1/02/19

3.

$$\langle x|2\rangle = \frac{\psi_0(x)}{\sqrt{2}}\left(\frac{2m\omega}{\hbar}x^2 - 1\right)$$

$$\langle 2|\psi_0\rangle = \int_{-\infty}^{+\infty} \langle 2|x\rangle\langle x|\psi_0\rangle\, dx$$

$$= \int_{-\infty}^{+\infty} \frac{\psi_0(x)}{\sqrt{2}}\left(\frac{2m\omega}{\hbar}x^2 - 1\right) e^{-\gamma x^2}\psi_0(x)\, dx$$

$$= \frac{1}{\sqrt{2}}\left[\underbrace{\int_{-\infty}^{+\infty} x^2\psi_0(x)^2 e^{-\gamma x^2}}\frac{2m\omega}{\hbar}dx - \underbrace{\int_{-\infty}^{+\infty}\psi_0(x)^2 e^{-\gamma x^2}dx}\right]$$

$$= \sqrt{\frac{m\omega}{\pi\hbar}}\int_{-\infty}^{+\infty} e^{-x^2\left(\gamma + \frac{m\omega}{\hbar}\right)} x^2\, dx$$

$$= \cancel{\sqrt{\frac{m\omega}{\pi\hbar}}}\sqrt{\frac{1}{\cancel{\frac{m\omega}{\hbar}}(\alpha+1)}}\;\frac{\hbar}{m\omega(\alpha+1)}\;\frac{\cancel{\sqrt{\pi}}}{2}$$

$$= \frac{\hbar}{2m\omega(\alpha+1)}\frac{1}{\sqrt{\alpha+1}} \qquad \alpha = \frac{\gamma\hbar}{m\omega}$$

$$= \frac{1}{2\gamma}\frac{\alpha}{\alpha+1}\frac{1}{\sqrt{\alpha+1}}$$

$$\sqrt{\frac{m\omega}{\pi\hbar}}\int_{-\infty}^{+\infty} e^{-x^2\left(\gamma+\frac{m\omega}{\hbar}\right)}dx$$

$$= \sqrt{\frac{m\omega}{\pi\hbar}}\sqrt{\frac{\pi}{\gamma+\frac{m\omega}{\hbar}}} = \sqrt{\frac{m\omega}{\gamma\hbar+m\omega}}$$

$$= \sqrt{\frac{1}{\frac{\gamma\hbar}{m\omega}+1}}$$

$$= \sqrt{\frac{1}{\alpha+1}}$$

$$= \frac{1}{\sqrt{2}}\sqrt{\frac{1}{\alpha+1}}\left[\frac{1}{2\gamma}\frac{\alpha}{\alpha+1} - 1\right]$$

$$= \frac{1}{\sqrt{2}}\sqrt{\frac{1}{\alpha+1}}\left[\frac{1}{2}\frac{\hbar/m\omega}{\gamma\hbar/m\omega+1} - 1\right]$$

$$= \frac{1}{\sqrt{2}}\sqrt{\frac{1}{\alpha+1}}\left[\frac{1}{2}\frac{\hbar}{\gamma\hbar+m\omega} - \frac{2(\gamma\hbar+m\omega)}{2(\gamma\hbar+m\omega)}\right]$$

$$= \frac{1}{\sqrt{2}}\sqrt{\frac{m\omega}{\gamma\hbar+m\omega}}\;\frac{\hbar\left(\frac{1}{2}-2\gamma\right)-2m\omega}{2(\gamma\hbar+m\omega)}$$

$$\langle 0|\psi_0\rangle = m\omega\sqrt{\frac{1}{\pi\hbar m\omega+\pi\hbar^2\gamma}}$$

$$= m\omega\sqrt{\frac{1}{\pi\hbar(m\omega+\hbar\gamma)}}$$

$$\Rightarrow \tilde{E}_0^{(1)} = \frac{\lambda\hbar}{2m\omega}\left(\langle 2|\psi_0\rangle + \langle 0|\psi_0\rangle\right)$$

... calcolo

5. Vogliamo che

$$\lambda\langle E_0^{(0)}|H' E_0^{(0)}\rangle \ll \frac{\hbar\omega}{2}$$

Quindi dovremmo calcolare

$$\langle E_0^{(0)}|H' E_0^{(0)}\rangle$$

...