$z^n = 1$ la molteplicità degli zeri.

$z_1, \dots, z_n$ distinte

$$z^n - 1 = (z - z_1)^{m_1} \cdots (z - z_n)^{m_n}$$

$k = n$

$$m_1 + \dots + m_n = n$$

$$m_1, \dots, m_n \in \mathbb{N}$$

$$m_1 = m_2 = \dots = m_n = 1$$

$$z^n = (x+iy)^n = \sum_{j=0}^{n} \binom{n}{j} i^j y^j x^{n-j}$$

$$i^j \begin{cases} \pm i & \text{se } j \text{ è dispari} \\ \pm 1 & \text{se } j \text{ è pari} \end{cases}$$

$$= \sum_{\substack{j=0 \\ j \text{ pari}}}^{n} \binom{n}{j} i^j y^j x^{n-j} + \sum_{\substack{j=0 \\ j \text{ dispari}}}^{n} \binom{n}{j} i^j y^j x^{n-j}$$

$$\{ j : 0 \le j \le n , j \text{ pari} \} = \{ 2k : 0 \le k \le [\tfrac{n}{2}] \}$$

$$\sum_{\substack{j=0 \\ j \text{ pari}}}^{n} \binom{n}{j} i^j y^j x^{n-j} = \sum_{k=0}^{[\frac{n}{2}]} \binom{n}{2k} (i^2)^k y^{2k} x^{n-2k}$$

$$\operatorname{Re}(x+iy)^n = \sum_{k=0}^{[\frac{n}{2}]} \binom{n}{2k} (-1)^k y^{2k} x^{n-2k}$$

$$\sum_{\substack{j=0 \\ j \text{ dispari}}}^{n} \binom{n}{j} i^j y^j x^{n-j} = \sum_{\substack{j=1 \\ j \text{ dispari}}}^{n} \binom{n}{j} i^j y^j x^{n-j}$$

$$\{ j : 1 \le j \le n, j \text{ dispari} \} = \{ 2k+1 : 0 \le k \le [\tfrac{n-1}{2}] \}$$

$$2k+1 \le n \qquad k \le \frac{n-1}{2}$$

$$\sum_{\substack{j=1 \\ j \text{ dispari}}}^{n} \binom{n}{j} i^j y^j x^{n-j} = i \underbrace{\boxed{\sum_{k=0}^{[\frac{n-1}{2}]} \binom{n}{2k+1} (-1)^k y^{2k+1} x^{n-2k-1}}}_{\operatorname{Im}(x+iy)^n}$$

$$\boxed{|L_1 - L_2| < \varepsilon \ \forall \varepsilon > 0 \Rightarrow L_1 = L_2}$$ $\ast$

Suppongo $L_1 \neq L_2$ e che $L_1 > L_2$.

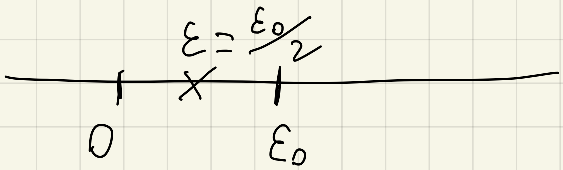
$$L_1 = L_2 + \varepsilon_0 \qquad \text{con } \varepsilon_0 > 0$$

$$|L_1 - L_2| = |\varepsilon_0| = \varepsilon_0 < \varepsilon \qquad \forall \varepsilon > 0$$

$0 = \inf \mathbb{R}_+$

1) Se $\varepsilon_0 < \varepsilon \ \forall \varepsilon > 0 \Rightarrow \varepsilon_0 \leq 0$ ✓ $\ast$

2) $\boxed{|L_1 - L_2| = 0 \Rightarrow L_1 = L_2}$ ✓



$\varepsilon < \varepsilon_0 < \varepsilon$

$\varepsilon < \varepsilon$ assurdo

$$f : (0, +\infty) \to \mathbb{R}$$

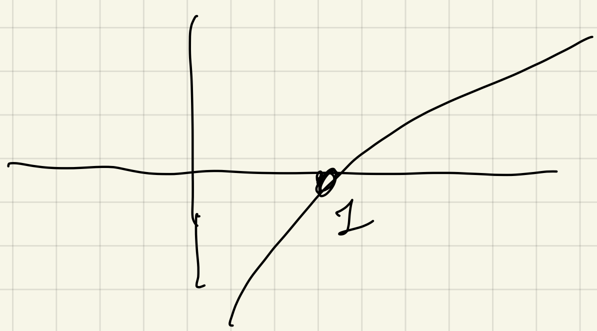
$$f(x) = x + \log_6 x$$

e' iniettiva.

$f(x)$ e' strettamente crescente.

1) x e' strettamente crescente

2) $\log_6 x$ e' strettamente crescente



$f_1, f_2 : (0, +\infty) \to \mathbb{R}$ f_1, f_2 strettamente crescenti, allora $f_1 + f_2$ e' strettamente crescente

$x_1 < x_2$

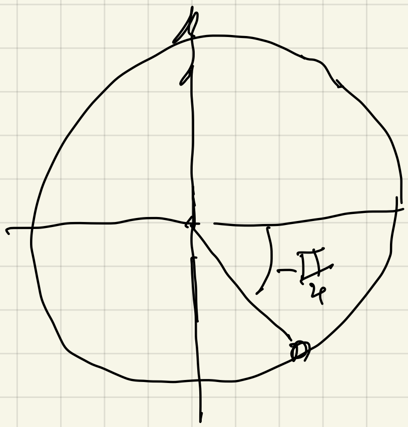
$$\left.\begin{array}{l} f_1(x_1) < f_1(x_2) \\ f_2(x_1) < f_2(x_2) \end{array}\right\} +$$

$$f_1(x_1) + f_2(x_1) < f_1(x_2) + f_2(x_2)$$

$$|1-i| = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\sqrt{2-2i} \qquad 1-i = \sqrt{2} \qquad \frac{1-i}{\sqrt{2}}$$

$$2-2i = 2(1-i) = 2\sqrt{2}\left(\frac{1-i}{\sqrt{2}}\right) =$$



$$= 2\sqrt{2}\,(\cos\vartheta + i\sin\vartheta)$$

$$\cos\vartheta = \frac{1}{\sqrt{2}} \qquad \vartheta = -\frac{\pi}{4}$$

$$\sin\vartheta = -\frac{1}{\sqrt{2}}$$

$$z^2 = 2^{\frac{3}{2}}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$$

$$z = r(\cos\vartheta + i\sin\vartheta)$$

DeMoivre

$$z^2 = \cancel{r^2}\left(\cos(2\vartheta) + i\sin(2\vartheta)\right) = \cancel{2^{\frac{3}{2}}}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$$

$$\boxed{r = 2^{\frac{3}{4}}}$$

$$\begin{cases} \cos(2\vartheta) = \cos\left(-\frac{\pi}{4}\right) \\ \sin(2\vartheta) = \sin\left(-\frac{\pi}{4}\right) \end{cases} \Rightarrow 2\vartheta = -\frac{\pi}{4} + 2\pi k \qquad k \in \mathbb{Z}$$

$$\vartheta = -\frac{\pi}{4} + \frac{2\pi k}{2} \qquad \boxed{k = 0, 1}$$

$$\sqrt{2-2i} = \sqrt{2}\ \sqrt{1-i} = \sqrt{2}\ (1-i)^{\frac{1}{2}}$$! Sbagliato!

$$\cancel{=}\sqrt{2} \qquad (\sqrt{2})^{\frac{1}{2}}\ \left(\cos\tfrac{1}{2}\vartheta + i\sin\tfrac{1}{2}\vartheta\right)$$

$$z = (1-i)^{\frac{1}{2}}$$

$$z^2 = 1-i$$

non è un polinomio in z.

$$\cancel{\boxed{z^4 + 2|z|^2 = 1}}$$

ho 8 zeri

$$|z|^2 = z\bar{z}$$

$$P(z) = \boxed{z^4 + a_3 z^3 + a_2 z^2 + a_1 z + a_0}$$

$$\underline{a_n} z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0$$

$$z^4 + 2|z|^2 = (x+iy)^4 + 2(x^2+y^2) = P(x,y)$$

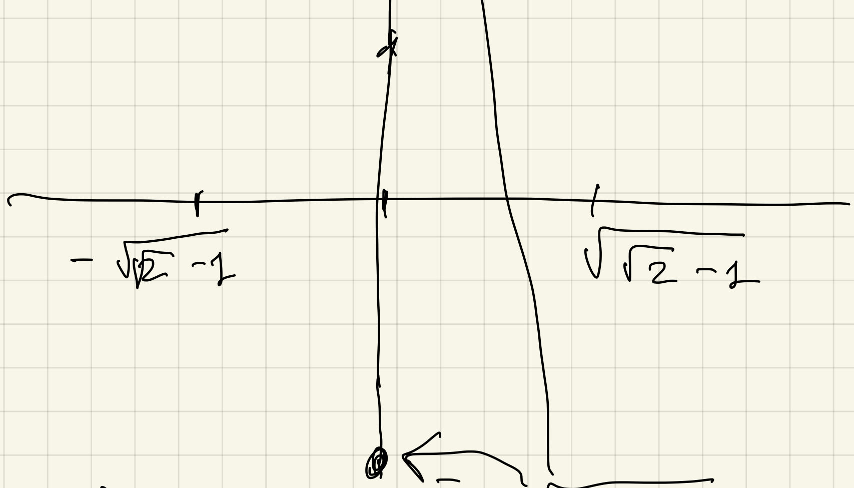
$$z^4 + 2z^2 = 1 \qquad w = z^2$$

$$w^2 + 2w - 1 = 0$$

$$w_\pm = -1 \pm \sqrt{1+1} = -1 \pm \sqrt{2}.$$

$$z^2 = \sqrt{2} - 1 \qquad z^2 = -\sqrt{2} - 1$$

$$z^2 = \sqrt{2} - 1 \iff z = \pm\sqrt{\sqrt{2} - 1}$$



$$z^2 = -\left(\sqrt{2}+1\right) \iff z = \pm i\sqrt{\sqrt{2}+1}$$

$X \subseteq \mathbb{R}$ $\quad X$ finito, $\quad X' = \emptyset$

$$\lim_{n\to\infty} \frac{b^n}{n!} = 0$$

$$\frac{b^n}{n!} = \frac{(1+b-1)^n}{n!} = \boxed{\frac{(1+a)^n}{n!}} \geq \frac{1+na}{n!} = \frac{1+n(b-1)}{n!} =$$

$\frac{1+na}{n!} \xrightarrow[n\to+\infty]{} 0$

$$= \frac{1+\cancel{n}(b-1)}{\cancel{(1+n)\,n}\ (n-1)!\ \cancel{(n+1)!}}$$

$$= \frac{1}{n!} + \frac{1}{(n-1)!}(b-1)$$

$$\leq \frac{1}{n} + \frac{1}{n-1}(b-1) \xrightarrow{n\to} 0$$

$b > 1$ $\qquad [b] \leq b < [b]+1$

per $n > [b]+1$

$$0 < \frac{b^n}{n!} = \frac{\overbrace{b \cdot b \cdots b}^{[b]+1}\ b \cdots\cdots b}{1 \cdot 2 \cdots ([b]+1)([b]+2)\cdots n} =$$

$$= \frac{b}{1}\,\frac{b}{2}\cdots\frac{b}{[b]+1}\,\frac{b}{[b]+2}\cdots\cdots\frac{b}{n}$$

$$\leq b\,\frac{b}{2}\cdots\frac{b}{[b]}\,\frac{b}{[b]+1}\,\frac{b}{[b]+1}\cdots\cdots\frac{b}{[b]+1}$$

$$= \left(b\,\frac{b}{2}\cdots\frac{b}{[b]}\right)\left(\frac{b}{[b]+1}\right)^{n-[b]}$$

$$0 < \frac{b^n}{n!} \leq \underbrace{\left(b\ \frac{b}{2} \cdots \frac{b}{[b]}\right)\left(\frac{b}{[b]+1}\right)^{-[b]}}\left(\frac{b}{[b]+1}\right)^n \xrightarrow{n\to\infty} 0$$

$$\text{Se } 0<a<1 \Rightarrow \lim_{n\to+\infty} a^n = 0 = \frac{1}{\lim_{n\to+\infty}\left(\frac{1}{a}\right)^n}$$

$$\Rightarrow \text{segue} \qquad \lim_{n\to+\infty}\left(\frac{1}{a}\right)^n = +\infty \qquad a^n = \frac{1}{\left(\frac{1}{a}\right)^n}$$

$$0 < \boxed{\frac{b}{[b]+1} < 1}$$

$$b < [b]+1$$

$$x_n > 0$$

$$\lim x_n$$

$$\lim_{n\to+\infty} \frac{x_{n+1}}{x_n} = r \qquad \text{e} \qquad r < 1$$

$$\Rightarrow \lim_{n\to+\infty} x_n = 0$$

$$\lim_{n \to +\infty} \frac{x_{n+1}}{x_n} =$$

$$\frac{\frac{b^{n+1}}{(n+1)!}}{\frac{b^n}{n!}} = \frac{b}{n+1} \longrightarrow 0 \qquad r = 0$$