1/02/19

3.

3. $\tilde{E}_n^{(1)} = \langle E_n^{(0)} | H' E_n^{(0)} \rangle$

$$|\tilde{E}_n^{(1)}\rangle = \sum_{m \neq n} \frac{\langle E_m^{(0)} | H' E_n^{(0)} \rangle}{E_n^{(0)} - E_m^{(0)}}$$

$$\int_a^b \frac{\partial}{\partial x} f(x) = f(b) - f(a)$$

4. $\tilde{E}_0^{(0)} = E_0 = \frac{\hbar\omega}{2}$

$\tilde{E}_0^{(1)} = \langle E_0^{(0)} | H' E_0^{(0)} \rangle$

$H' = \langle x | \lambda \hat{x} \psi_\gamma \rangle$

$$L = \frac{1}{2}\sqrt{\frac{2\hbar}{m\omega}} (\hat{a} + \hat{a}^+)$$

$$\Rightarrow \tilde{E}_n^{(1)} = \lambda\sqrt{\frac{\hbar}{2m\omega}} \left(\langle 0 | \hat{a} \psi_\gamma | 0 \rangle + \langle 0 | \hat{a}^+ \psi_\gamma | 0 \rangle \right)$$

$$= \lambda\sqrt{\frac{\hbar}{2m\omega}} \left(\langle \hat{a}^+ 0 | \psi_\gamma | 0 \rangle + \langle \hat{a} 0 | \psi_\gamma | 0 \rangle \right)$$

$$= \lambda\sqrt{\frac{\hbar}{2m\omega}} \left(\langle 1 | \psi_\gamma | 0 \rangle \right)$$

$$= \lambda\sqrt{\frac{\hbar}{2m\omega}} \int_{-\infty}^{+\infty} \langle 1 | x \rangle \langle x | \psi_\gamma | 0 \rangle \, dx$$

$\hat{a}^+ |0\rangle = |1\rangle$

$$\langle x | \hat{a}^+ \psi_0 \rangle = \langle x | \sqrt{\frac{m\omega}{2\hbar}} \hat{x} \psi_0 - i \frac{1}{\sqrt{2m\omega\hbar}} \hat{p} \psi_0 \rangle$$

$$= \sqrt{\frac{m\omega}{2\hbar}} x \psi_0 - \hbar \partial_x \psi_0 \frac{1}{\sqrt{2m\omega\hbar}}$$

$$= \sqrt{\frac{m\omega}{2\hbar}} x \psi_0(x) - \sqrt{\frac{\hbar}{2m\omega}} \left(-\frac{m\omega}{\hbar} x \right) \psi_0(x)$$

$$= x \psi_0(x) \left(\sqrt{\frac{m\omega}{2\hbar}} + \sqrt{\frac{m\omega}{2\hbar}} \right)$$

$$= \sqrt{\frac{2m\omega}{\hbar}} \, x \psi_0(x)$$

$$= \lambda\sqrt{\frac{\hbar}{2m\omega}} \int_{-\infty}^{+\infty} \sqrt{\frac{2m\omega}{\hbar}} \, x \psi_0(x) e^{-\gamma x^2} \psi_0(x) \, dx$$

$$= \lambda \int_{-\infty}^{+\infty} x \psi_0(x) e^{-\gamma x^2} dx$$

$$= \lambda \int_{-\infty}^{+\infty} x \sqrt{\frac{m\omega}{\pi\hbar}} e^{-x^2(\gamma + \frac{m\omega}{2\hbar})} dx$$

$= 0$ (dispari)