MARCHESIN
STEFANO

$\mathbb{C}^d \quad d \geq 2 \qquad H = \hbar\omega P_\psi = \hbar\,\omega\,|\psi\rangle\langle\psi| \qquad \langle\psi|\psi\rangle = 1$

$U_t = e^{-\frac{it}{\hbar}H} = e^{-i\omega t P_\psi} = \sum_{n=1}^{\infty}(-i\omega t)^n \frac{1}{n!} P_\psi$

$P_\psi = \sum_{i=1}^{d} |\psi_i^H\rangle\langle\psi_i^H| \, |\langle\psi_i^H|\psi\rangle|^2$

$H = H^+ = \sum_i E_i |E_i\rangle\langle E_i| = \hbar\omega P_\psi = \hbar\omega \sum_{i=1}^{d} |\psi_i^H\rangle\langle\psi_i^H| \, |\langle\psi_i^H|\psi\rangle|^2$

$U_t = \sum_{n=1}^{\infty}\left(-\frac{it}{\hbar}\right)^n \frac{1}{n!}\left(\hbar\omega\sum_{i=1}^{d}|\langle\psi_i^H|\psi\rangle|^2 \; P_i^H\right)^n$

$= \sum_{i=1}^{d}\left(\sum_{n=1}^{\infty}\left(-i\omega t\,|\langle\psi_i^H|\psi\rangle|^2\right)^n \frac{1}{n!}\right) P_i^H$

2) $H = \hbar\omega P_\psi \qquad U_t = \sum_{n=0}^{\infty}\left(-\frac{it}{\hbar}\right)^n \frac{1}{n!} H^n = \left(\sum_{n=0}^{\infty}(-i\omega t)\frac{1}{n!}\right) \overset{\text{IDEMPOTENTE}}{P_\psi} = e^{-i\omega t} P_\psi$

1) studio $P_\psi \qquad P_\psi|\phi\rangle = \langle\psi|\phi\rangle|\psi\rangle \qquad \forall|\phi\rangle$

def $P_\psi = P_1$ 1° elemento ~~base~~ ONB $\quad P_1 = \langle\psi_1|\phi\rangle|\psi_1\rangle$

?

con $|\phi\rangle = \sum_{i=1}^{d}\langle\psi_i|\phi\rangle|\psi_i\rangle$

$P_1|\phi\rangle = |\psi_1\rangle\langle\psi_1|\left(\sum_{i=1}^{d}\langle\psi_i|\phi\rangle|\psi_i\rangle\right) = \sum_{i=1}^{d}\langle\psi_i|\phi\rangle\underbrace{\langle\psi_1|\psi_i\rangle}_{\delta_{1i}}|\psi_1\rangle$

$= \langle\psi_1|\phi\rangle|\psi_1\rangle$

$P_1|\phi\rangle = \lambda|\phi\rangle = \langle\psi_1|\phi\rangle|\psi_1\rangle \qquad \lambda = \langle\psi_1|\phi\rangle \Rightarrow |\phi\rangle = \lambda|\psi_1\rangle$

$\Rightarrow P_1|\phi\rangle = \lambda^2|\psi_1\rangle = \lambda|\phi\rangle \Rightarrow \boxed{\lambda = 1 \quad |\phi\rangle = |\psi_1\rangle}$

3) $\text{Prob}(|\phi_t\rangle = |\chi\rangle) = |\langle\chi|\phi_t\rangle|^2 \quad$ con $\phi_t = U_t\,\phi_{t=0} \qquad \phi_{t=0} = \phi$

$\forall\chi$