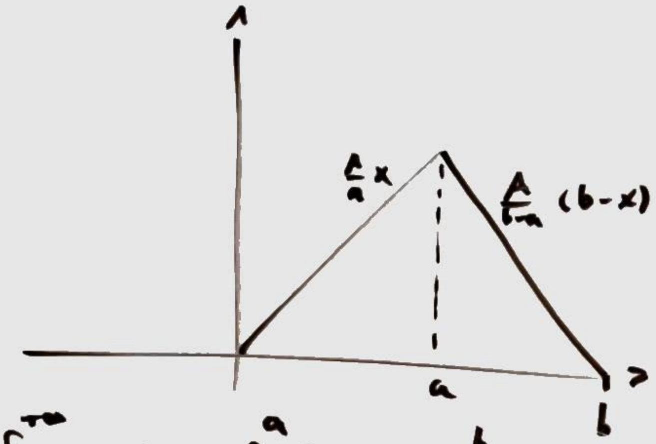
1/02/19

1) 1.



2.
$$1 = \int_{-\infty}^{+\infty} |\psi(x)|^2 = \int_0^a \psi^2(x) + \int_a^b \psi^2(x)$$
$$= \left(\frac{A}{a}\right)^2 \frac{a^3}{3} + \left(\frac{A}{b-a}\right)^2 \left[b^2(b-a) + \frac{b^3-a^3}{3} - b(b^2-a^2) \right]$$
$$= \alpha \frac{a^3}{3} + \frac{A^2}{b-a}\left[b^2 + \frac{b^2+a^2+ab}{3} - \underset{=b^2+ba}{b(b+a)} \right]$$
$$= \alpha \frac{a^3}{3} + \beta \left[\frac{a^2}{3} + \frac{b^2}{3} - \frac{2}{3}ab \right]$$
$$= \alpha \frac{a^3}{3} + \frac{A^2}{3}(b-a)$$
$$\Rightarrow 1 = A^2\left(\frac{a}{3} + \frac{b}{3} - \frac{a}{3}\right) = A^2 \frac{b}{3}$$
$$\Rightarrow A = \sqrt{\frac{3}{b}}$$

3. $A = |x\rangle\langle x|$ con $x < a$

$$P(\text{nel punto } y) = |\langle y|\psi\rangle|^2$$
$$= |\psi(y)|^2$$
$$\Rightarrow P = \int_0^a \psi(y)^2\, dy = \left(\frac{A}{a}\right)^2 \frac{a^3}{3} = \frac{3}{b}\frac{a}{3} = \frac{a}{b}$$

4.
$$\langle \hat{x} \rangle = \langle \psi | \hat{x} \psi \rangle = \int_{\mathbb{R}} \langle \psi | y \rangle \langle y | \hat{x} \psi \rangle \, dy$$
$$= \int_{\mathbb{R}} dy \; \psi(y)\, y\, \psi(y)\, dy$$
$$= \int_0^b dy\; y\, \psi(y)^2$$
$$= \int_0^a dy\, y\, \psi^2(y) + \int_a^b dy\; y\; \psi(y)^2$$

1/02/19

$$= \left(\frac{A}{a}\right)^2 \frac{x^4}{4}\Big|_0^a + \left(\frac{A}{b-a}\right)^2 \left[\frac{b^2-a^2}{2} b^2 + \frac{b^4-a^4}{4} - 2b \frac{b^3-a^3}{3}\right]$$

$$= \left(\frac{Aa}{2}\right)^2 + \beta \left[\frac{3}{4} b^4 - \frac{a^2b^2}{2} - \frac{a^4}{4} - \frac{2}{3} b^4 - \frac{2}{3} ba^3\right]$$

$$= \alpha + \beta \left[\frac{1}{12} b^4 - \frac{a^2b^2}{2} - \frac{2}{3} ba^3 - \frac{a^4}{4}\right]$$

$$= \alpha + \frac{1}{12}\beta \left(b^4 - 6a^2b^2 - 8ba^3 - 3a^4\right)$$

$$= \frac{3}{2}\frac{a^2}{b} + \frac{1}{4}\frac{1}{b}\frac{1}{b-a}\left(b^4 - 6a^2b^2 - 8ba^3 - 3a^4\right)$$

5. $\psi^2(x)$ e' la d.d.p. della v.a. posizione della particella

$\rightarrow$ la probabilità e' massima in

$$x : \psi(x) = \sup_{y \in \mathbb{R}} \psi(y)$$

cioe' in $x = a$.