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Esercizio meccanica

1. $H = \hbar\omega P_{\psi_1}$ $\qquad |\psi_1\rangle \in \mathbb{C}^d$ normaliz. $\qquad$ uso sempre $|\psi\rangle = |\psi_1\rangle$

2. $H|\tilde{\psi}\rangle = \lambda|\tilde{\psi}\rangle$ $\qquad$ prendiamo una base ortonorm di cui $|\psi_1\rangle$ è il 1° vettore
↳ sono anche ortog.

$\hbar\omega P_{\psi_1}|a_1\psi_1 + \dots\rangle = \lambda|a_1\psi_1 + \dots\rangle$

$a_1\hbar\omega|\psi_1\rangle = \lambda a_1|\psi_1\rangle$

$\lambda_1 = \hbar\omega$ $\quad$ autovalore con degenerazione 1 e autovettore $a_1|\psi_1\rangle$

$H|\tilde{\psi}\rangle = 0\cdot|\tilde{\psi}\rangle$

$\hbar\omega P_{\psi_1}|\tilde{\psi}\rangle = 0$

$\hbar\omega\left(|\psi_1\rangle\langle\psi_1|a_1\psi_1\rangle + |\psi_1\rangle\langle\psi_1|a_2\psi_2\rangle + \dots + |\psi_1\rangle\langle\psi_1|a_d\psi_d\rangle\right) = 0$

$\lambda_2 = 0$ $\quad$ autovalore con degenerazione $d-1$, autovettori sono i vettori di base tranne $|\psi_1\rangle$

3. $U_t = e^{-\frac{i}{\hbar}tH} = e^{-\frac{i}{\hbar}t\hbar\omega}|\psi_1\rangle\langle\psi_1| + \sum_{i=2}^{d} e^{0}|\psi_i\rangle\langle\psi_i|$

$\qquad = e^{-it\omega}P_{\psi_1} + \sum_{i=2}^{d} P_{\psi_i}$ $\quad$ ...usando il teorema spettrale

$$U_t = e^{-\frac{i}{\hbar}tH} = \sum_{K=0}^{\infty}\frac{1}{K!}\left(-\frac{i}{\hbar}t\right)^K H^K = \sum_{K=0}^{\infty}\frac{1}{K!}\left(-\frac{i}{\hbar}t\right)^K\left(\sum_i \lambda_i P_{\psi_i}\right)^K$$

$$= \sum_{K=0}^{\infty}\frac{1}{K!}\left(-\frac{i}{\hbar}t\right)^K\left(\sum_i \lambda_i^K P_{\psi_i}\right) = \sum_{K=0}^{\infty}\frac{1}{K!}\left(-\frac{i}{\hbar}t\right)^K\left(\lambda_1^K P_{\psi_1} + \lambda_2^K\sum_{i=2}^{d}P_{\psi_i}\right)$$

$$= \sum_{K=0}^{\infty}\frac{1}{K!}\left(-\frac{it\lambda_1}{\hbar}\right)^K P_{\psi_1} + \sum_{K=0}^{\infty}\frac{1}{K!}\left(-\frac{it\lambda_2}{\hbar}\right)^K\sum_{i=2}^{d}P_{\psi_i}$$

$$= e^{-it\omega}P_{\psi_1} + \sum_{i=2}^{d}P_{\psi_i}$$

4. ① $|\langle\chi|U_t\phi\rangle|^2 = |\langle\chi|e^{-it\omega}P_{\psi_1}\phi + \sum_{i=2}^{d}P_{\psi_i}\phi\rangle|^2$

② $|\langle\psi_1|U_t\phi\rangle|^2 = |\langle\psi_1|\dots\rangle|^2 = |e^{-it\omega}\langle\psi_1|\phi\rangle|^2 = |\langle\psi_1|\phi\rangle|^2$

③ $|\langle\psi_1^\perp|U_t\phi\rangle|^2 = |\langle\psi_1^\perp|\dots\rangle|^2 = |\langle\sum_{i=2}^{d}P_{\psi_i}\psi_1^\perp|\phi\rangle|^2$

$\qquad = |\sum_{i=2}^{d}\langle\phi|\psi_i\rangle\langle\psi_i|\psi_1^\perp\rangle|^2$