1) autovalore $\lambda = \hbar\omega$ di degenerazione $= 1$

$\rightarrow Sp(H) = \{\hbar\omega\}$

2) $U_t = e^{-\frac{i}{\hbar}tH}$ con $H = \hbar\omega P_\psi \rightarrow U_t = e^{-i\omega t P_\psi}$

Per il teorema spettrale $U_t = \sum_j e^{-i\omega t} |E_j\rangle\langle E_j| = e^{-i\omega t} P_\psi$

Con la serie esponenziale $U_t = \sum_{k=0}^{\infty} \left(\frac{-it}{\hbar}\right)^k \frac{H^k}{k!} =$

$= \sum_{k=0}^{\infty} \left(\frac{-it}{\hbar}\right)^k \frac{(\hbar\omega P_\psi)^k}{k!} = \sum_{k=0}^{\infty} \frac{(-it\omega P_\psi)^k}{k!} =$ il proiettore è idempotente

$= P_\psi \sum_{k=0}^{\infty} \frac{(-it\omega)^k}{k!} = e^{-i\omega t} P_\psi$

3) $Prob(|\Phi_t\rangle = |x\rangle) = |\langle x|\Phi_t\rangle|^2 = |\langle x|U_t\Phi\rangle|^2 =$

$= |\langle x|(e^{-i\omega t} P_\psi|\Phi\rangle)\rangle|^2 = |e^{-i\omega t}\langle x|(\langle\psi|\Phi\rangle|\psi\rangle)\rangle|^2 =$

$= |e^{-i\omega t}\langle\psi|\Phi\rangle\langle x|\psi\rangle|^2$

$Prob(|\Phi_t\rangle = |\psi\rangle) = |\langle\psi|\Phi_t\rangle|^2 = |\langle\psi|U_t\Phi\rangle|^2 \overset{\text{come prima}}{=} \dots =$

$= |e^{-i\omega t}\langle\psi|\Phi\rangle\overbrace{\langle\psi|\psi\rangle}^{=1}|^2 = |e^{-i\omega t}\langle\psi|\Phi\rangle|^2$

$Prob(|\Phi_t\rangle = |\psi_\perp\rangle) = |\langle\psi_\perp|\Phi_t\rangle|^2 = \dots \overset{\text{come prima}}{=}$

$= |e^{-i\omega t}\langle\psi|\Phi\rangle\overbrace{\langle\psi_\perp|\psi\rangle}^{=0}|^2 = 0$