$$A_1 = I_{4\times 4}$$

semipl. STAB

$$A_2 = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

INST.

$$A_3 = \left[\begin{array}{cc|cc} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

INST.

$$x_{k+1} = A x_k$$

$$\Lambda_A \{+1\} \qquad p(\lambda) = (\lambda - 1)^4$$

$$\begin{cases} x_1(k+1) = x_2(k) \\ x_2(k+1) = -x_1(k) \end{cases} \qquad A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$\bar{x}_{eq}$? stab. eq? stab. sistemul? $J_A = ?$

$$\begin{cases} \bar{x}_1 = \bar{x}_2 \\ \bar{x}_2 = -\bar{x}_1 \end{cases} \qquad \bar{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$V(x_1, x_2) = [x_1 \ x_2] \, P \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

P simm. def. pozitiva

$$V(\bar{x}) = 0$$

$$V(x) > 0 \quad \forall x \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \bar{x}$$

$$\Delta V(x) \leq 0$$

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \quad \begin{cases} p_{11} > 0 \\ p_{11} \cdot p_{22} - p_{12}^2 > 0 \end{cases}$$

$$P = I_{2\times 2} \qquad V(x) = x_1^2 + x_2^2 = x^T I x$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$V(0) = 0 = 0^2 + 0^2 \qquad V(x) = x_1^2 + x_2^2 \neq 0$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Delta V = V[\underbrace{x(k+1)}_{f(x(k)) = A\,x(k)}] - V(x(k))$$

$$\begin{cases} x_1(k+1) = x_2(k) \\ x_2(k+1) = -x_1(k) \end{cases}$$

$$= [x_1(k+1)]^2 + [x_2(k+1)]^2 - [x_1(k)]^2 - [x_2(k)]^2$$

$$\cancel{[x_2(k)]^2} + \underbrace{[-x_1(k)]^2}_{\cancel{+x_1^2(k)}} - \cancel{x_1^2(k)} - \cancel{x_2^2(k)}$$

$$= 0 \quad \forall \; x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

ΔV semidef. negativa $\Rightarrow \bar{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ semplt stabile

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$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad p_A(\lambda) = \det(\lambda I - A)$$

$$= (\lambda^2 + 1)$$

$$\lambda_1 = +j$$
$$\lambda_2 = -j$$

coppie di autovalori c.c. $|\lambda_{1,2}| = 1$

molteplicità 1 $\Rightarrow$ stab. semplice

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad J_A = ? \qquad J_A = \begin{bmatrix} +j & 0 \\ 0 & -j \end{bmatrix}$$

$$T = ? \qquad T = \begin{bmatrix} -j & +j \\ +1 & +1 \end{bmatrix}$$

stab. semplice

$$A^T P A - P = -Q$$

$\forall Q$ def. pos. $\exists P$ def. pos. risolve

$$V = x^T P x \Rightarrow \Delta V = -x^T Q x$$

$$P = I \qquad V(0) = 0 \qquad V(x) > 0 \quad \forall x \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Delta V = -x^T Q x \qquad A^T \underbrace{P}_{I} A - \underbrace{P}_{I} = -Q$$

$$A^T I A - I = -Q$$

$$A^T \cdot I \cdot A - I = I - I = \cancel{0} \Rightarrow Q = \cancel{0}$$

$$x(k+1) = A\,x(k) \qquad A = \begin{bmatrix} 0 & -1/2 \\ 1 & 0 \end{bmatrix}$$

$$A^T P A - P = -Q$$

Q simmetrica def pos $\quad Q = \begin{bmatrix} q_1 & \bar{q} \\ \bar{q} & q_2 \end{bmatrix}$

$$\begin{cases} q_1 > 0 \\ q_1 q_2 - \bar{q}^2 > 0 \end{cases}$$

$Q = I$

$$A^T P A - P = -I$$

$$P = \begin{bmatrix} p_1 & \bar{p} \\ \bar{p} & p_2 \end{bmatrix} \quad \begin{cases} p_1 > 0 \\ p_1 p_2 - \bar{p}^2 > 0 \end{cases}$$

$$\begin{bmatrix} 0 & -1 \\ -1/2 & 0 \end{bmatrix} \begin{bmatrix} p_1 & \bar{p} \\ \bar{p} & p_2 \end{bmatrix} \begin{bmatrix} 0 & -1/2 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} p_1 & \bar{p} \\ \bar{p} & p_2 \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} (p_2 - p_1) & -\frac{\bar{p}}{2} \\ -\frac{\bar{p}}{2} & \frac{p_1}{4} - p_2 \end{bmatrix} = -I$$

$$\begin{cases} p_2 - p_1 = -1 \\ \bar{p} = 0 \\ \frac{p_1}{4} - p_2 = -1 \end{cases} \qquad P = \begin{bmatrix} 8/3 & 0 \\ 0 & 5/3 \end{bmatrix}$$

def pos !

$$x(k+1) = A\,x(k) \qquad A = \begin{bmatrix} 1/3 & 1 \\ 0 & 3 \end{bmatrix}$$

$Q = I$

$$A^T P A - P = -I$$

$$P = \begin{bmatrix} p_1 & \bar{p} \\ \bar{p} & p_2 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} -\frac{8}{9} p_1 & \frac{1}{3} p_1 \\ \frac{1}{3} p_1 & p_1 + 6\bar{p} + 8 p_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$