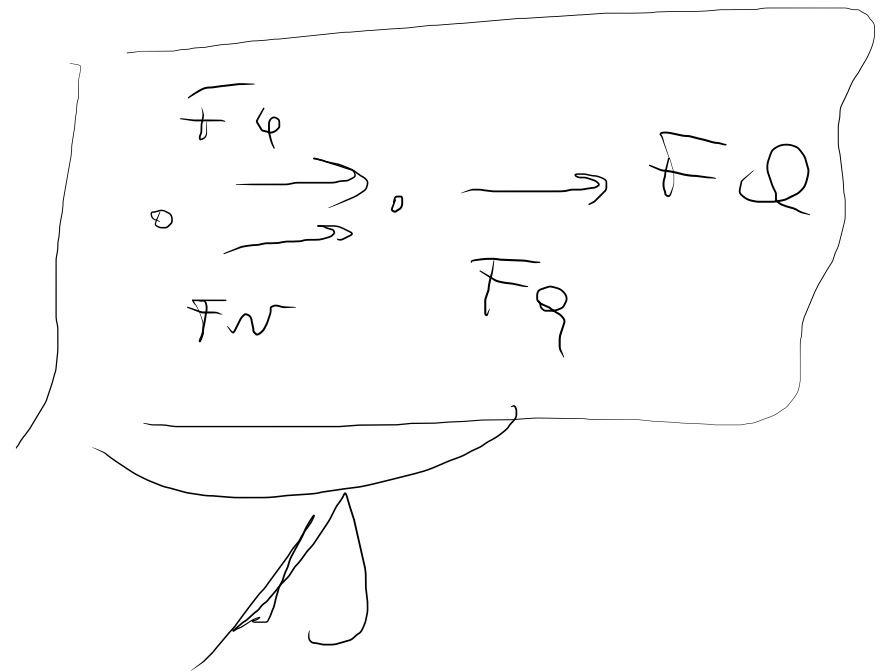
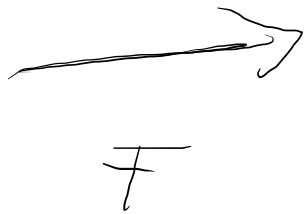
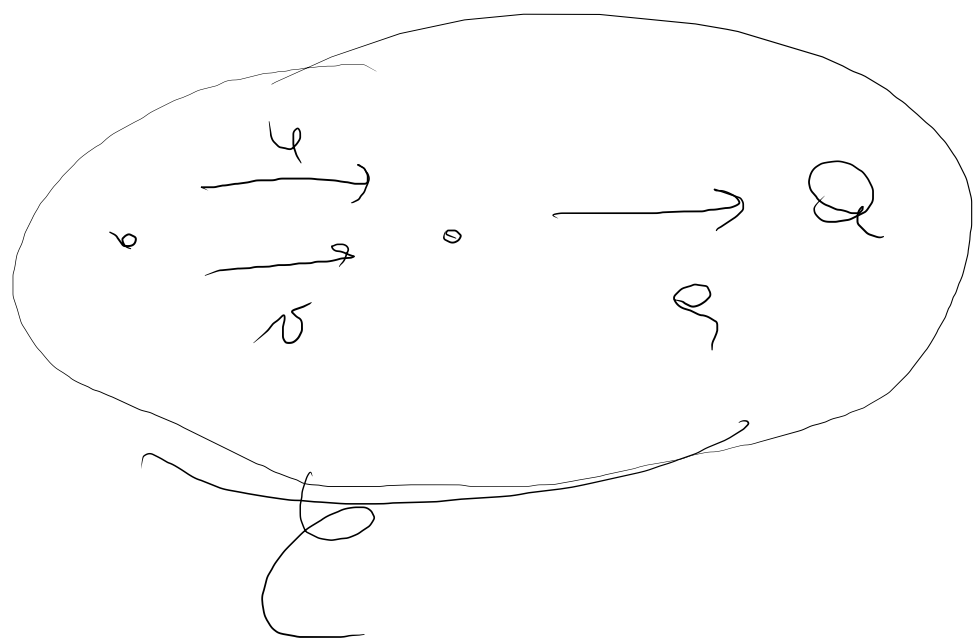


Def See ABSOLUTE coequalizer in \mathcal{C} is

a coequalizer $(Q, q) = \text{Coep}(u, v) \quad \triangleright$ that

$\forall F: \mathcal{C} \rightarrow \mathcal{D}$ then (FQ, Fq) is again

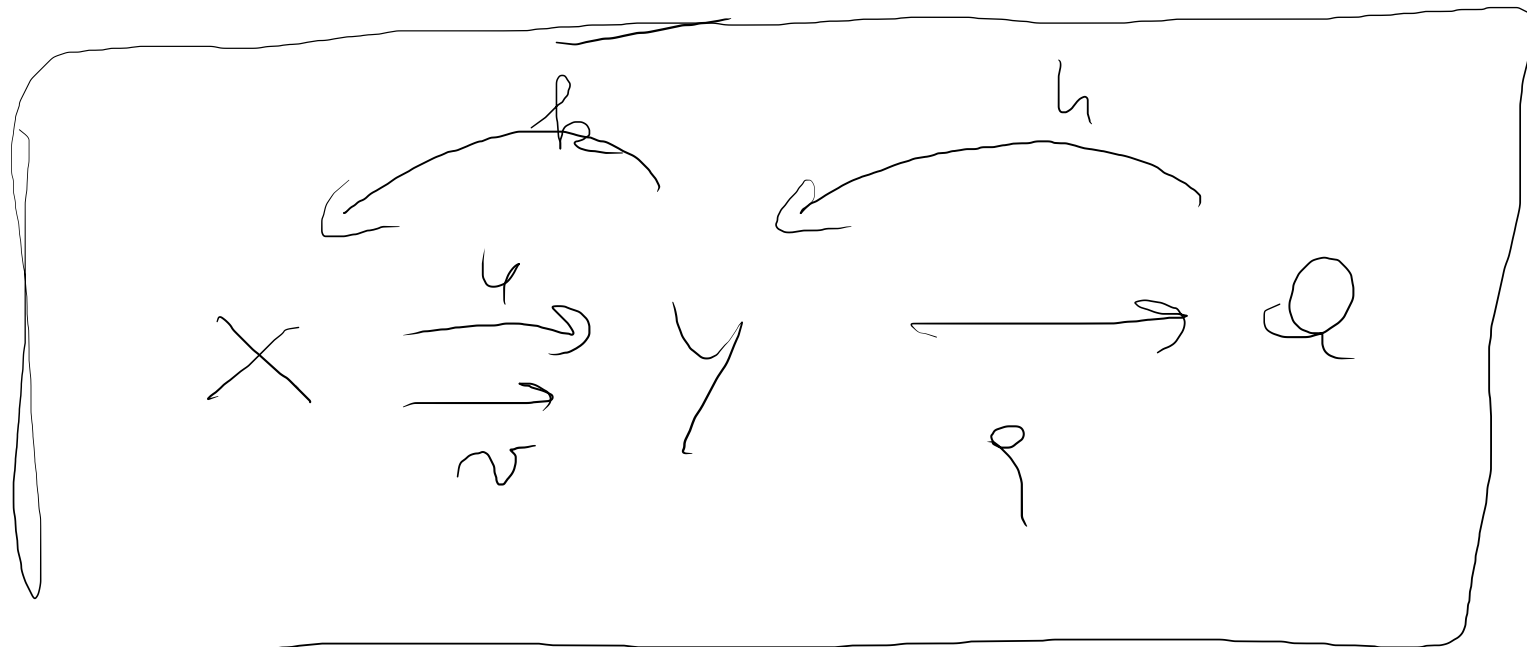
a coequalizer



Def

A SPLIT Sequence

is



split

• $q \circ u = q \circ v$

• $u \circ h = h \circ g$

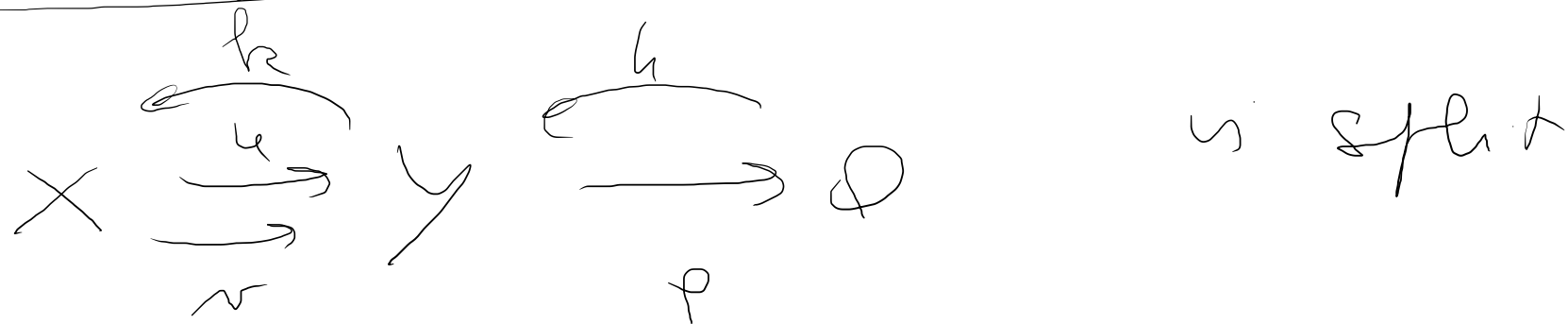
• $q \circ h = h \circ q$

• $v \circ h = h \circ q$

Lemma 1

Any SPLIT sep $u \in \text{COEQUAL}$

Proof



we must have that $(Q, q) = \text{Coeq}(u, v)$

$$q \circ u = q \circ v \quad \checkmark$$

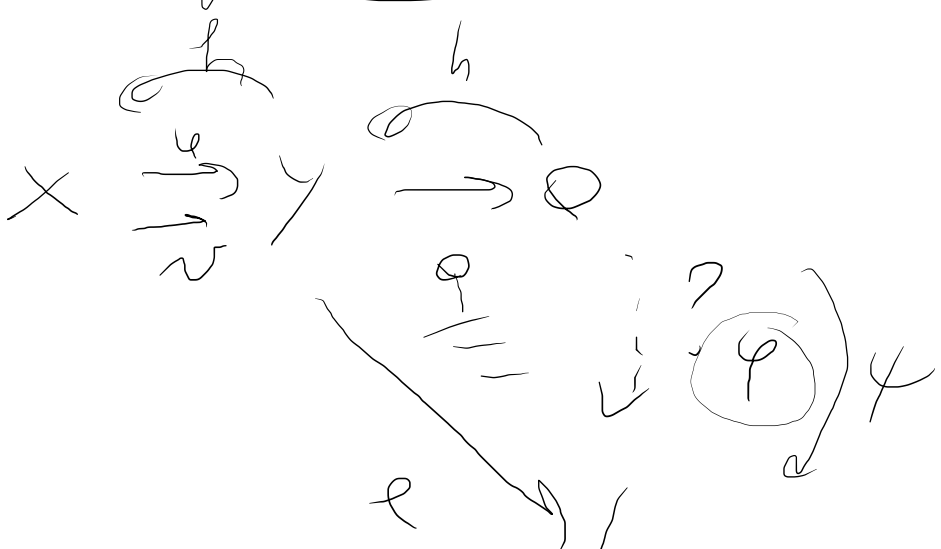
we must verify the universal property

$\forall (L, e: Y \rightarrow L)$ with $e \circ u = e \circ v \rightarrow$

$\Rightarrow \boxed{\exists! \varphi: \mathbb{Q} \rightarrow L}$ such +

ph?

$\boxed{l = \varphi \varrho}$



$\boxed{\varphi = lh}$
DET

to prove $\boxed{l = \varphi \varrho}$

$\rightarrow l[h\varrho] = l[\varrho k] =$
 $= l[uk] = l[1y] = l$

φ is unique - if φ is such

$\boxed{\varphi \varrho = l} \Rightarrow \boxed{\varphi = \varphi}$

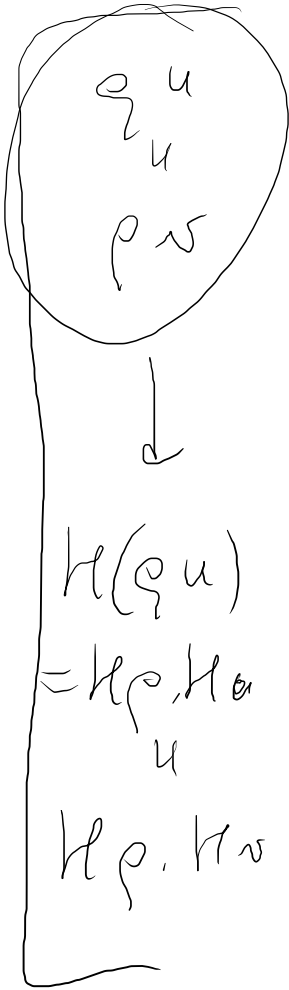
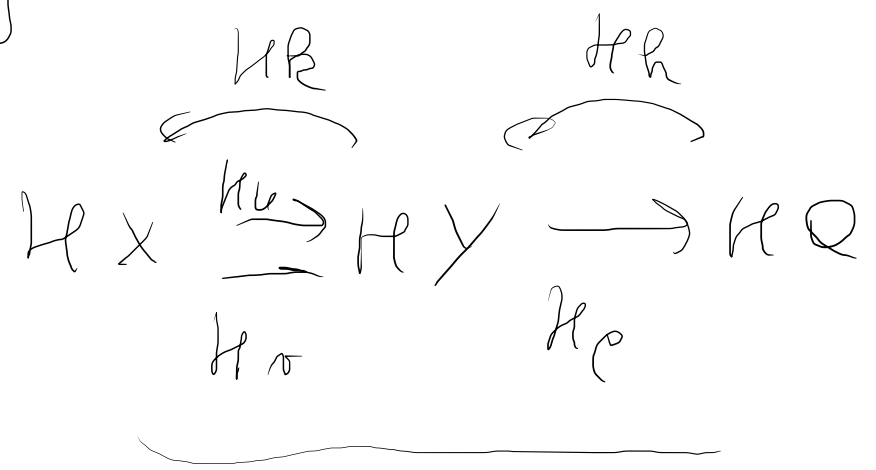
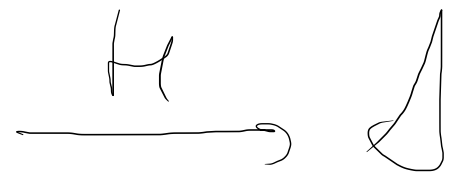
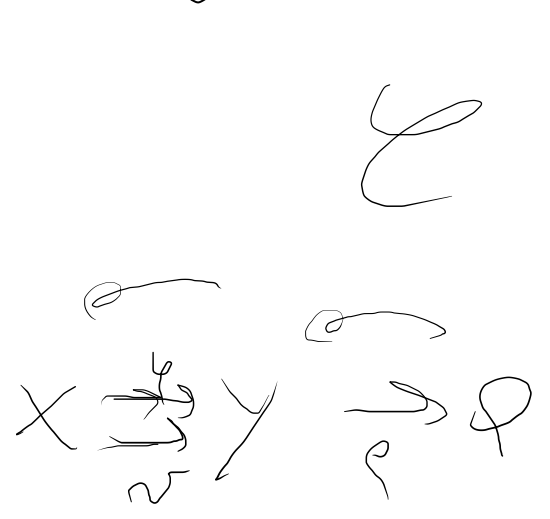
$\varphi \varrho = \varphi \varrho \rightarrow \varphi \varrho h = \varphi \varrho h \rightarrow$
 $\rightarrow \varphi \cdot 1_{\mathbb{Q}} = \varphi \cdot 1_{\mathbb{Q}} \rightarrow \boxed{\varphi = \varphi}$

Lemma 2

split coop $\xrightarrow{?}$ absolute coop
 \nearrow Lemma 1

split sequence

Proof



$u \in \mathcal{L}$ is split
4 equations apply

$u \in \mathcal{L}$
the 4 equations
are still
valid

any funct preserves compositions and identities

So the 4 equations of a split algebra

are preserved (they are solid w/d)

→ the sequence w/d is split

→ so it is a coalgebra

→ (η, η) is an absolute coop

Lemma 3

$\mathcal{L}^T \text{ on } T$ (T, η, μ) is a triple on \mathcal{L}

\mathcal{L}^T category of T-algebras

\forall T-algebra $(X, h: TX \rightarrow X) \exists$ a canonical

associated split sequence

that is called the

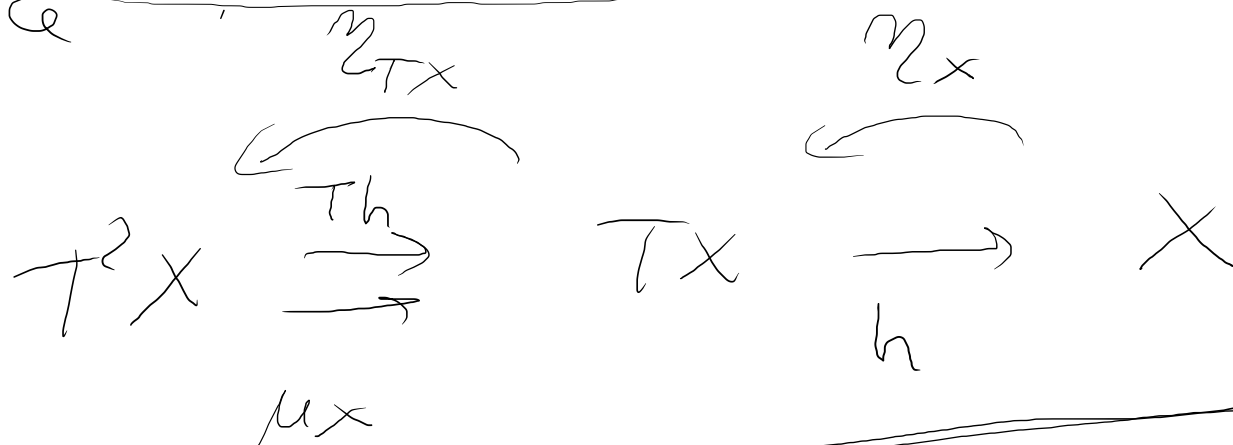
of the T-algebra

"presentation"

$$\forall (x, h) \in \mathcal{E}^T$$

we can define a split

reference



\mathcal{E} is
SPLIT
coop

$$h \cdot Th = h \cdot \mu_x$$

ax 2 of T algebra

hyp: (x, h) is a T algebra

$$h \cdot \eta_x = \text{id}_x$$

ax 1 of T algebra

$$\mu_x \circ \eta_{TX} = \text{id}_{TX}$$

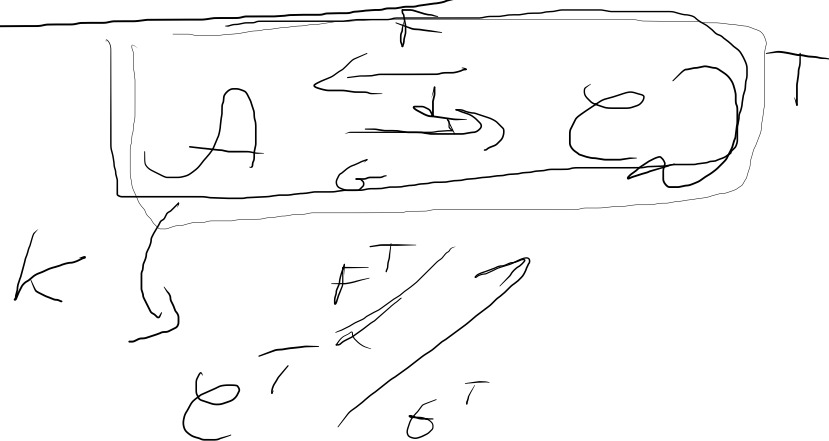
ax 1 of type

$$Th \cdot \eta_{TX} = \eta_x \cdot h$$

naturality of η_x

Characterization theorem

Let be



T the universal type
 and E^T the category
 of T -algebras

The following points are equivalent:

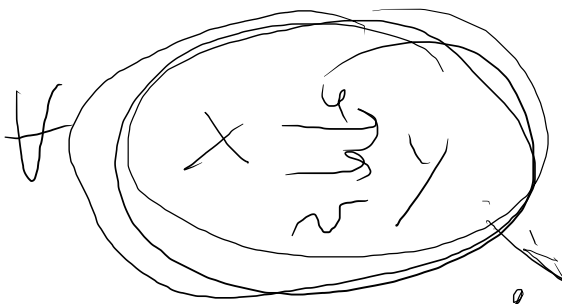
1) A is algebraic on E (that means K is an iso)

2) G creates absolute coequalizers ~~Δ~~

3) G creates split coeq. ~~Δ~~

G creates coop

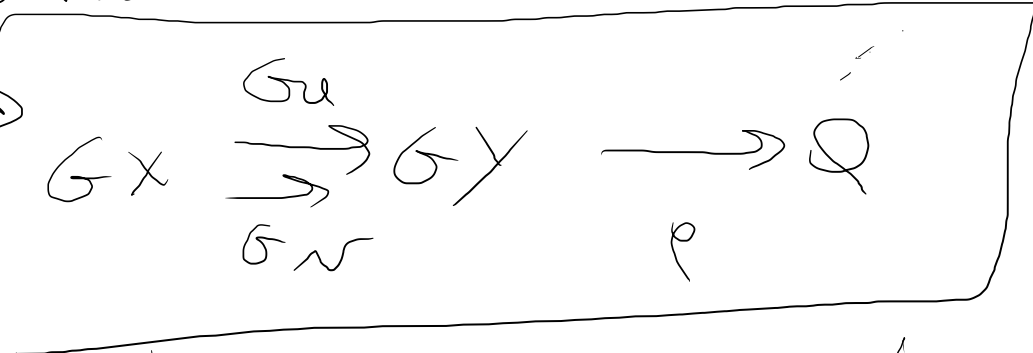
$A \xrightarrow{G} \mathcal{C}$



$w \in A$

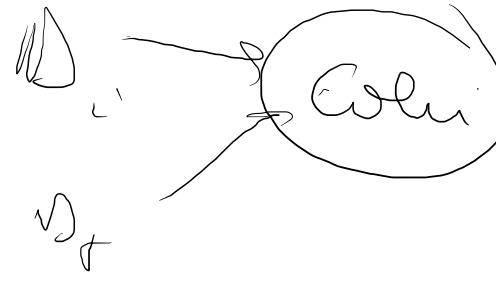


s. the



has a cop in \mathcal{C}

then



① $\exists! T \in A \quad t: y \rightarrow Q \quad \text{s. the}$

$G(T) = Q \quad G(t) = p$

② $(T, t) = \text{Coeq}(y, v)$

G creates ABSOLUTE (SPLIT) comp in \mathcal{E}

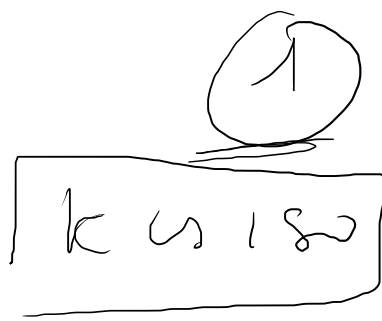


If (u, v) is that (Gx, Gy) has an ABSOLUTE comp

then $\exists (T, t)$ where

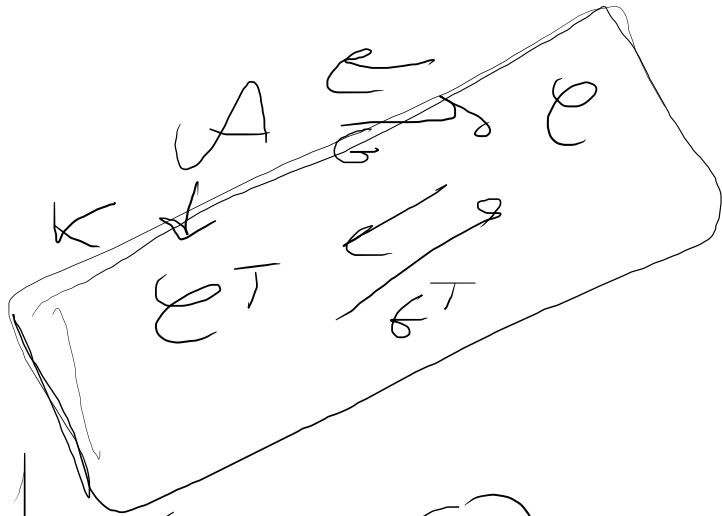
$$\text{cost}(T, t) = \text{Comp}(u, v)$$

Proof



② G creates obs. cop.

k is an iso → G and G^T are
isomorphic.



to have ②

G creates obs. cop.

we can have

G^T creates obs. cop.



\mathcal{E}^T

$R = (Q, ?)$



$u \in \mathcal{E}$

If (u, v) s.t. $(G^T(u), G^T(v))$ has an obscure coop

$(Q, p) = \text{Coop}(u, v) \quad u \in \mathcal{E}$

then $\textcircled{1} \exists ! R \in \mathcal{E}^T \quad (y, B) \xrightarrow{r} R \quad \Bigg| \quad G^T(R) = Q$

$\textcircled{2} \underline{(R, \eta)} = \text{Coop}(u, v) \quad u \in \mathcal{E}^T$

we want prove : \exists a structure of T-algebra on Q
 s.t. q is a T-alp. morphism.

$$(X, h) \xrightarrow{u} (Y, k) \in \mathcal{T}$$

I rewrite in \mathcal{C}

\mathcal{C} is a
 morph of T-alp
 \mathcal{C}

$$TX \xrightarrow{Tu} TY \xrightarrow{Tq} TQ$$

is a
~~cop~~

$$\begin{array}{ccc} TX & \xrightarrow{Tu} & TY & \xrightarrow{Tq} & TQ \\ \downarrow h & \cong & \downarrow k & & \downarrow \xi \\ X & \xrightarrow{u} & Y & \xrightarrow{q} & Q \end{array}$$

cop
 in \mathcal{C}

$$k \circ Tu = v \circ h$$

$$k \circ Tu = u \circ h$$

in \mathcal{C}

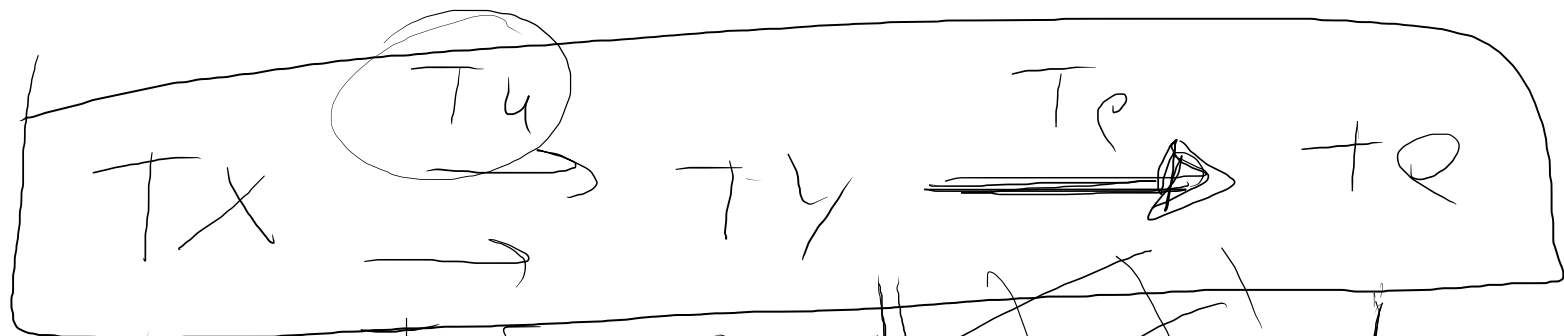
q is an absolute
 cop $\rightarrow Tp = \text{Coeq}(Tu, Tv)$

we must define ξ in such a way

that $(Q, \xi : TQ \rightarrow Q)$ is a T-algebra

\bullet $q : (X, \mu) \rightarrow (Q, \xi)$ is a morph. of T-algebras

\bullet (Q, ξ) is the coop of $(u, v) \in \mathcal{C}^T$



surj. part
 of T_p is
 coequalizer

$$q \circ h \circ Tu = p \circ h \circ Tv$$

$\rightarrow \exists' \xi \circ \text{incl}$
 then $\{ T_p = p \cdot h$

we must have that $(Q, \xi: TQ \rightarrow Q)$ is a T-algebra

axioms are valid

ax1

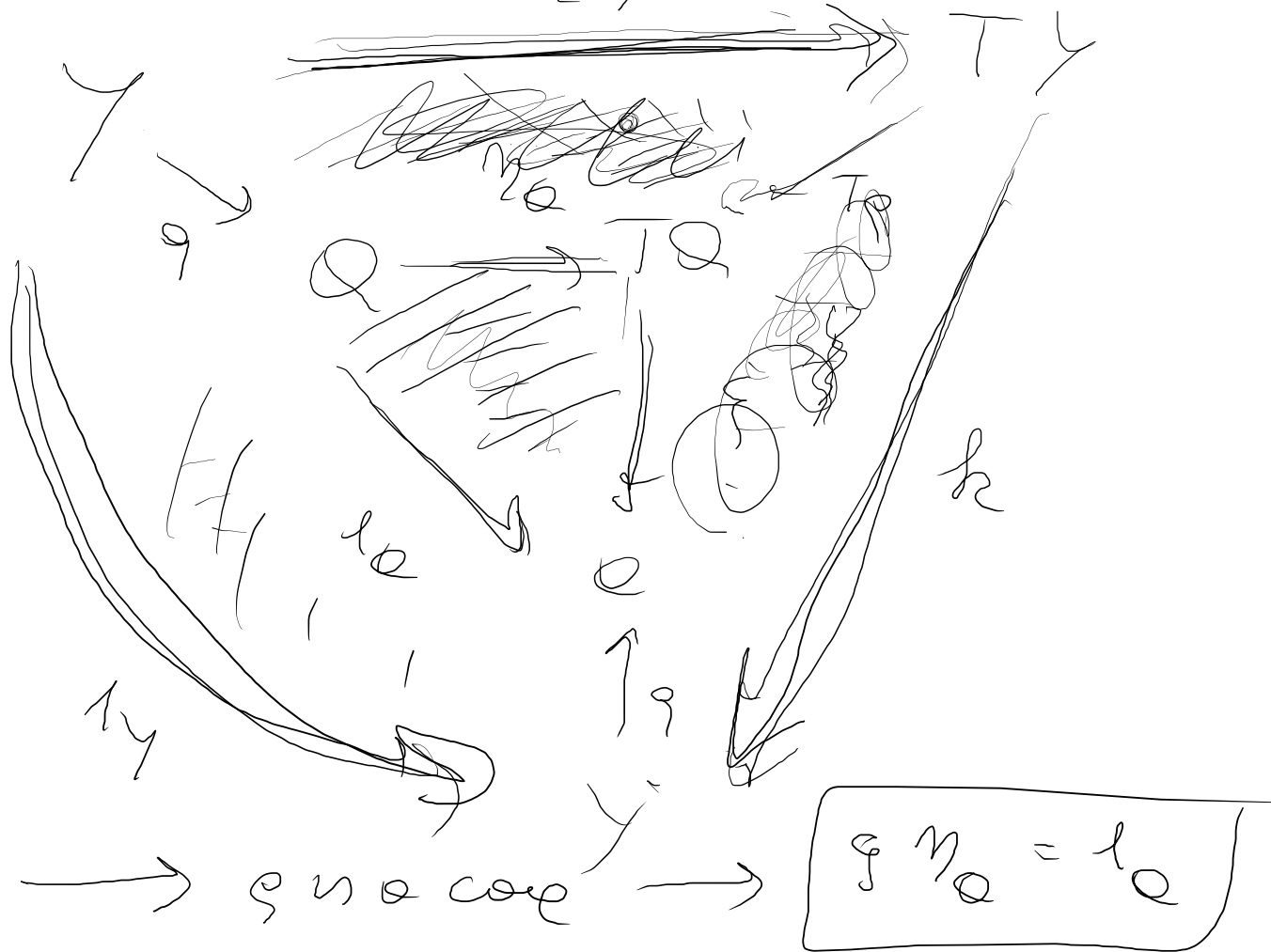
$$\xi \eta_Q = \tau_Q$$

$$\boxed{\xi \eta_Q = \tau_Q}$$

$$= \xi \tau_Q \eta_Q$$

$$= \xi \tau_Q \eta_Q$$

$$= \xi \cdot \tau_Q = \tau_Q$$



$$\boxed{(Y, h) \text{ is a T-algebra}}$$

↓ ?

$$\boxed{\xi \cdot \eta_Q = \tau_Q}$$

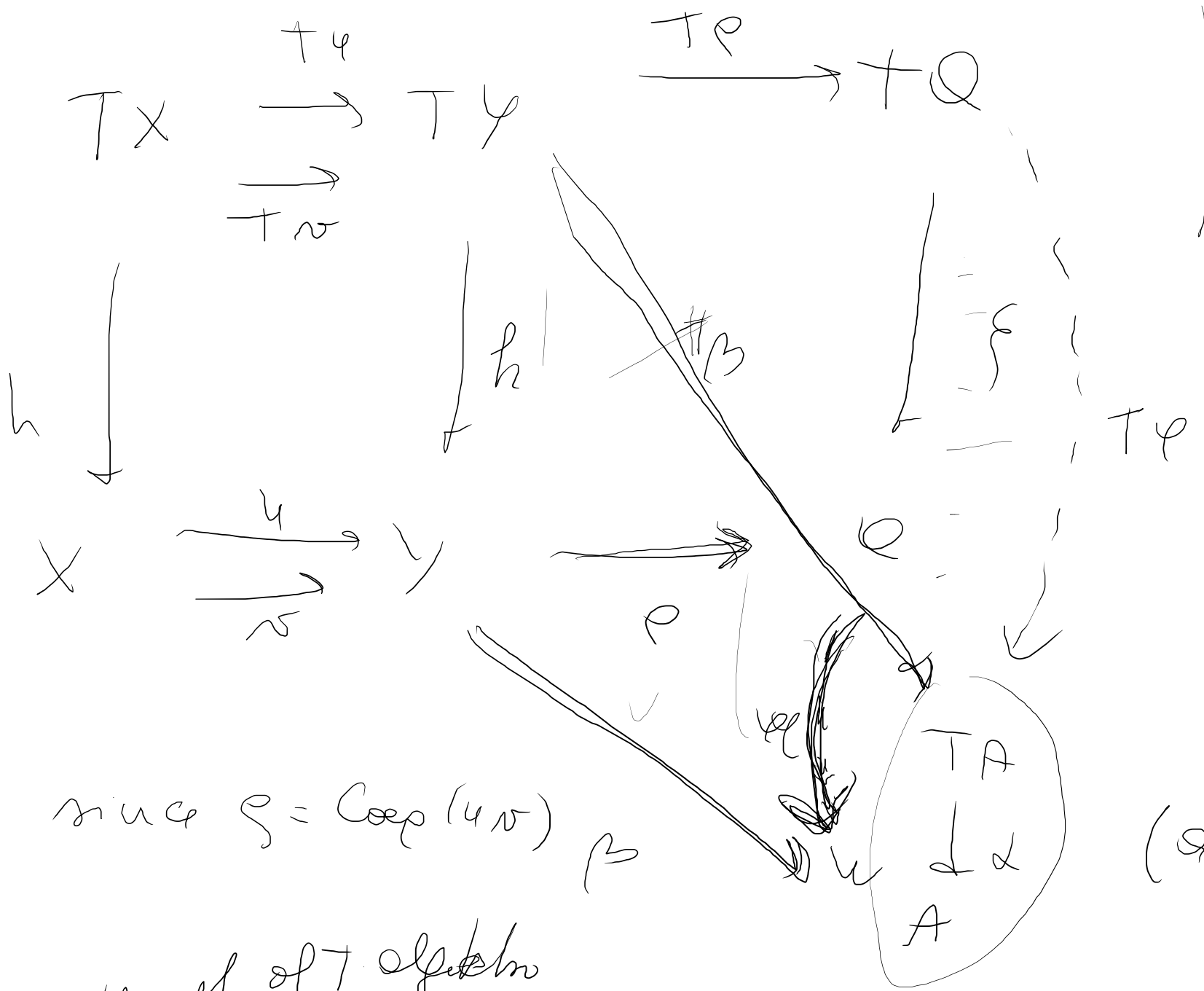
$V(Q, \mathfrak{g})$

to have

is a T. algebra

V_p is a unital algebra of T. of

$$(Q, \mathfrak{g}) = \text{Coep}(u, v) \quad u, v \in T$$



$$(A, \alpha) \in \mathcal{E}^T$$

$$\beta: (Y, h) \rightarrow (A, \alpha)$$

s. that

$$\beta u = \beta v$$

in T algebra

$$\exists! \varphi:$$

$$(Q, \rho) \rightarrow (A, \alpha)$$

with

$$\boxed{\beta = \varphi \rho}$$

$\varphi \in \mathcal{E}$ since $\rho = \text{Cosp}(\psi, \rho)$
 φ is a morphism of T algebra

φ is a morph of T algebra

\leftrightarrow

$$\varphi \circ \xi = \alpha \circ T\varphi$$

$$\begin{array}{ccc} TQ & \xrightarrow{T\rho} & TA \\ \xi \downarrow & \parallel & \downarrow \alpha \\ \rho & \xrightarrow{\varphi} & A \end{array}$$

$$\begin{array}{ccccc} & & T\rho & & \\ & & \xrightarrow{\quad} & & \\ T\varphi & \xrightarrow{T\rho} & TQ & \xrightarrow{T\varphi} & TA \\ \downarrow \xi & \circlearrowleft & \downarrow \xi & \rightsquigarrow & \downarrow \alpha \\ \rho & \xrightarrow{\varphi} & \rho & \xrightarrow{\varphi} & A \end{array}$$

$\underbrace{\hspace{10em}}_{\beta}$

ρ is a morph of T -alg

$T\rho$ is a morph

$$\begin{array}{ccc} \varphi \circ \xi & \cancel{T\rho} & \\ \cup & & T\rho \\ \alpha \circ T\varphi & \cancel{T\rho} & \end{array}$$

2 \Rightarrow 3

obvious

\mathbb{G} creates
ob coep

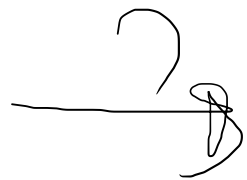


\mathbb{G} creates
split coep

any split coep is absolute

3 \Rightarrow 1

\mathbb{G} creates
split coep



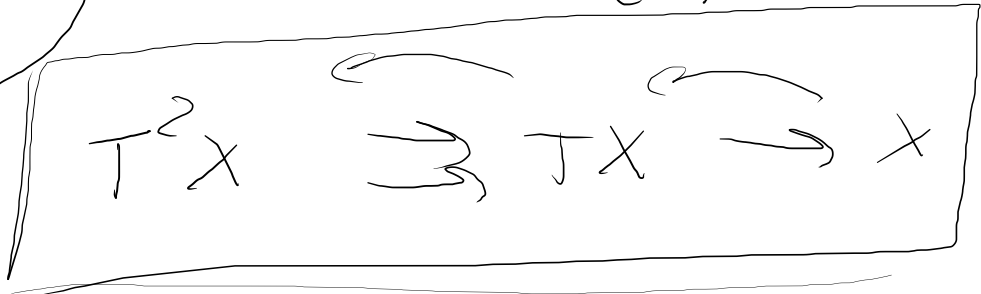
A is algebraic

K is an ISO

$A \xrightarrow{\kappa} \mathcal{E}^T$ we will construct a function H inverse of κ



$\forall (x, h) \in \mathcal{E}^T$ we must define $H(x, h)$



is split

\mathcal{L}_3

$T = GF$

$$T^2 X \xrightarrow[\mu_X]{\eta_{TX}} TX \xrightarrow[h]{\eta_X} X$$

$(X, h: TX \rightarrow X)$

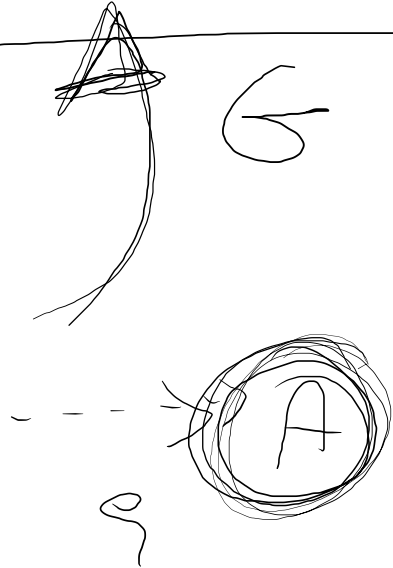
$$GF GF X \xrightarrow[\circlearrowleft \eta_{GF X}]{GFh} GF X \xrightarrow[h]{\eta_X} X$$

split sep

$in \mathcal{C}$

G create split
cop

$$FGFX \xrightarrow[Fh]{Fh} FX$$



$\exists A \in \mathcal{A} \quad \rho$

$GA = X$

$A = \overline{\text{Cosp}(\dots)}$

\mathcal{A}

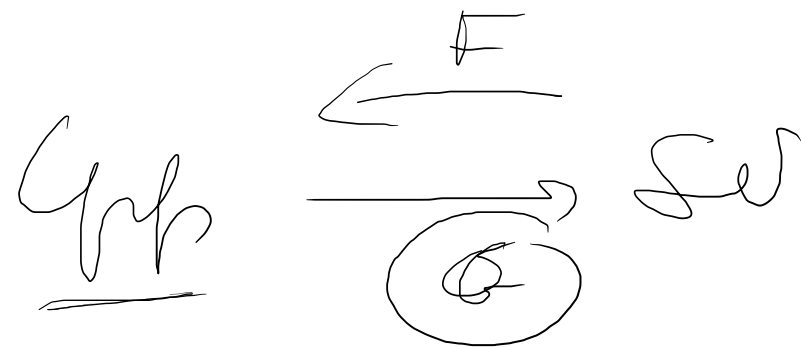
Def $H(x, h) \equiv A$

$$H: \mathcal{E}T \rightarrow A$$

H is a functor

h and k are inverse functors

$\rightarrow A \cong \mathcal{E}T$ A is Algebraic



G via coeq ASS.

