Orbit Dynamics and Analysis

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Kepler's laws of planetary motion

1. Orbits are ellipses with sun at one focus.
2. The line joining the planet to the sun sweeps equal areas in equal times.
3. The square of the period of a planet is proportional to the cube of its mean distance to the sun.

Newton's law of gravity:

The gravitational force acting upon a satellite orbiting the earth

$$\vec{F}_G = -\frac{GM_{earth}m}{r^2}\left(\frac{\vec{r}}{r}\right)$$

where G is the universal constant of gravity, M_{earth} is the mass of the Earth, m is the mass of the satellite and r is the position vector of the satellite relative to the centre of Earth.

Conservation of energy and angular momentum

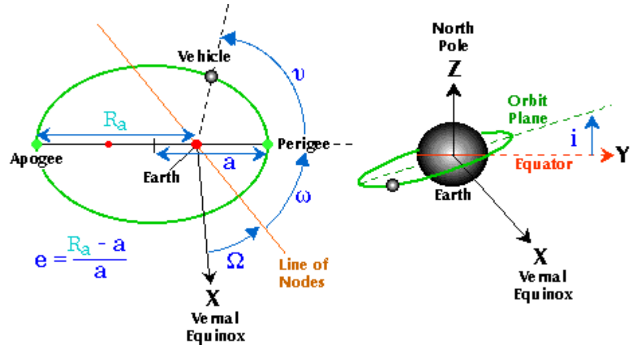
Introducing $\mu = GM_{earth}$ and the normalising with respect to the mass of the orbiting object, we can consider the specific energy (potential energy is referred to an infinitely distant point):

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} = CONSTANT$$, i.e. energy is CONSERVED

Also;

$$\vec{h} = \vec{r} \times \vec{v} = CONSTANT$$, i.e. angular momentum is CONSERVED.

Classical orbital elements



Semi major axis (size):	a
Eccentricity (shape):	e
Inclination (orientation):	i
Right ascension of the ascending node (orientation):	Ω
Argument of perigee (location):	ω
True anomaly (location):	υ

Some simple derived parameters

Radius of perigee: $r_p = a(1-e)$

Radius of apogee: $r_a = a(1+e)$

Orbital period: $P = 2\pi\sqrt{a^3/\mu}$ [s]

Mean motion: $\omega_0 = \sqrt{\mu/a^3}$ [rad/s]

Orbit perturbations:

-Non-spherical earth perturbations in degrees per day:

Right ascension: $\dot{\Omega}_{J_2} = -2.06474 \times 10^{14} a^{-3.5} \cos(i)(1-e^2)^{-2}$

Argument of perigee: $\dot{\omega}_{J_2} = 1.03237 \times 10^{14} a^{-3.5} (4 - 5\sin^2(i))(1-e^2)^{-2}$

-Lunar and solar perturbations in degrees per day

Moon impact on Right ascension $\dot{\Omega}_{moon} = -0.00338\cos(i)/n$

Sun impact on Right ascension $\dot{\Omega}_{sun} = -0.00154\cos(i)/n$

Moon impact on arg. of perigee $\dot{\omega}_{moon} = 0.00169\left(4-5\sin^2(i)\right)/n$

Sun impact on arg. Of perigee $\dot{\omega}_{sun} = 0.00077\left(4-5\sin^2(i)\right)/n$

Where n is the number of orbit revolutions per day.

-Atmospheric drag for <u>near circular</u> orbits:

Change in velocity: $\Delta V = \pi(C_D A/m)\rho a V$ (m/s per orbit)

Change in semimajor axis: $\Delta a = -2\pi(C_D A/m)\rho a^2$ (m per orbit)

Change in period: $\Delta P = -6\pi^2(C_D A/m)\rho a^2/V$ (s per orbit)

$(C_D A/m)$ is the ballistic coef and ρ is the density of the atmosphere.

-Solar pressure

$$Acceleration_{solar\ preassure} \approx 4.5\times10^{-6}(1+r)A/m$$

Where A is the crosssection area exposed to the sun and m is mass.

Hohmann transfer

1. Find the following quantities: r_1, a_{xfr} and r_2, which are the semi major axis of the parking orbit, the transfer orbit and the mission orbit respectively.
2. Find velocity in the parking orbit: $V_1 = \sqrt{\mu/r_1}$
3. Find velocity at perigee in transfer orbit: $V_{xfr1} = \sqrt{2\left(\frac{-\mu}{2a_{xfr}} + \frac{\mu}{r_1}\right)}$
4. Find velocity at apogee of transfer orbit: $V_{xfr2} = \sqrt{2\left(\frac{-\mu}{2a_{xfr}} + \frac{\mu}{r_2}\right)}$
5. Find velocity in the mission orbit. $V_2 = \sqrt{\mu/r_{21}}$
6. Find total ΔV required: $\Delta V_{tot} = (V_{xfr1} - V_1) + (V_2 - V_{xfr2})$

Orbit plane change

Simple plane change (thrust perpendicular to orbit plane):

$$\Delta V = 2V\sin(\theta/2)$$

Combined plane change (simultaneous plane and velocity change):

$$\Delta V = \sqrt{V_{initial}^2 + V_{final}^2 - 2V_{initial}V_{final}\cos\theta}$$

In-plane positioning

ΔV requirement depends on how fast you wish to reposition, not on the reposition angle:

$$\omega_{drift} = 1080(\Delta V/V) \quad \text{[degrees/orbit]}$$

$$\Delta V_{total} = \Delta V_{start} + \Delta V_{stop} = 2(\omega_{drift}V)/1080$$

Orbit maintenance

Drag: $\Delta V = \pi\left(\frac{C_D A}{m}\right)\rho a V$ (m/s per orbit)

North-South: $\Delta V_{moon} = 102.67\cos\alpha\sin\alpha \quad [m/s \quad per\ year]$

$\approx 36.93 \quad [m/s \quad per\ year\ for\ i=0]$

East-West: $\Delta V_{sun} = 40.17\cos\gamma\sin\gamma \quad [m/s \quad per\ year]$

$\approx 14.45 \quad [m/s \quad per\ year\ for\ i=0]$

Where α is the angle between the orbital plane and the moon's orbit and γ is the angle between the orbital plane and the ecliptic plane.

Deorbiting

In principle, peform a burn to lower perigee to 0 km. This yields:

$$\Delta V_{deorbit} \approx V\left(1-\sqrt{2R_E/(2R_E+r)}\right)$$

For GEO orbits, graveyard is further out (due to collision avoidance considerations). Guideline is to raise orbit by:

$$235 + 1000\,rA/m \qquad \text{[km]}$$

r is the reflection coefficient (0 for absorption, 1 for reflection at normal incidence, 0.4 for diffuse reflection)