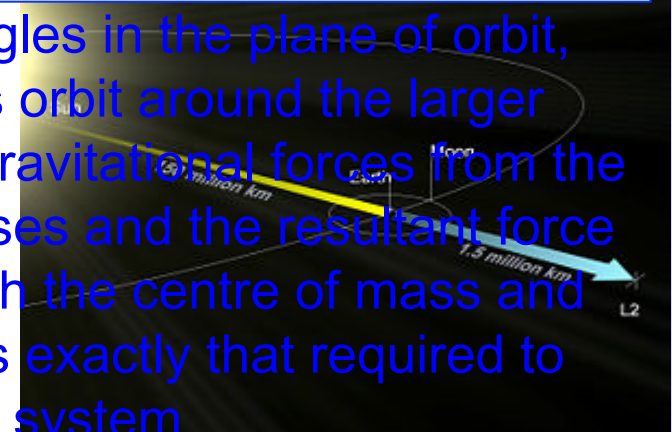
defined by and between the 2 large masses  $M1$  and  $M2$ . It is understood of the Lagrangian points where the gravitational forces effectively cancel each other out. L1 and L2 are located on the line defined by the 2 large masses, while L3, L4, and L5 are located at the vertices of the 2 equilateral triangles in the plane of orbit.



- ✦ L3: lies on the line defined by the 2 large masses, the combined pull of Earth and Sun again causes the satellite to have the same period as the Earth

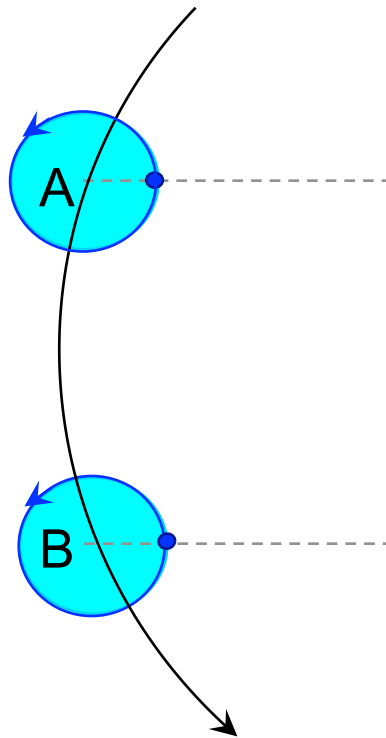


L4 and L5 are the vertices of the 2 equilateral triangles in the plane of orbit. At L4 (and L5) the smaller mass wrt its orbit around the larger mass is in a 1:1 resonance with the larger mass. If the 2 masses are equal  $\Rightarrow$  gravitational forces from the 2 masses are equal  $\Rightarrow$  the resultant force is exactly that required to maintain equilibrium with the rest of the system.



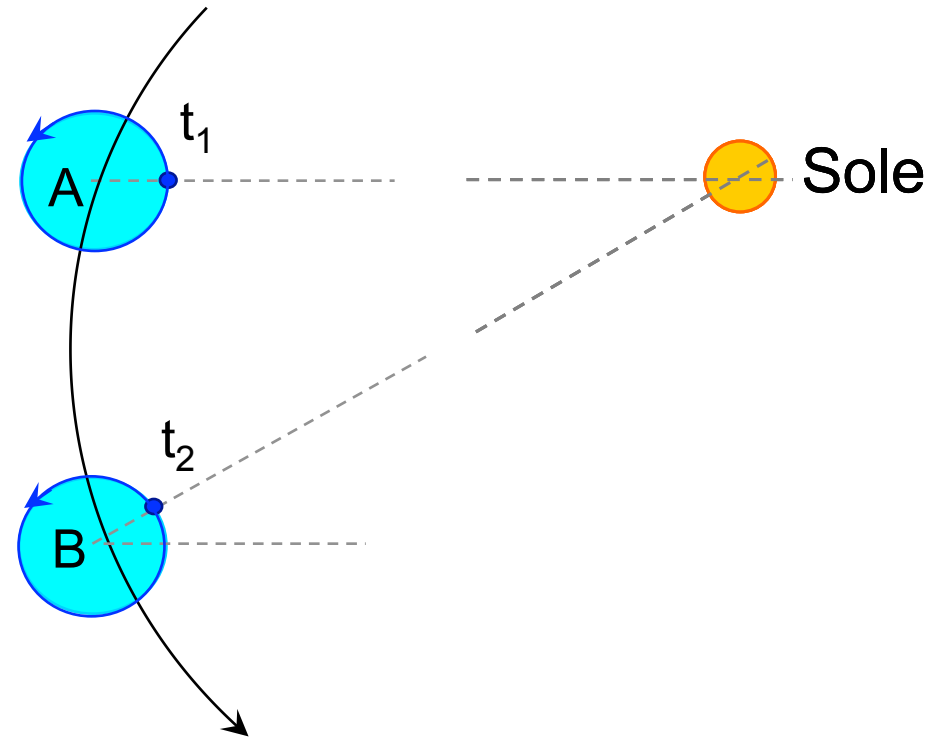
# Giorno siderale e sinodico

Giorno Siderale



Stelle  
fisse

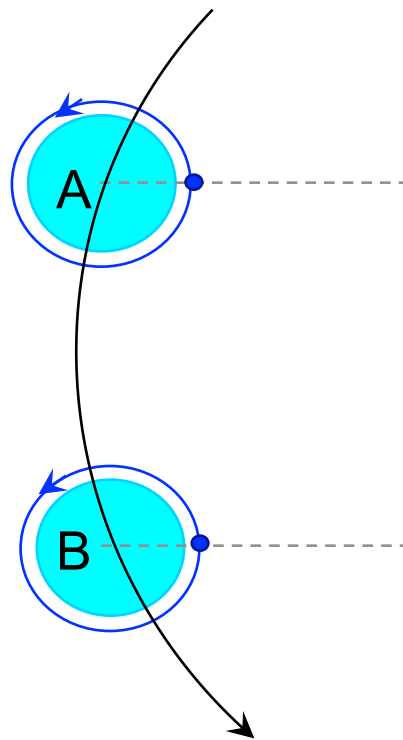
Giorno Sinodico  
(giorno solare apparente)



Sole

# Periodo siderale e sinodico

Periodo Siderale  $\tau_S$

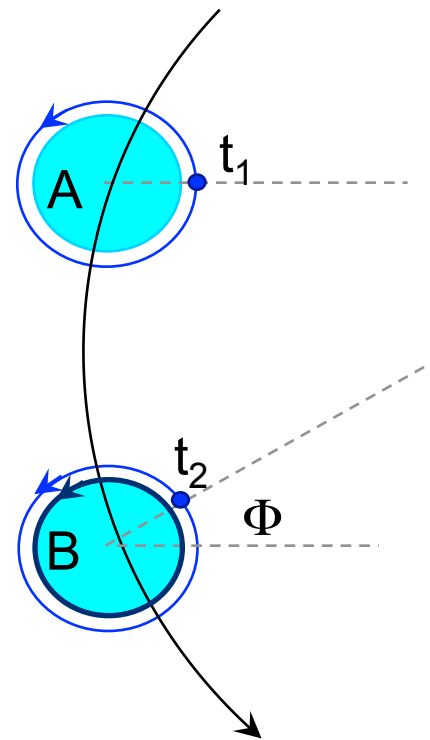


Stelle  
fisse

$$2\pi + \Phi = 2\pi/\tau_S (t_2 - t_1)$$

$$\Phi = 2\pi/\tau_M (t_2 - t_1)$$

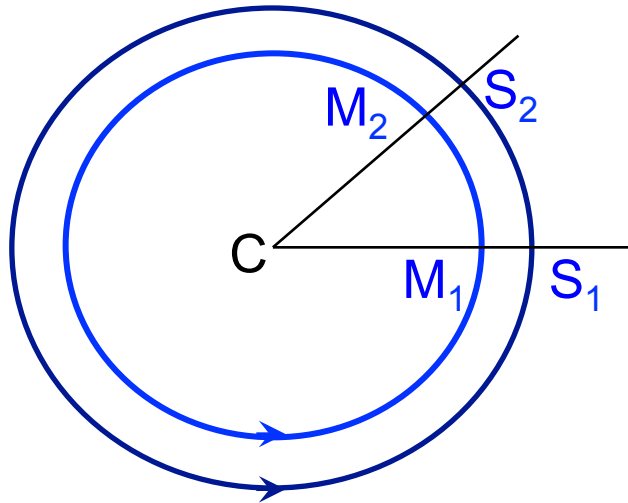
Periodo Sinodico  $\tau_{SS}$



LEO

$$\Rightarrow 1/(t_2 - t_1) = 1/\tau_S - 1/\tau_M = 1/\tau_{SS}$$

# Terra e pianeti esterni e Luna



$$1/\tau_M - 1/\tau_S = 1/\tau_{SS}$$

Congiunzione S, T, P (C,M,S):  
periodo sinodico

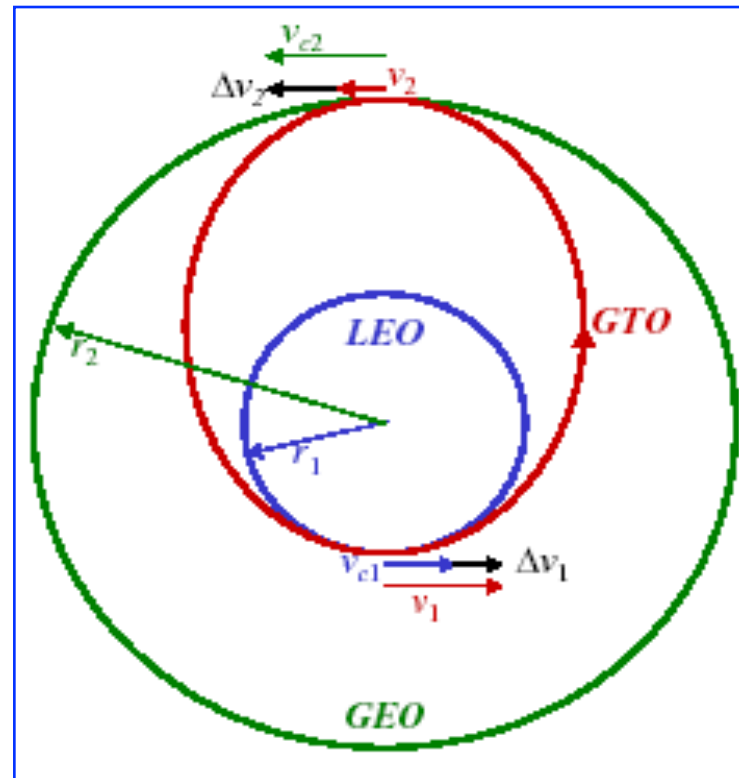
$$\Rightarrow 1 - 1/\tau_S = 1/\tau_{SS}$$

$$\Rightarrow 1/\tau_P = 1 - 1/T_P$$

Congiunzione T, L, S (C,M,S): “Luna  
Nuova”, periodo sinodico

$$\tau_S = 365 \text{ giorni}, \tau_L = 27.3 \text{ giorni}, \Rightarrow \\ \tau_{SS} = 29.5 \text{ giorni}$$

# Trasferimento LEO-GEO: GTO 1/3

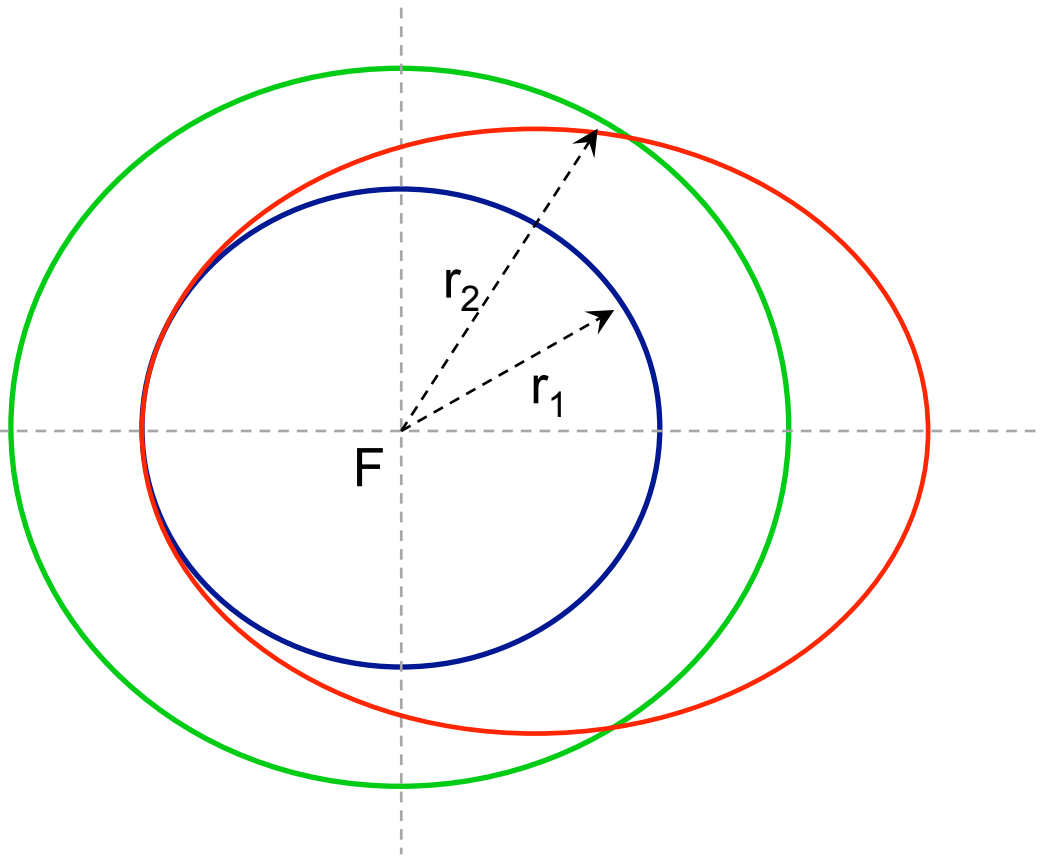


Ellissi di  
Hohmann



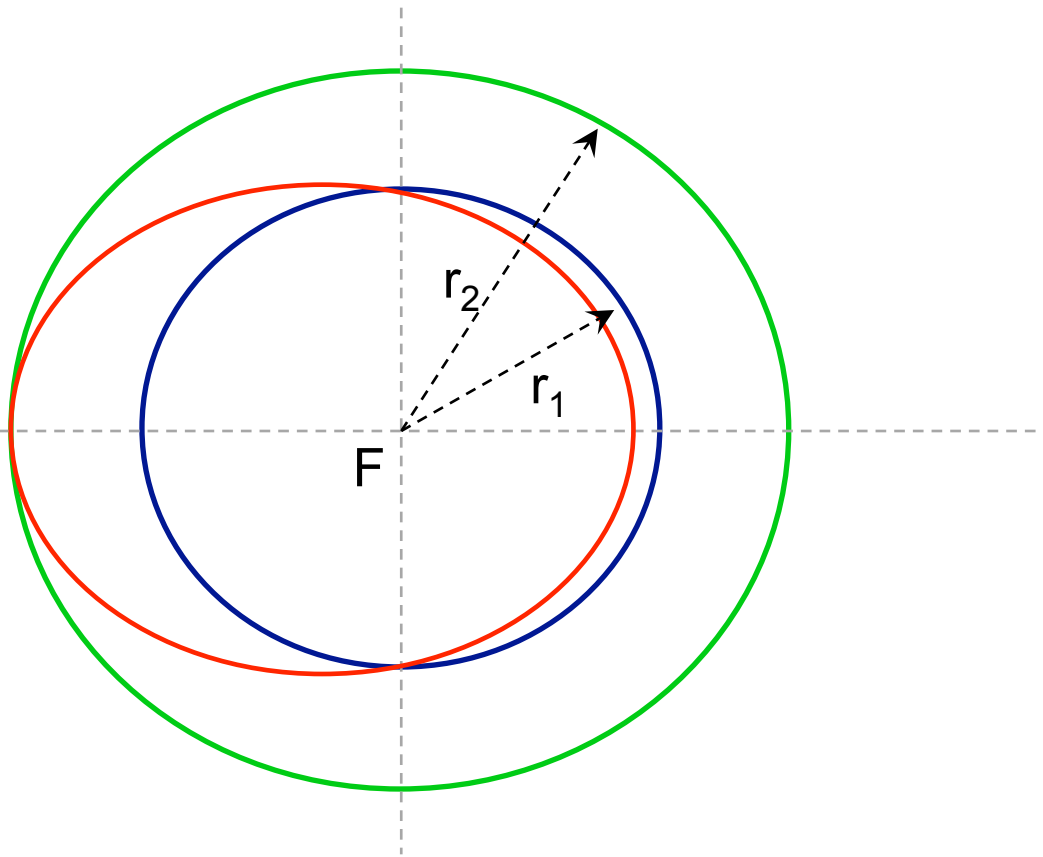
# Trasferimenti possibili

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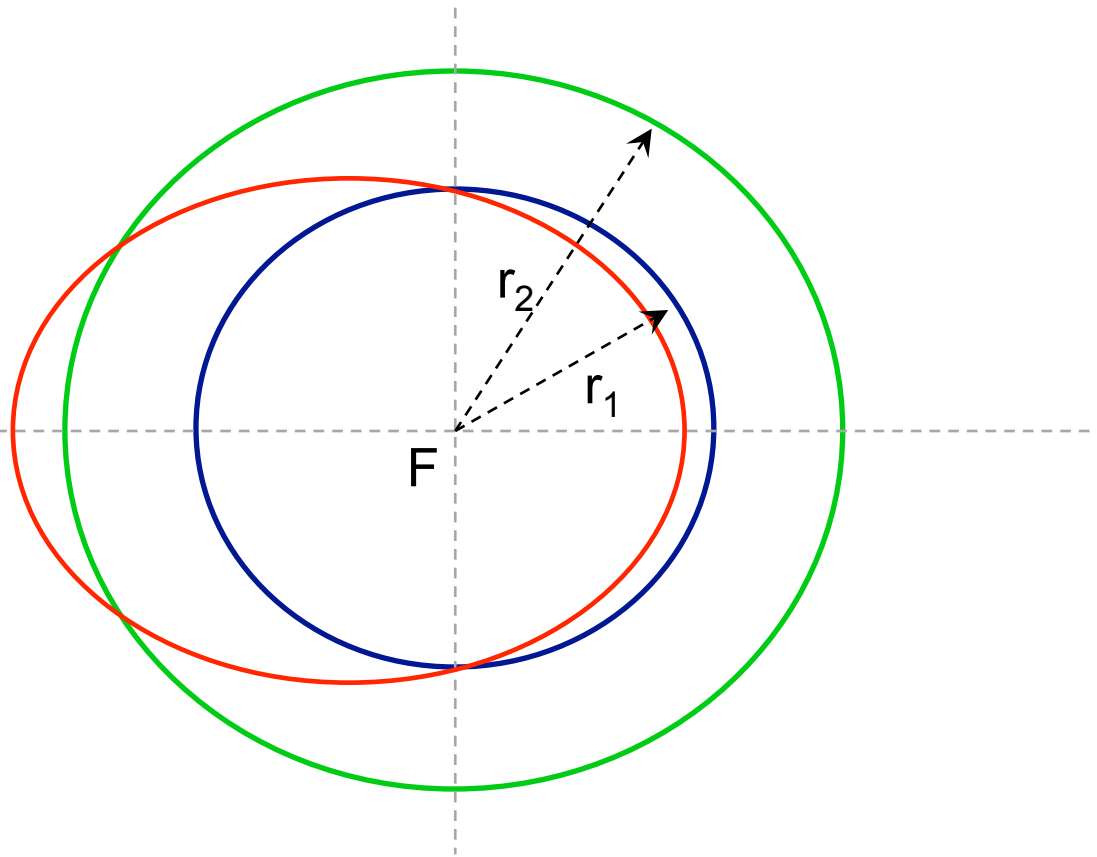
# Trasferimenti possibili

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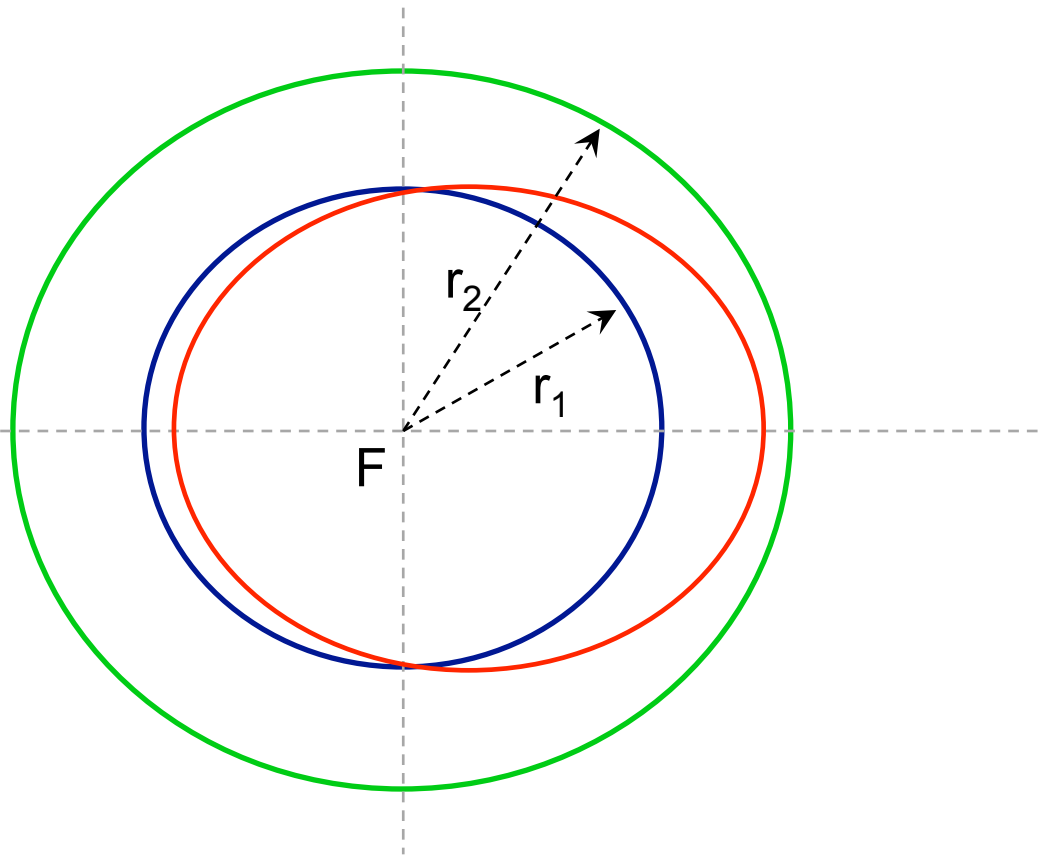
# Trasferimenti possibili

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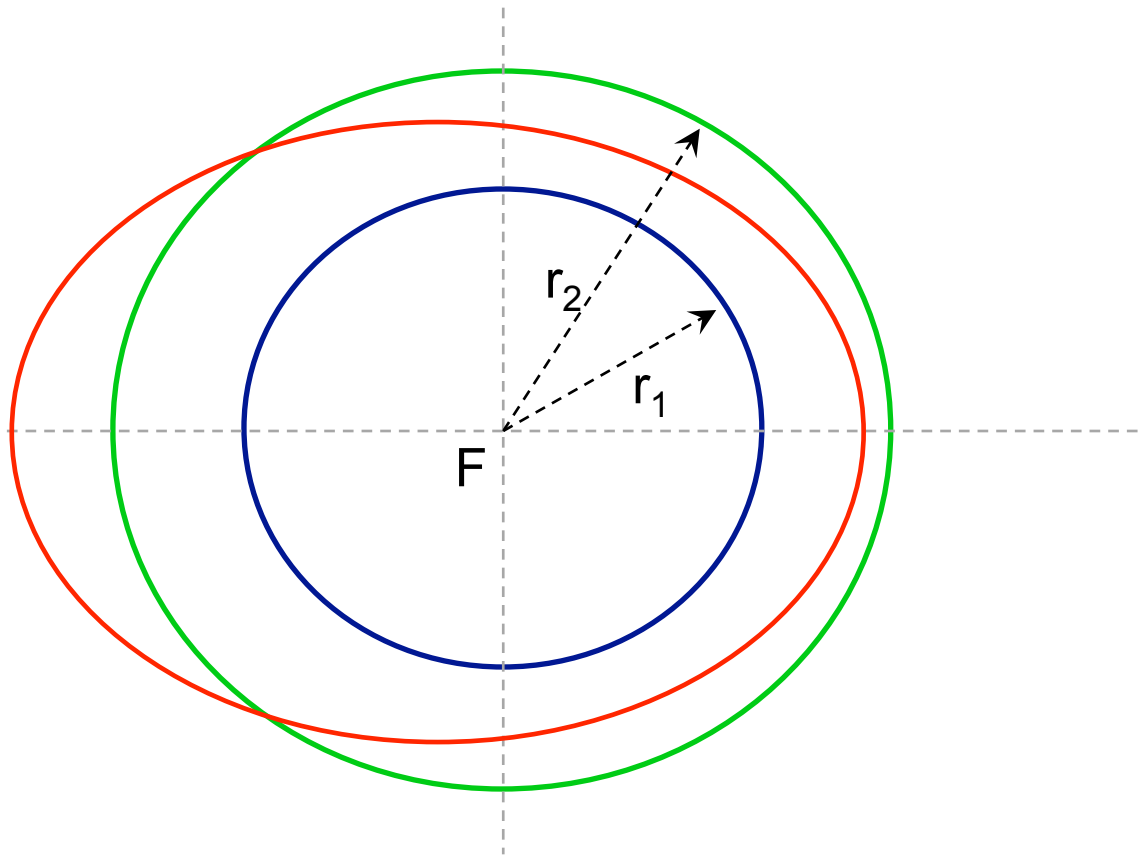
# Trasferimenti possibili

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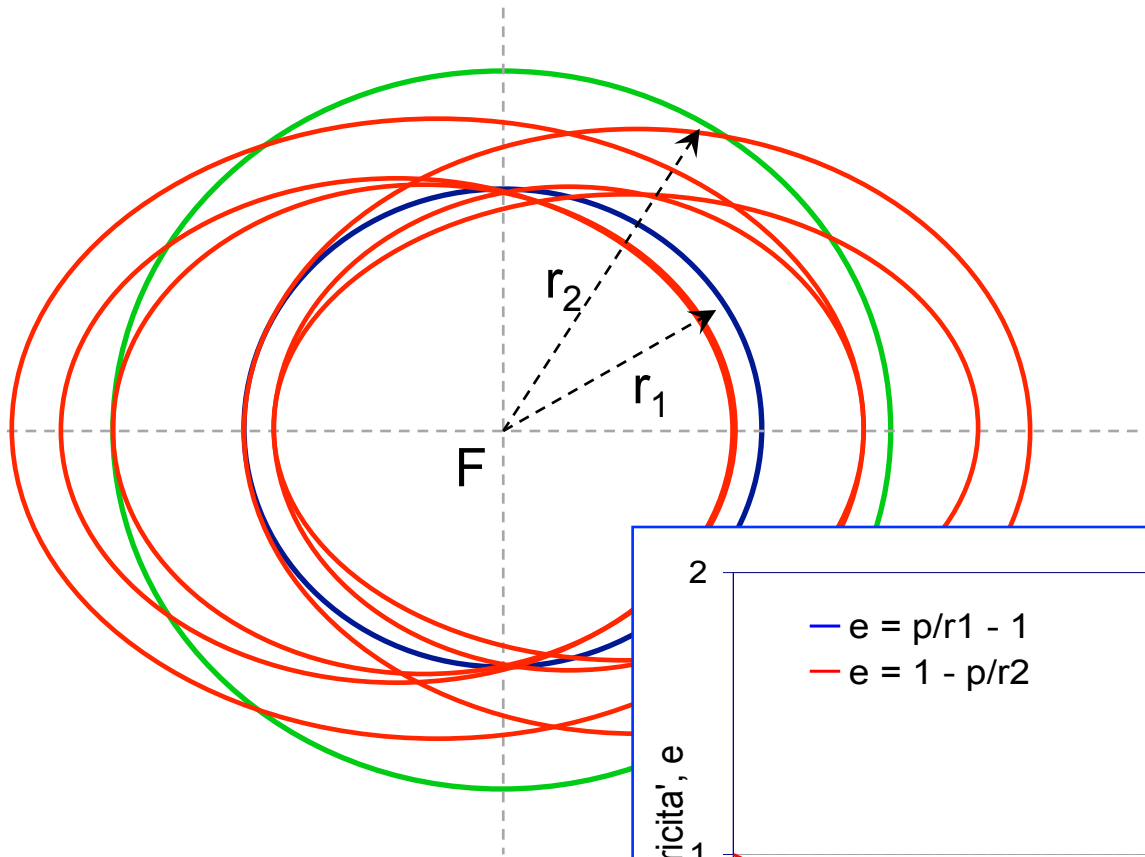


# Trasferimenti possibili

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# Trasferimenti possibili

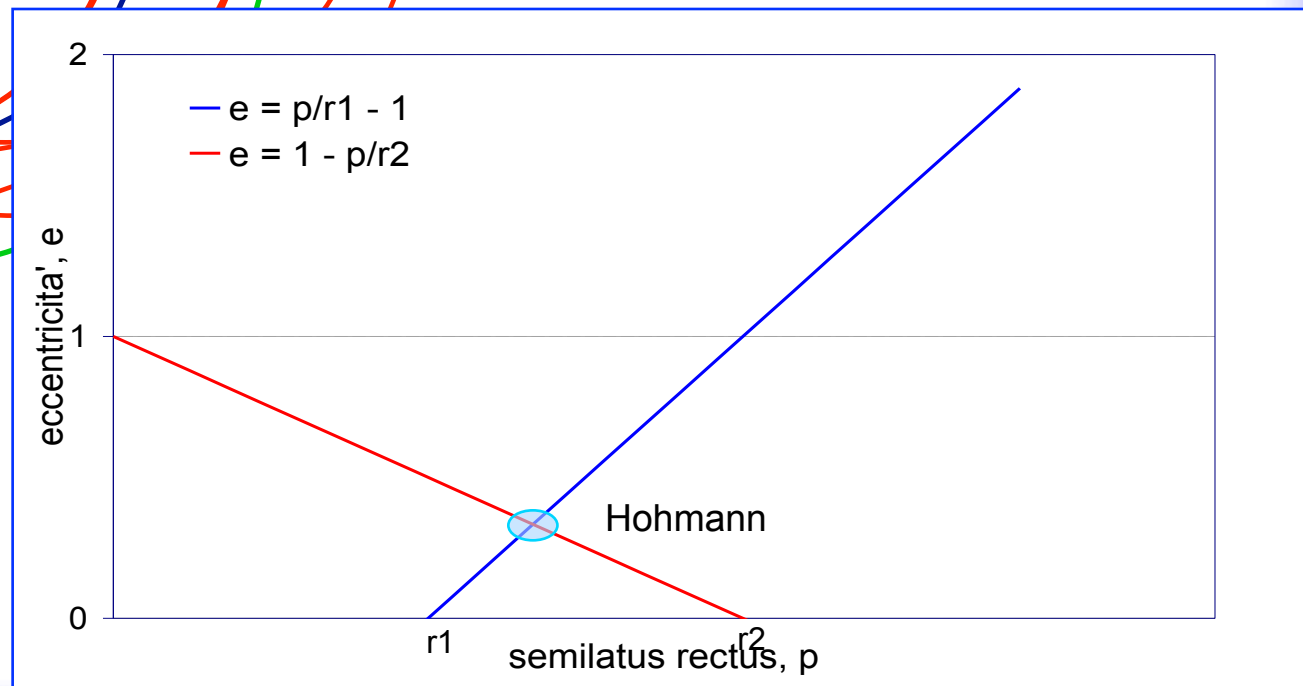


$$\Rightarrow r_{\text{peri}} = p / (1+e) \leq r_1$$

$$\Rightarrow r_{\text{apo}} = p / (1-e) \geq r_2$$

$$\Rightarrow e \geq p/r_1 - 1$$

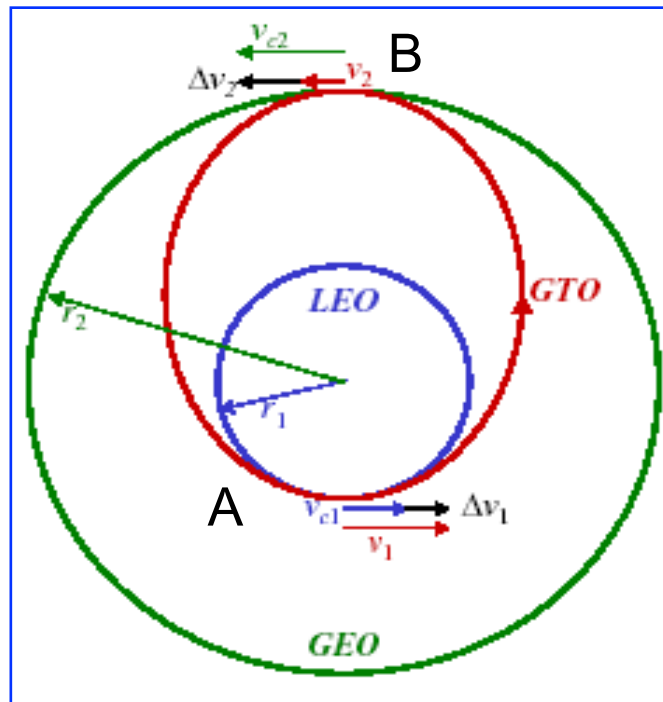
$$\Rightarrow e \geq 1 - p/r_2$$



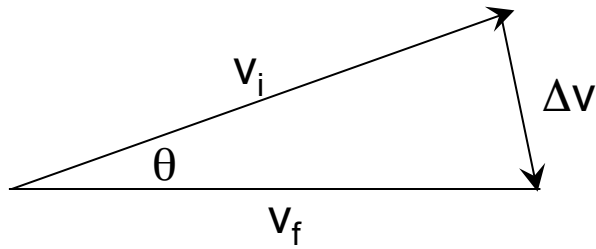
# Trasferimento LEO-GEO: GTO 2/3

	<b>LEO</b>		<b>GEO</b>
h	400 km	h	35781 km
radius	6778 km	radius	42160 km
v	7.669 km/s	v	3.075 km/s

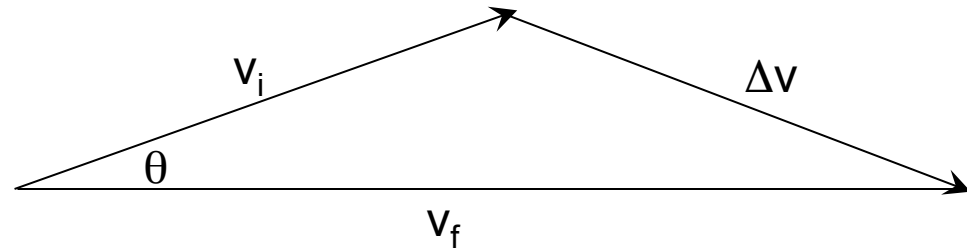
	<b>ELLISSE</b>
a	24469 km
$r_A$	6778 km
$r_B$	42160 km
$v'_A$	10.066 km/s
$v'_B$	1.618 km/s
$\tau$	19046 s
	317.4 min
$\Delta v_A$	2.397 km/s
$\Delta v_B$	1.456 km/s
$\Delta v_{TOT}$	3.854 km/s



# Trasferimento di piano



$$\Delta v_{\text{plane}}/2 = v_i \sin \theta/2$$



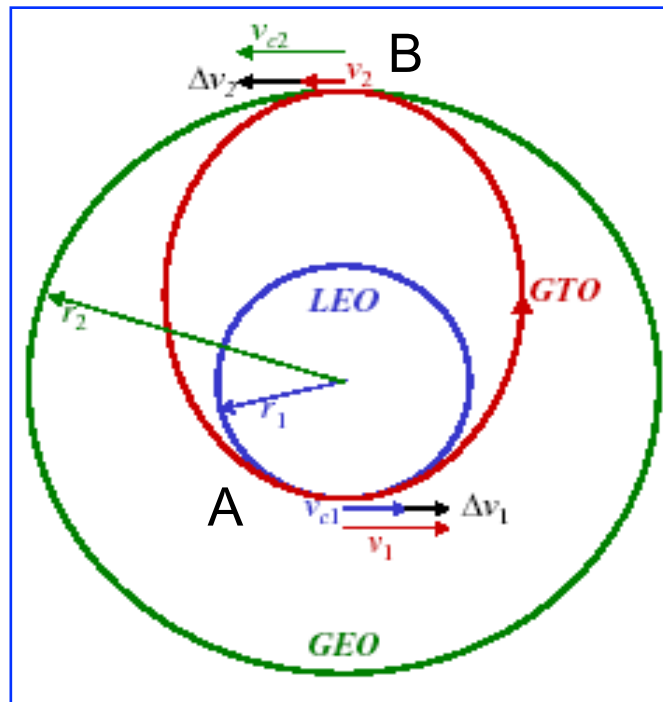
$$\Delta v_{\text{plane,if}}^2 = v_i^2 + v_f^2 - 2 v_i v_f \cos \theta$$



# Trasferimento LEO-GEO: GTO 2/3

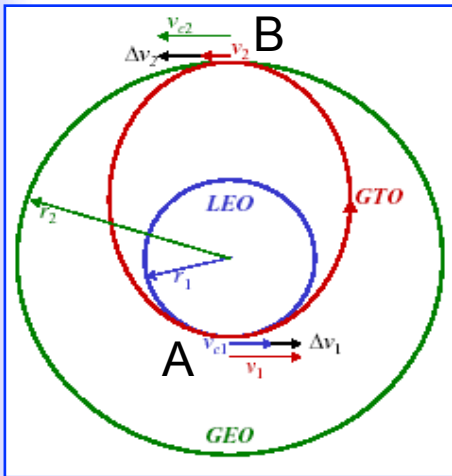
	<b>LEO</b>		<b>GEO</b>
h	400 km	h	35781 km
radius	6778 km	radius	42160 km
v	7.669 km/s	v	3.075 km/s

	<b>ELLISSE</b>
a	24469 km
$r_A$	6778 km
$r_B$	42160 km
$v'_A$	10.066 km/s
$v'_B$	1.618 km/s
$\tau$	19046 s
	317.4 min
$\Delta v_A$	2.397 km/s
$\Delta v_B$	1.456 km/s
$\Delta v_{TOT}$	3.854 km/s



# Trasferimento LEO-GEO: GTO 3/3

LEO		GEO		ELLISSE	
h	400 km	h	35781 km	a	24469 km
radius	6778 km	radius	42160 km	$r_A$	6778 km
v	7.669 km/s	v	3.075 km/s	$r_B$	42160 km
i	28 deg	i	0 deg	$v'_A$	10.066 km/s
	0.4887 rad		0 rad	$v'_B$	1.618 km/s
				$\tau$	19046 s
					317.4 min
				$\Delta v_A$	2.397 km/s
				$\Delta v_B$	1.456 km/s
				$\Delta v_{TOT}$	3.854 km/s
				$\Delta v_{plane}$	1.4877 km/s
				$\Delta v_{TOT}$	5.342 km/s
				$\Delta v_{plane,B}$	1.8128 km/s
				$\Delta v_{TOT}$	4.210 km/s

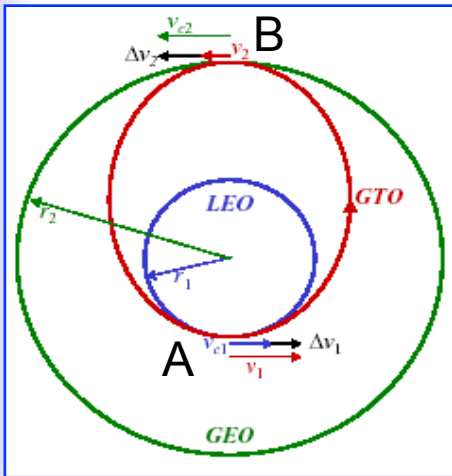


Tutto trasferimento  
effettuato in B

$\Delta v_{plane}$	1.4877 km/s
$\Delta v_{TOT}$	5.342 km/s
$\Delta v_{plane,B}$	1.8128 km/s
$\Delta v_{TOT}$	4.210 km/s

# Trasferimento LEO-GEO: GTO 3/3

LEO		GEO		ELLISSE	
h	400 km	h	35781 km	a	24469 km
radius	6778 km	radius	42160 km	$r_A$	6778 km
v	7.669 km/s	v	3.075 km/s	$r_B$	42160 km
i	28 deg	i	0 deg	$v'_A$	10.066 km/s
	0.4887 rad		0 rad	$v'_B$	1.618 km/s
				$\tau$	19046 s
					317.4 min
				$\Delta v_A$	2.397 km/s
				$\Delta v_B$	1.456 km/s
				$\Delta v_{TOT}$	3.854 km/s
				$\Delta v_{plane}$	3.710 km/s
				$\Delta v_{TOT}$	7.564 km/s
				$\Delta v_{plane,A}$	4.880 km/s
				$\Delta v_{TOT}$	6.337 km/s



Tutto trasferimento effettuato in A



$\Delta v_{plane}$	3.710 km/s
$\Delta v_{TOT}$	7.564 km/s
$\Delta v_{plane,A}$	4.880 km/s
$\Delta v_{TOT}$	6.337 km/s

# Aggiustamenti di orbita

## Attrito Atmosferico

LEO

$$\Delta v_{\text{rev}} = \pi (C_D A / m) \rho a v$$

GEO,  $i=0^\circ$

## Triassialità della Terra

$$\Delta v_{\text{anno}} = 1.715 \sin(2|\text{long} - \text{long}_s|) \text{ m/s} \quad (\text{long}_s = 75^\circ/225^\circ \text{ E})$$

## Forze Gravitazionali del Sole e della Luna

$$\Delta v_{\text{Luna,anno}} = 102.67 \cos \alpha \sin \alpha \text{ m/s /anno} \quad \sim 36.93 \text{ m/s /anno}$$

$$\Delta v_{\text{Sole,anno}} = 40.17 \cos \gamma \sin \gamma \text{ m/s /anno} \quad \sim 14.45 \text{ m/s /anno}$$

# Esercizio

# Shuttle

◆ LEO	$h=350 \text{ km},$	$i=0^\circ,$	$e=0$
	Sun-synchronous $h = h_p$	$i=28^\circ,$	$e=0$
		$e=0.1$	
		$i=96.85^\circ,$	$e=0$
◆ MEO	$h=10\,000 \text{ km},$	$i=28^\circ,$	$e=0$
	$h=20\,222 \text{ km},$	$i=60^\circ,$	$e=0$
◆ GEO	$h=35\,782 \text{ km},$	$i=0^\circ,$	$e=0$
		$i=23^\circ,$	$e=0$
		$i=60^\circ,$	$e=0$
◆ Molniya	$a=26\,600 \text{ km},$	$i=63.4^\circ,$	$e=0.75$

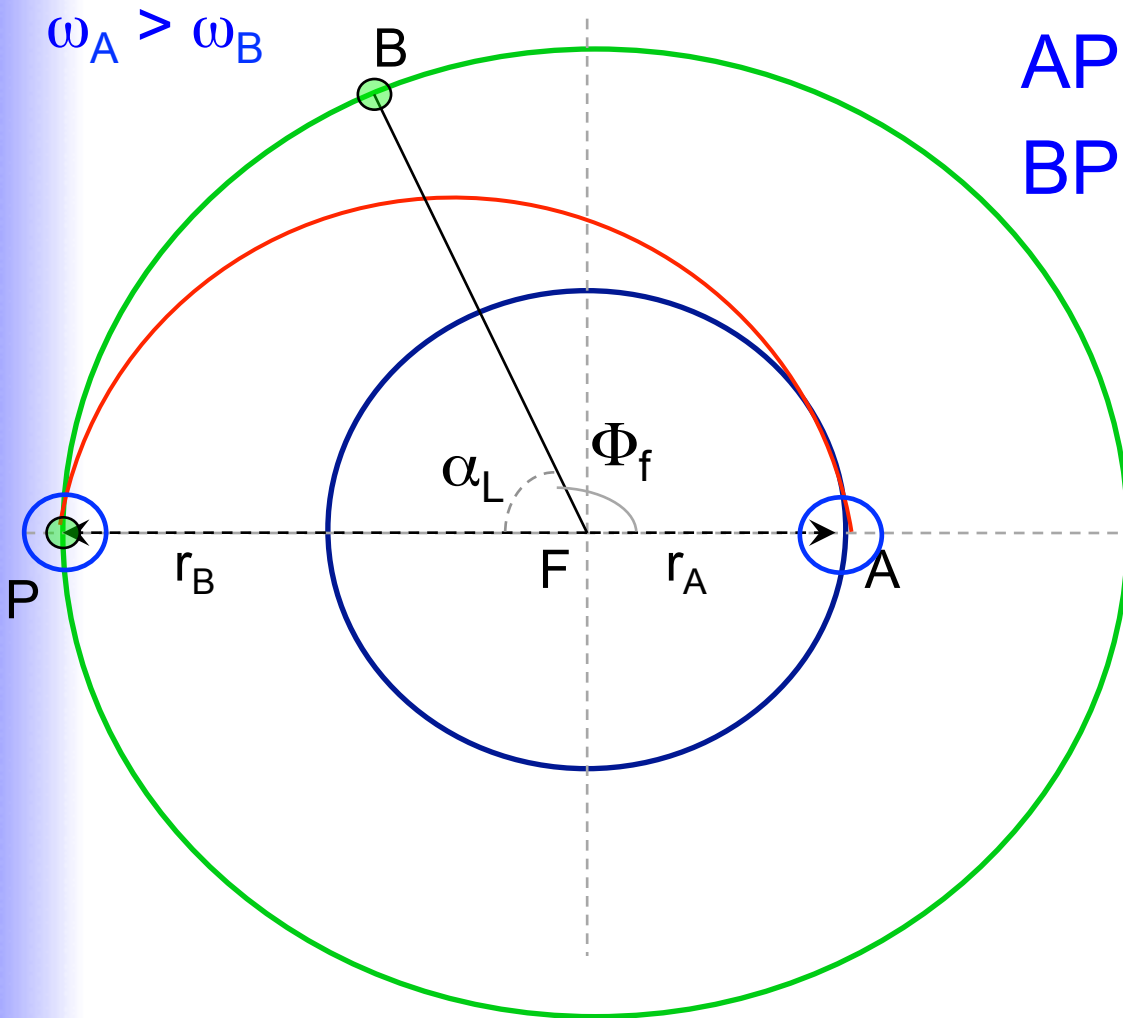
GPS

# Parametri Orbitali

ORBIT	CG Terrestre	Luna	Sole
Shuttle	(a=6700 km,e=0.0,i=28 <sup>o</sup> )		
$\Omega'$	-7.350	-0.00019	-0.00008 gradi/giorno
$\omega'$	12.050	0.00031	0.00014 gradi/giorno
Sun-Synchronous	(a=6728 km,e=0.0,i=96.85 <sup>o</sup> )		
$\Omega'$	0.986	0.00003	0.00001 gradi/giorno
$\omega'$	-4.890	-0.00010	-0.00005 gradi/giorno
GPS	(a=26600 km,e=0.0,i=60 <sup>o</sup> )		
$\Omega'$	-0.033	-0.00085	-0.00038 gradi/giorno
$\omega'$	0.008	0.00021	0.00010 gradi/giorno
Molniya	(a=26600 km,e=0.75,i=63.4 <sup>o</sup> )		
$\Omega'$	-0.157	-0.00076	-0.00034 gradi/giorno
$\omega'$	0.000	0.00000	0.00000 gradi/giorno
Geosynchronous	(a=42160 km,e=0.0,i=0 <sup>o</sup> )		
$\Omega'$	-0.013	-0.00338	-0.00154 gradi/giorno
$\omega'$	0.025	0.00676	0.00307 gradi/giorno

- ✦ Calcolare velocita' orbitale e di fuga (nel caso di orbite ellittiche, al/da perigeo e apogeo), periodo orbitale, energia cinetica, potenziale e totale

# Appuntamenti in orbita



$$AP: \tau_A = \pi \sqrt{a^3/\mu} = \tau_B = \tau_H$$

$$BP: \tau_B = \alpha_L/\omega_B = \alpha_L \sqrt{r_B^3/\mu}$$

$$\alpha_L = \pi - \Phi_f$$

$$t_w \omega_A - t_w \omega_B = \Phi_i - \Phi_f (+2k\pi)$$

$$(\Phi_i = 0)$$

$$t_w = (\alpha_L - \pi + 2k\pi) / (\omega_A - \omega_B)$$

$$= (\tau_H \omega_B - \pi + 2k\pi) / (\omega_A - \omega_B)$$

$$t_{\text{tot}} = t_w + \tau_H + t_o$$

# Parametri Ellisse

$$v_t = v \cos \gamma (*)$$

$$v_r = v \sin \gamma$$

$\gamma=0$  per orbite  
circolari ( $v=v_t$ ,  $v_r=0$ )

