PAOLO DA ROLD

2) $|\phi\rangle \in \mathbb{C}^d \qquad P_\psi = |\psi\rangle\langle\psi|$

$H|\phi\rangle = \hbar\omega P_\psi|\phi\rangle = \hbar\omega\langle\psi|\phi\rangle|\psi\rangle$

L'unico autovalore è $\hbar\omega$ con autovettore $|\psi\rangle$ quindi solo se $|\phi\rangle = |\psi\rangle$

3) $U_t = e^{-\frac{it}{\hbar}H} = e^{-\frac{it}{\hbar}\hbar\omega P_\psi} = e^{-it\omega P_\psi} = \sum_{k=0}^{\infty} \frac{(-it\omega)^k}{k!} P_\psi \qquad P_\psi^k = P_\psi$

4) a) $\text{Prob}(|\phi_t\rangle = |\chi\rangle) = |\langle\chi_0|\psi_t\rangle|^2 \qquad |\phi_t\rangle = U_t|\phi\rangle$

$= \sum_{k=0}^{\infty} \frac{(-it\omega)^k}{k!} P_\psi|\phi\rangle$

$= \sum_{k=0}^{\infty} \frac{(-it\omega)^k}{k!}\langle\psi|\phi\rangle|\psi\rangle$

b) $\text{Prob}(|\phi_t\rangle = |\psi\rangle) = |\langle\psi|\phi_t\rangle|^2$

c) $\text{Prob}(|\phi_t\rangle = |\psi_\perp\rangle) = |\langle\psi_\perp|\phi_t\rangle|^2$

1) $|\langle\chi|\psi_t\rangle|^2 = \langle\chi|\psi_t\rangle\overline{\langle\psi_t|\chi_t\rangle}$

$= \sum_{k=0}^{\infty} \frac{(-it\omega)^k}{k!}\langle\psi|\phi\rangle\langle\chi|\psi\rangle \cdot \sum_{m=0}^{\infty} \frac{(it\omega)^m}{m!}\langle\psi|\phi\rangle\overline{\langle\psi|\chi\rangle}$

$= e^{-it\omega}\langle\psi|\phi\rangle\langle\chi|\psi\rangle e^{it\omega}\langle\psi|\phi\rangle\overline{\langle\psi|\chi\rangle}$

$= \langle\psi|\phi\rangle^2|\langle\chi|\psi\rangle|^2$

2) $|\langle\psi|\phi_t\rangle|^2 = \langle\psi|\phi\rangle^2|\langle\psi|\psi\rangle|^2 = \langle\psi|\phi\rangle^2$

3) $|\langle\psi_\perp|\phi_t\rangle|^2 = \langle\psi|\phi\rangle^2|\underbrace{\langle\psi_\perp|\psi\rangle}_{=0}|^2 = 0$