1/02/19

3\. 1. $H_0 = \frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2$

2\. $\hat{a}|0\rangle = 0$

$$= \sqrt{\frac{m\omega}{2\hbar}}\,\hat{x}|0\rangle + i\sqrt{\frac{1}{2m\omega\hbar}}\,\hat{p}|0\rangle$$

$\downarrow$ posizioni

$$\sqrt{\frac{m\omega}{2\hbar}}\,x\,\psi_0(x) = i\,\frac{i\hbar\,\partial_x\psi_0(x)}{\sqrt{2m\omega\hbar}} = -\frac{\hbar\,\partial_x\psi_0(x)}{\sqrt{2m\omega\hbar}}$$

$$-\frac{m\omega}{\hbar}x = \frac{\partial_x\psi_0(x)}{\psi_0(x)} \longrightarrow -\frac{m\omega}{\hbar}\frac{x^2-x_0^2}{2} = \ln\psi_0(x) - c$$

$$\ln\psi_0(x) = -\frac{m\omega}{\hbar}\frac{x^2}{2} - d + c$$

$$\Rightarrow \psi_0(x) = c'\,e^{-\frac{m\omega}{\hbar}\frac{x^2}{2}}$$

Normalizzazione:

$$1 = c'^2\int_{-\infty}^{+\infty} e^{-\frac{m\omega}{\hbar}x^2} = c'^2\sqrt{\frac{\hbar}{m\omega}}\int_{-\infty}^{+\infty} e^{-y^2}\,dy = c'^2\sqrt{\frac{\hbar\pi}{m\omega}}$$

$y = ax \qquad y^2 = (ax)^2$

$dy = a\,dx$

$\Rightarrow dx = \frac{dy}{a}$

$$c' = \sqrt{\frac{m\omega}{\hbar\pi}}^{\,1/2} = \sqrt[4]{\frac{m\omega}{\pi\hbar}}$$

$$\Rightarrow \psi_0(x) = \sqrt[4]{\frac{m\omega}{\pi\hbar}}\,e^{-\frac{m\omega}{2\hbar}x^2}$$

$$H_0\psi_0(x) = \frac{(-i\hbar)^2}{2m}\partial_x^2\psi_0(x) + \frac{m\omega^2}{2}x^2\psi_0(x)$$

$$= -\frac{\hbar^2}{2m}\partial_x\left(-\frac{m\omega}{\hbar}x\,\psi_0(x)\right) + \frac{m\omega^2}{2}x^2\psi_0(x)$$

$$= \frac{\hbar\omega}{2}\psi_0(x)\left(1 - x\frac{m\omega}{\hbar}x\right) + \frac{m\omega^2}{2}x^2\psi_0(x)$$

$$= \frac{\hbar\omega}{2}\psi_0(x) - \frac{m\omega^2}{2}x^2\psi_0(x) + \frac{m\omega^2}{2}x^2\psi_0(x)$$

$$= \frac{\hbar\omega}{2}\psi_0(x)$$