UNIVERSITÀ
DEGLI STUDI DI TRIESTE

A.A. 2020-2021

Corso di Laurea Magistrale in GEOSCIENZE

Metodi Elettromagnetici in Geofisica (6 CFU) - MEMAG -

UD-2B: EM induction, conduction and polarization

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EM waves in geologic materials

The previously discussed equations (in this form strictly valid only when material properties are assumed isotropic, frequency-independent and linear) can be simplified by considering SPECIFIC FREQUENCY RANGES.

$$\underbrace{\nabla \times \nabla \times \boldsymbol{E}}_{A} + \underbrace{\mu\sigma \frac{\partial \boldsymbol{E}}{\partial t}}_{B} + \underbrace{\mu\varepsilon \frac{\partial^2 \boldsymbol{E}}{\partial t^2}}_{C} = 0 \qquad \nabla \times \nabla \times \boldsymbol{H} + \mu\sigma \frac{\partial \boldsymbol{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \boldsymbol{H}}{\partial t^2} = 0$$

A **B** **C**

The relative importance of each term can be estimated by the ratios:

$$\left|\frac{B}{A}\right| = \mu\sigma \frac{\Delta x^2}{\Delta t}$$

$$\left|\frac{C}{A}\right| \approx \mu\varepsilon \frac{\Delta x^2}{\Delta t^2}$$

and for a low loss environment we should have $\left|\frac{B}{A}\right| << \left|\frac{C}{A}\right|$ so: $\left|\frac{B}{C}\right| = \frac{\sigma \Delta t}{\varepsilon} << 1$

The solution of ABC eq., when reduced in the scalar form: $\frac{\partial^2}{\partial \beta^2} f(\beta,t) - \mu\sigma \frac{\partial}{\partial t} f(\beta,t) - \mu\varepsilon \frac{\partial^2}{\partial t^2} f(\beta,t) \equiv 0$

with $\beta = \mathbf{r} \cdot \mathbf{k}$

Is approximated by perturbing our wave solution by defining: $f(\beta,t) = p_1(\beta \pm vt)\, p_2(\beta)$

where p_1 is a solution in the zero loss case, namely

$$\frac{\partial^2 p_1}{\partial \beta^2} = \mu\varepsilon \frac{\partial^2 p_1}{\partial t^2}, \qquad p_2(\beta) = e^{\pm\alpha\beta} \quad \rightarrow \quad \alpha = \frac{\mu\sigma v}{2}$$

EM waves in geologic materials

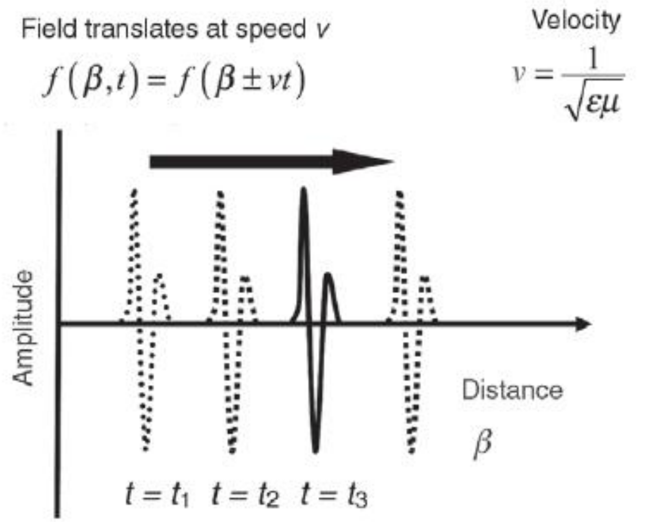


Figure 6. A function of the form $f(\beta \pm vt)$ represents an event which moves spatially at a velocity v. Such functions are solutions to the scalar wave equation.

More realistic solutions are damped waves:

$$f(\beta,t)=p_1(\beta\pm vt)e^{\pm\alpha\beta}$$

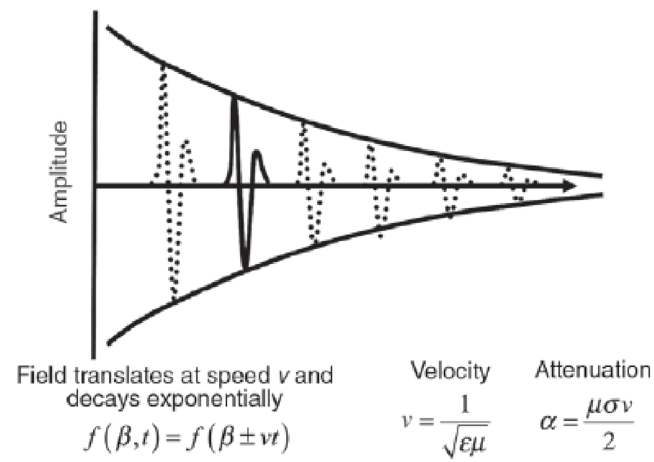


Figure 7. EM fields propagate as spatially damped waves when electrical losses are small. The signal amplitude decays exponentially in the direction of field translation while the field shape remains invariant.

Damped waves decay esponentially in the direction the waves are travelling, being α **the attenuation coefficient** and β **the phase constant**.

They can be combined into the **PROPAGATION CONSTANT or WAVENUMBER "*k*"** as:

$ik = a + ib$ (notice that it is complex in the general case – conductive dielectric!)

EM waves in geologic materials

The propagation constant is related to the electrical properties of the materials by the relation:

$$k = \sqrt{\omega^2 \varepsilon\mu + i\omega\mu\sigma}$$

For low loss materials, $\omega\mu\sigma \ll \omega^2\varepsilon\mu$ or $\frac{\sigma}{\omega\varepsilon} \ll 1$ (Low loss criterion)

The parameters α and β can be related to ε, σ and μ giving the following expressions:

$$\alpha = \omega\sqrt{\left[\frac{\mu\varepsilon'}{2}\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right]} \qquad \beta = \omega\sqrt{\left[\frac{\mu\varepsilon'}{2}\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]}$$

In which The dimensionless factor $\frac{\varepsilon''}{\varepsilon'}$ is often referred as "loss tangent" (tanδ) and for general lossy materials is frequency dependent and equal to:

$$\tan\delta = \frac{\sigma' + \omega\varepsilon''}{\omega\varepsilon' - \sigma''}$$

An interesting approximation of this formula is given by:

$$\tan\delta = \frac{\sigma_{DC}}{\omega\varepsilon_0\varepsilon_r} + \frac{\varepsilon''}{\varepsilon'}$$

CONDUCTION LOSSES (purple circle) DIPOLAR LOSSED (orange circle)

Conduction in geologic materials

$$\nabla \times \nabla \times \boldsymbol{E} + \mu\sigma \frac{\partial \boldsymbol{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0$$

A **B** **C**

We already pointed out the physical meaning of different terms in the ABC equation (as well as of its corresponding one related to the magnetic field).
When the fields are sinusoidal in time, it reduces into:

$$\nabla \times \nabla \times \mathbf{E} - i\omega\mu\sigma\mathbf{E} - \omega^2\varepsilon\mu\mathbf{E} = 0$$

For LOW FREQUENCY ($f<10^5$Hz) ➔ **$\omega^2\varepsilon\mu << \omega\mu\sigma$** so:

$$\nabla^2 \boldsymbol{E} \cong \mu\sigma \frac{\partial \boldsymbol{E}}{\partial t}$$

A **B**

Conduction currents dominate ➔ INDUCTION REGIME

For HIGH FREQUENCY ($f>10^7$Hz) and/or $\sigma \cong 0$ ➔ **$\omega^2\varepsilon\mu >> \omega\mu\sigma$** so:

$$\nabla^2 \boldsymbol{E} \cong \varepsilon\mu \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

A **C**

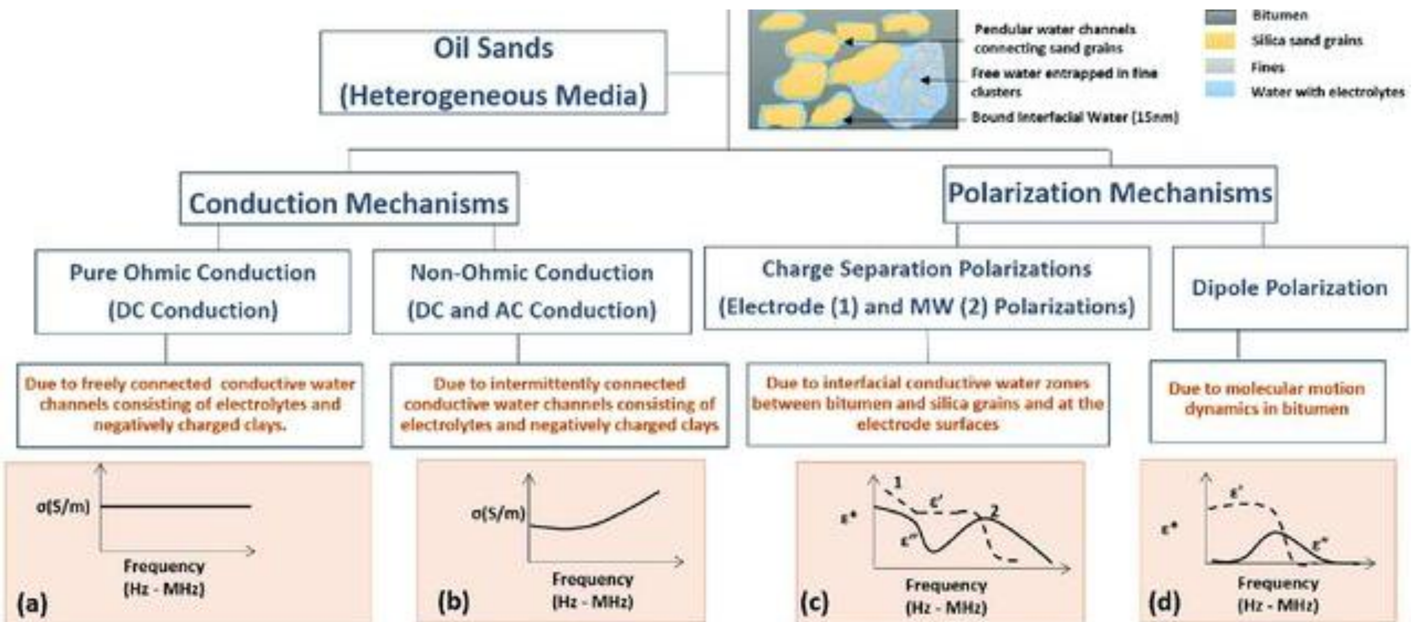
Displacement (polarization) currents dominate ➔ PROPAGATION REGIME

Conduction in geologic materials

In rocks and soils there are 3 conduction mechanisms, all affecting the EM travelling through media, namely:

1) **Electronic (ohmic) conduction**
2) **Electrolytic (ionic) conduction**
3) **Dielectric (polarization) conduction**

An example for an heterogeneous medium



Abraham et al., 2016

1) Electronic (ohmic) conduction

1) *When there are electrons free to move (delocalized electrons)*

$\Delta V = RI$ *Ohm's law ➔ "Ohmic" materials ➔ mainly METALS*

In differential form:

$$\boldsymbol{E} = -\frac{\partial V}{\partial \boldsymbol{r}} = \boldsymbol{J}\rho = \frac{\boldsymbol{I}}{S}\rho \quad ➔ \quad \rho = \sigma^{-1} = \frac{\boldsymbol{E}}{\boldsymbol{J}} \quad [\Omega m]$$

So, in an electric conductor with lenght "l" and section"S":

$$\Delta V = \frac{I}{S}\rho\int_0^l \partial l = \frac{I}{S}\rho l = RI \quad \text{da cui ➔} \quad R = \rho\frac{l}{S} \quad \text{o} \quad \rho = R\frac{S}{l} \quad [\Omega m]$$

The electrical resistivity ρ (reciprocal of the electrical conductivity σ) is an intrinsic, intensive property of a material (while the electric resistance R is estensive since it depends also by geometrical factors) describing the proportionality between the electrical field ***E*** *and the density of current* ***J***.

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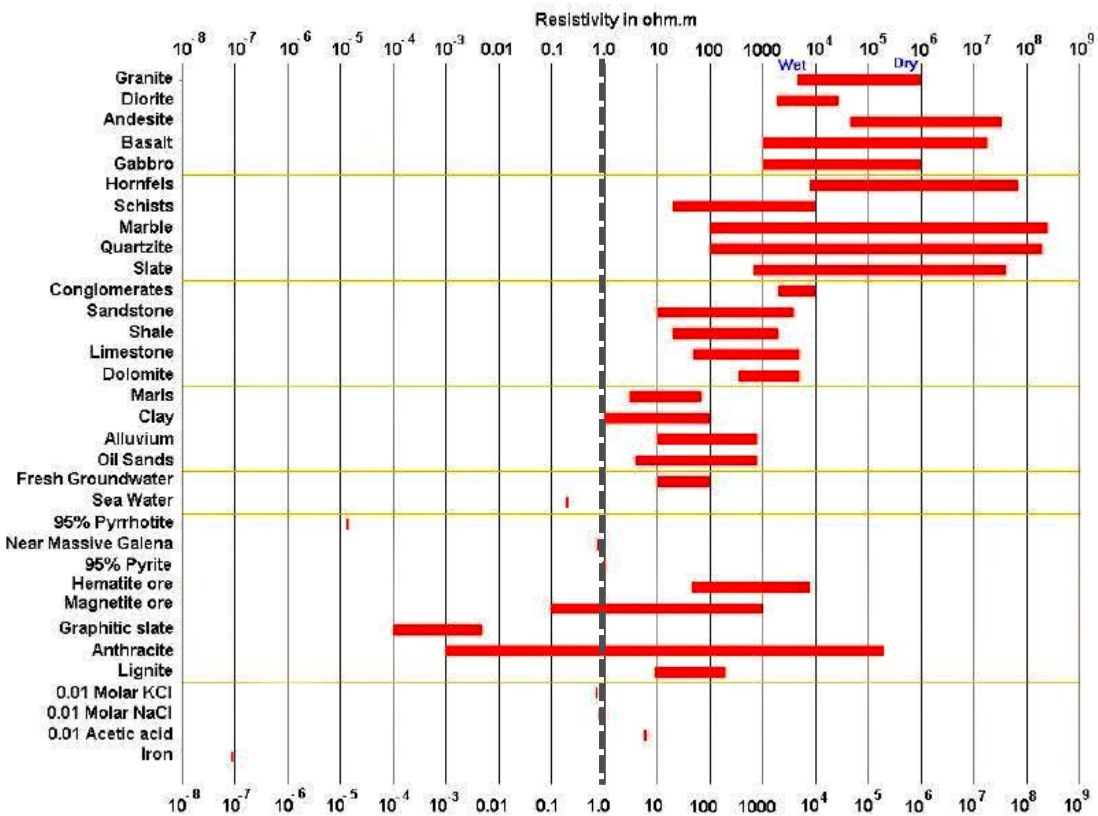
1) Electronic (ohmic) conduction

*In the geoelectrical method the subsurface resistivity ρ is measured by injecting a current **I** and measuring the potential difference ΔV, being the geometry of the experiment known.*



Loke, 2013

2) Electrolytic (ionic) conduction

From the previous diagram, most of rocks are dielectrics when in dry conditions because the most common minerals forming rocks are dielectrics. Relevant exceptions are native metals, some sulfates and oxides containing Fe, Ti, Pb, Cu,… with extreme differences due to their chemical structure (e.g. magnetite Fe_3O_4 is a very good conductor, while hematite Fe_2O_3 is nearly an insulator).

Material	Nominal resistivity (Ω m)
Sulphides:	
Chalcopyrite	1.2×10^{-5}–3×10^{-1}
Pyrite	2.9×10^{-5}–1.5
Pyrrhotite	7.5×10^{-6}–5×10^{-2}
Galena	3×10^{-5}–3×10^{2}
Sphalerite	1.5×10^{7}
Oxides:	
Hematite	3.5×10^{-3}–10^{7}
Limonite	10^{3}–10^{7}
Magnetite	5×10^{-5}–5.7×10^{3}
Ilmenite	10^{-3}–5×10
Quartz	3×10^{2}–10^{6}
Rock salt	3×10–10^{13}
Anthracite	10^{-3}–2×10^{5}
Lignite	9–2×10^{2}

Water plays an even more important role!

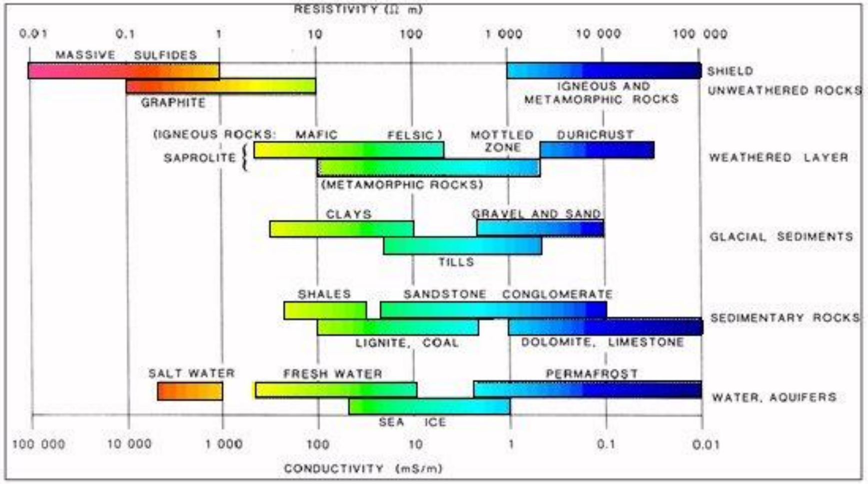
Overall resistivity decreases when water is present ➔ ionic conduction ➔ rocks can be considered as SOLID ELECTROLYTES.

Speed of the ions is noticeably lower than the electrons one, but a net charge transfer will occur due to actual mass movement.

Often an approximation to ohmic conductors is considered.

Conductivity of fluids depends upon quantity and mobility (velocity) of charge carriers. Mobility depends on viscosity of fluid (hence temperature) and diameter of charge carriers.

Fluid conductivity depends also upon temperature because the mobility of the ions in solution increases with temperature. This behavior is opposite to that of metallic conductors which involve electronic conduction rather than ionic conduction, and exhibit resistivity increases with temperature.



2) Electrolytic (ionic) conduction

Especially for sedimentary rocks in which minerals are mainly insulating carbonates and silicates, when water is present it influences the overall resistivity MORE THAN the resistivity of the grain skeleton.

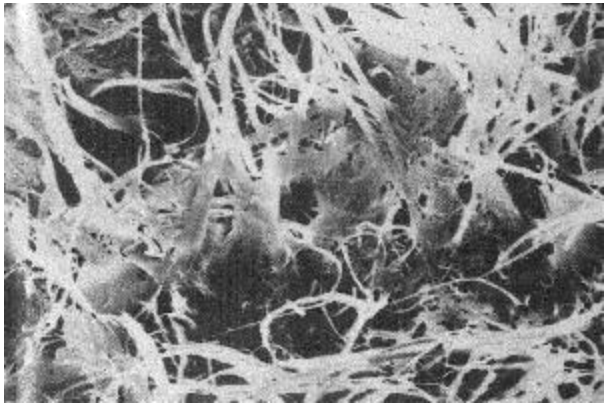
$$\rho_e = a\Phi^{-m} S^{-n} \rho_w$$

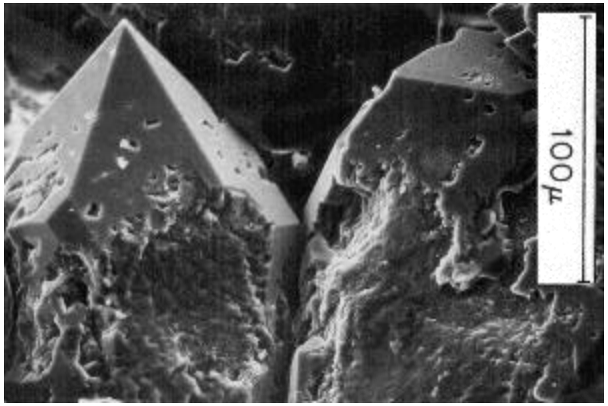
Archie's law (1942)

Φ is the porosity
S is the degree of saturation (volume fraction of pores with water)
a [0.5⩽a⩾2.5]; m [1.3⩽m⩾2.5]; n≊2
The ratio ρ_e/ρ_w is reported as Formation Factor (F)

Any fine grained mineral exhibits a certain Cation Exchange Capacity (CEC). That is, charges (positive) can be adsorbed (attached to the surface) onto the slightly negatively charged surface, and these can subsequently be exchanged or dissolved. Since clay has a huge surface area to volume ratio, it has a very high exchange capacity ➔ clays can dramatically increase the conductivity especially of fresh waters.

Illite (a clay mineral) with total surface area of **100m²/g**



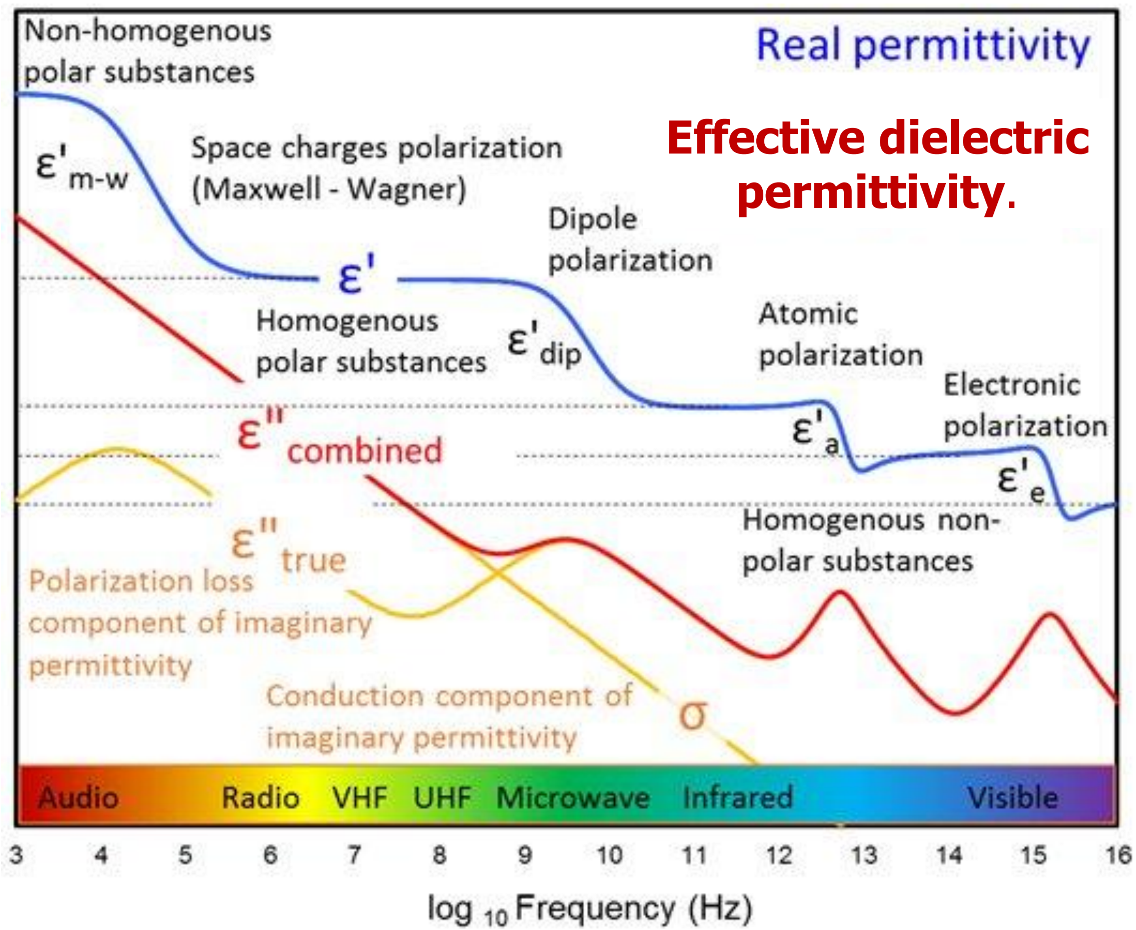


Quartz overgrowths in sandstone with total surface area of **0.1m²/g**

Knigth, 2006

3) Dielectric conduction (polarization)

The electrical behavior of the material (even when not mixed and when isotropic) depends by which is the time required for each polarization mechanism and it is anyway complex.



Guéguen and Palciauskas, 1994

Multiple processes contribute to the dielectric response of rock including dipolar, atomic and electronic polarizations. Additional processes resulting from the liberation of ions into the fluid through hydration are typically very slow but contribute significantly at low frequency.

The imaginary dielectric permittivity is comprised of two components:
The polarization loss component (in orange) results from work done in driving electrical polarization processes. The conduction loss (also in orange) is the work done in "friction" between charges (mainly ions) during electro-migration. The dipolar loss and conduction loss are combined into one single **effective dielectric permittivity**.

1), 2), 3) overall conduction

A dielectric medium in a time variable EM field suffers a displacement of the electrons from their respective nucleus. At macroscopic level such a behavior makes the material "POLARIZED" and DISPLACEMENT CURRENTS take place. The fundamental physical parameter involved is the DIELECTRIC PERMITTIVITY, as already discussed… ***D**=ε**E***

This mechanism become relevant when the contribution of both electronic and ionic conduction is negligible → dielectrics

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} = \boldsymbol{J_0} + \boldsymbol{J_C} + \boldsymbol{J_P}$$

$$J_C = \sigma E \qquad J_P = \frac{\partial D}{\partial t} = i\omega\varepsilon E$$

The POLARIZATION effects are not only at electrons level, but the mechanism can involve atoms or molecules, being the effect maximum for polar molecules (e.g. water) for which the value of ε is maximum considering the geologic media.

1), 2), 3) overall conduction

The electrical conduction within rocks and soils is a COMPLEX mechanism possibly including 1), 2) and 3).
Often, one (or two) mechanism(s) is/are dominant and the other(s) can be disregarded.

The strongest dependency is related to the FREQUENCY of the EM field used in the experiment and to the geologic materials characteristics (chemical, mineralogical, physical, granulometric,…)

Fluids (usually WATER) have a prominent effect at all frequencies depending by the their **TEMPERATURE, PHYSICAL STATUS** (frozen, unfrozen, steam), **CHEMICAL COMPOUNDS** (ions type, concentration,…), **TYPE** (connate, adsorbed, free,…), …

Many empirical relations linking different electrical properties to water content/characteristics are available, among the others:

$$\rho_e = a\Phi^{-m} S^{-n} \rho_w$$ Archie's law (1942)

$$\theta = -5.3\cdot 10^{-2} + 2.92\cdot 10^{-2}\varepsilon_r - 5.5\cdot 10^{-4}\varepsilon_r^2 + 4.3\cdot 10^{-6}\varepsilon_r^3$$ Topp's eq. (1980)

$$\varepsilon_r = \left(\theta\sqrt{\varepsilon_{r,w}} + (1-\phi)\sqrt{\varepsilon_{r,s}} + (\phi-\theta)\sqrt{\varepsilon_{r,a}}\right)^2$$ CRIM eq. (1974)

ϕ = porosity; θ = volumetric water content; subscripts *w, s, a* are respectively for water, soil and air

EM methods and their applicability

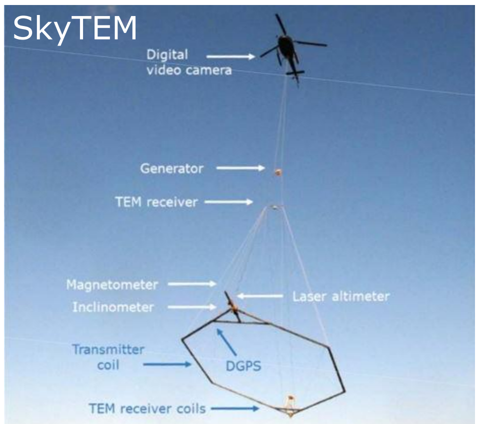
Group	Method	Source characteristics	Frequency	Parameters measured	Principal application
Electrical	**ERT/IP**	**Electric current injected**	**DC or <50Hz**	**ΔV→ρ M**	**ρ mapping (water, voids,...) chargeability - clay**
Low f EM	VLF	far-field source, radio transmitter	single	tilt angle	mapping structure
	AMT	natural EM source	broadband	*E*, *B*	resistivity sounding
	CSAMT	intermediate-field, electric bipole	broadband	*E*, *B*, *Ø*	resistivity sounding
	HLEM (Slingram)	near-field source, mobile loop	variable	*B*	mapping, shallow sounding vary x(T,R)
	GCM (ground-conductivity meter)	near-field source, mobile loop	single	*B*	mapping, simple sounding vary x(T,R)
	TDEM/TEM	near-field source, mobile loop, x(T,R) variable	time domain	d*B*/dt	mapping, vary *x*(T,R); sounding, vary time
		central loop, x(T,R) fixed		d*B*/dt	sounding, vary time-window
High f EM	**GPR**	**Antennas coupled with the ground**	**UHB**	**reflections**	**many possible applications**
	TDR	**Coaxial line (waveguide)**	**UHB**	**reflections (v_{EM}, ε, σ)**	**very shallow ε water content**

– f +

EM methods and their applicability

The **different mechanisms** exploited by the different geophysical methods allow a **wide range of applications** not only with sources and sensors directly coupled with the ground.

Helicopter-borne
Time-Domain EM system



Fixed Resistivity monitoring

Borehole GPR

Ground based
Freq.-Domain EM system

Resistivity water survey

Helicopter-borne GPR


Questions?