

ESERCIZIO M.Q.

1 sistema quantistico $H: \mathbb{C}^d$, $d \geq 2$, evolve secondo hamiltoniana

$$H = \kappa \omega P_\varphi = \kappa \omega |\varphi\rangle\langle\varphi|$$

dove P_φ proietta sullo stato normalizzato $|\varphi\rangle \in \mathbb{C}^d$ $\langle\varphi|\varphi\rangle = 1$

2) Trovare lo SPETTRO di H , i suoi autovettori e la degenerazione degli autovettori.

$$H |E_e\rangle = E_e |E_e\rangle$$

$$\kappa \omega P_\varphi |E_e\rangle = E_e |E_e\rangle$$

$$\kappa \omega P_\varphi |\varphi\rangle = \kappa \omega |\varphi\rangle$$

AUTOVETTORI E_e

~~det $(\kappa \omega P_\varphi - E_e I) = 0$~~

$$E_1 = \kappa \omega \quad \text{deg} = 1$$

$$|E_1\rangle = |\varphi\rangle$$

\mathbb{B}

3) operatore di evoluzione temporale associato ad H usando:

- teorema spettrale
- serie esponenziale

$$i\hbar \partial_t |\psi_t\rangle = H |\psi_t\rangle$$

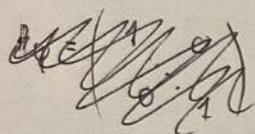
$$|\psi\rangle \xrightarrow{U_t} |\psi_t\rangle = U_t |\psi\rangle = e^{-i/\hbar t \cdot H} |\psi\rangle$$

$$= \sum_{K=0}^{\infty} \left(-\frac{it}{\hbar}\right)^K \frac{H^K}{K!}$$

$E_j = \text{autovalore}$

$$\bullet U_t = e^{-i/\hbar t H} = \sum_j e^{-i/\hbar t E_j} |E_j\rangle \langle E_j| \quad E_j = \hbar\omega \quad |E_j\rangle = \text{autovettore}$$

$$\bullet U_t = e^{-i/\hbar t \hbar\omega P_\psi} = e^{-it\omega P_\psi} = \sum_{K=0}^{\infty} \left(-it\omega\right)^K \frac{1}{K!} P_\psi^K$$



$$P_\psi = |\psi\rangle \langle \psi|$$

$$P_\psi^K = P_\psi$$

~~Ex. base~~

$$\Rightarrow U_t = e^{-i\omega t} |\psi\rangle \langle \psi| = e^{-i\omega t} P_\psi$$

n°4 $|\phi\rangle$ a $t=0$;

• $t > 0 \rightarrow |\chi\rangle \in e^d$

• $|\psi\rangle$

• $|\psi\rangle^\perp$

$$\text{Prob}(|\phi_t\rangle = |\psi\rangle^\perp) = |\langle \psi^\perp | \phi_t \rangle|^2 = |\langle \psi^\perp | U_t \phi \rangle|^2$$

$$\text{Prob}(|\phi_t\rangle = |\chi\rangle) = |\langle \chi | \phi_t \rangle|^2 = |\langle \chi | U_t \phi \rangle|^2$$

$$\text{Prob}(|\phi_t\rangle = |\psi\rangle) = |\langle \psi | \phi_t \rangle|^2 = |\langle \psi | U_t \phi \rangle|^2 = |\langle \psi | e^{-i/\hbar t H} \phi \rangle|^2 = \langle \psi | e^{-i\omega t P_\psi} \phi \rangle^2$$