Image Processing for Physicists

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Overview

- Definition of resolution
- Imaging systems:
  - Linear transfer model
  - Noise

Resolution

"the smallest detail that can be distinguished"

- No unique definition
  - Numerical aperture ← microscopy, photography
  - Pixel size
  - Other criteria (PSF, MTF) e.g. astronomy
- What is "detail"?
- What is "distinguish"?

Resolution

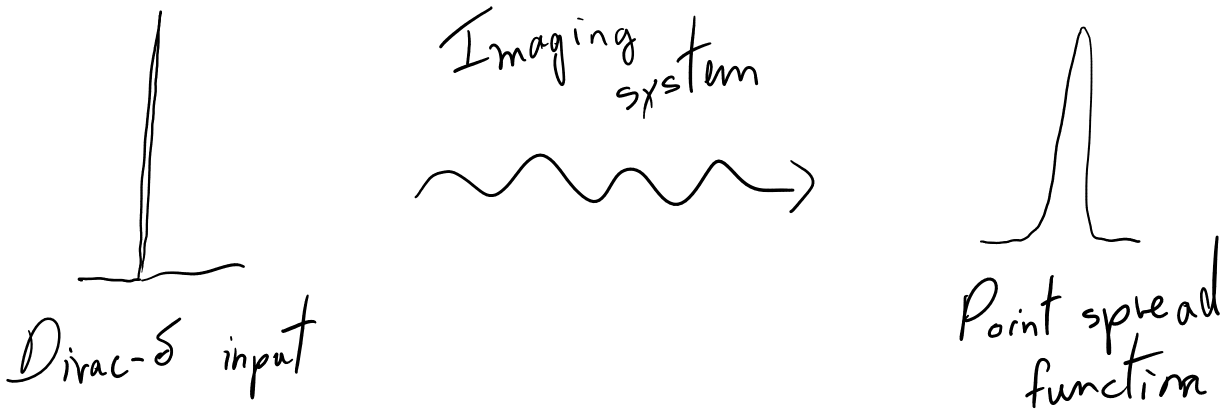
1280 x 1280

640 x 640

- **not** simply given by pixel size (i.e. sampling rate)
- light quality, optics quality, detector quality, algorithm quality, noise, ...

Linear $\underbrace{\text{translation-invariant}}$ systems

- Point spread function ("impulse response")

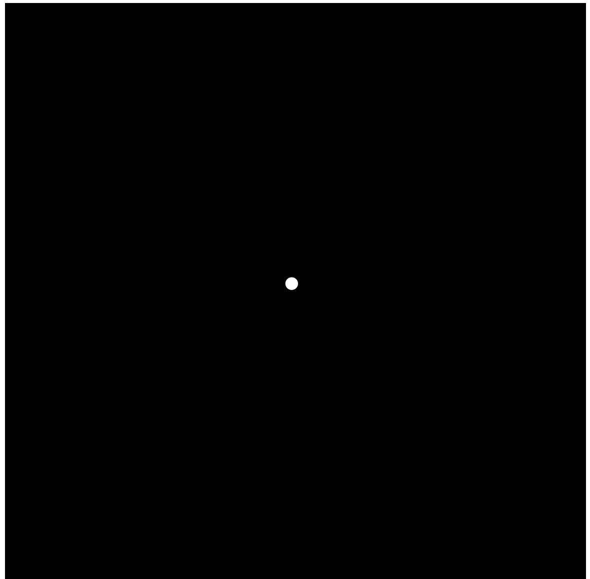


- $\underline{\text{LTI}}$ system: convolution with PSF

$$f(x,y) = \int_A dx'dy' \; f(x',y') \underbrace{\delta(x-x')\,\delta(y-y')}_{\text{system}}$$

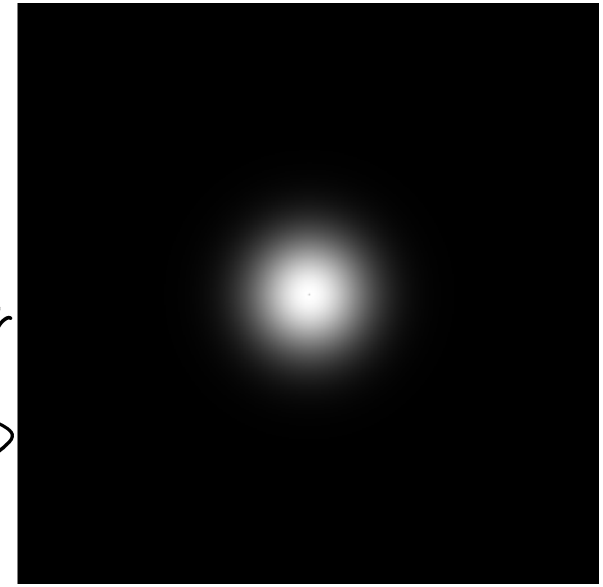
$$\int_A dx'dy' \; f(x',y') \; \overset{\text{PSF}}{h}(x-x', y-y') = f * h$$

Point spread function



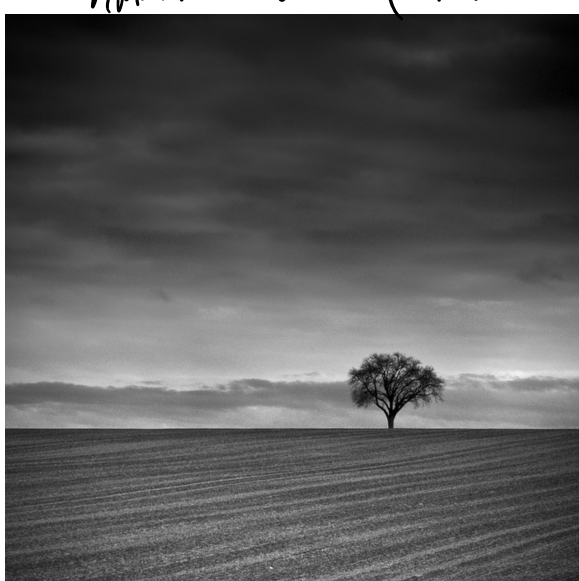
point source

Imaging



PSF (impulse response of optical system)

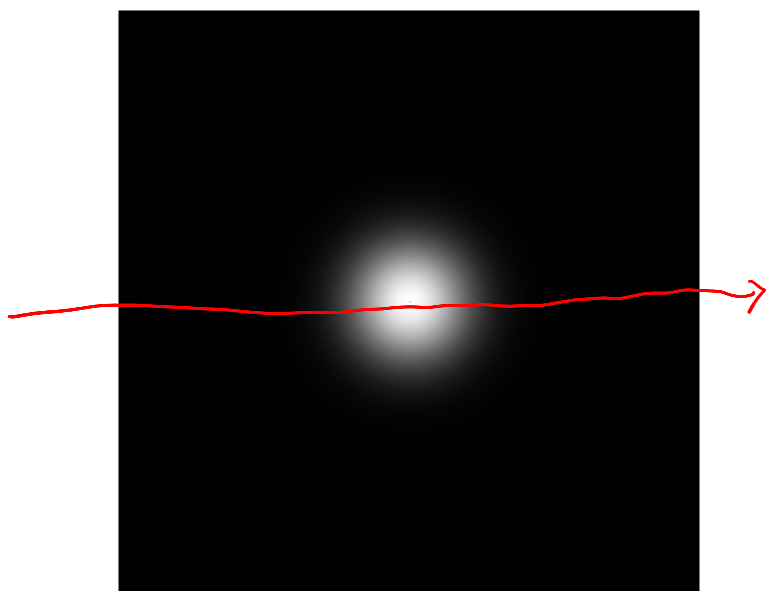
natural scene ("true" image)

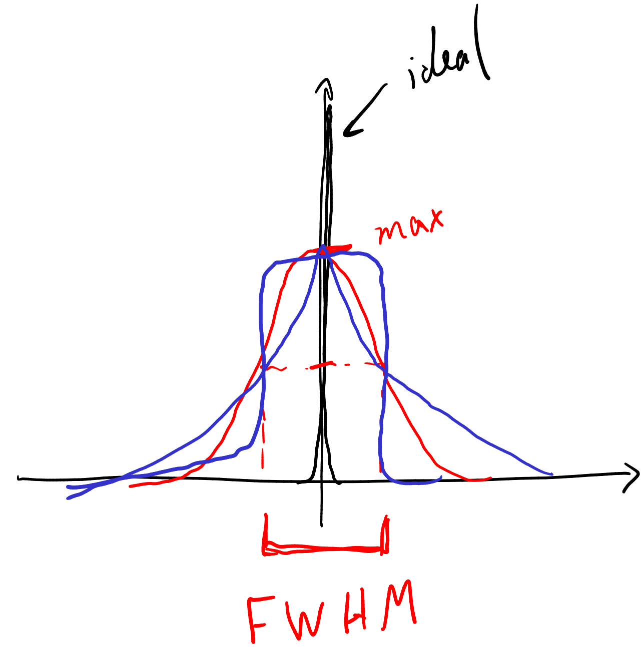


resulting image

PSF and resolution

ideal: $\delta(x-x') \longrightarrow \delta(x-x')$





"full width half maximum"

Measurement of the PSF

- Direct measurement from impulse

generate sharp point → measure output

astronomy: easy: pick a bright star!

- Line-spread function

"knife-edge"

imaging system ⟶

$$\mathcal{H}(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases}$$

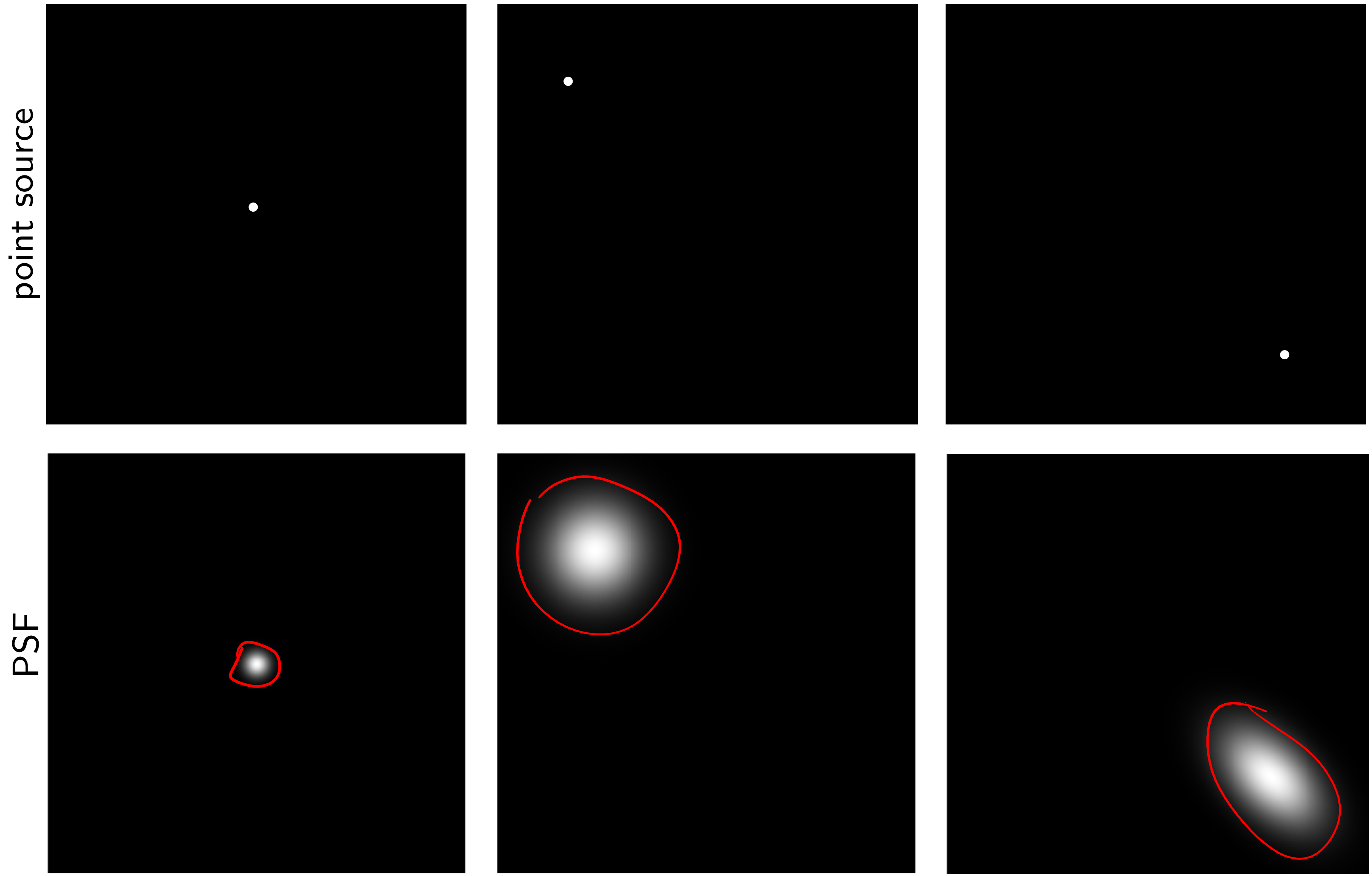
Heaviside step function

connection with PSF:

$$\frac{\partial \mathcal{H}}{\partial x} = \delta(x)$$

$\Rightarrow$ PSF = derivative of knife-edge measurement

PSF and translation invariance



- Not translation invariant $\rightarrow$ PSF depends on position $\rightarrow$ not a convolution
- Useful to model system imperfections, lens aberrations, ...

The Fourier picture

$$\mathcal{F}\{f * h\} = F(u) \cdot H(u)$$

describes how an oscillating signal changes through the imaging system

H : F.T. of PSF = Optical transfer function

Linear transformation $\rightarrow$ diagonalized

$e^{2\pi i x u}$ is an eigenvector of the linear imaging system with eigenvalue $H(u)$

Optical transfer function

Response of a system to an oscillating signal with well-defined frequency

$$OTF(u) = \mathcal{F}\{PSF\}$$

in general complex-valued

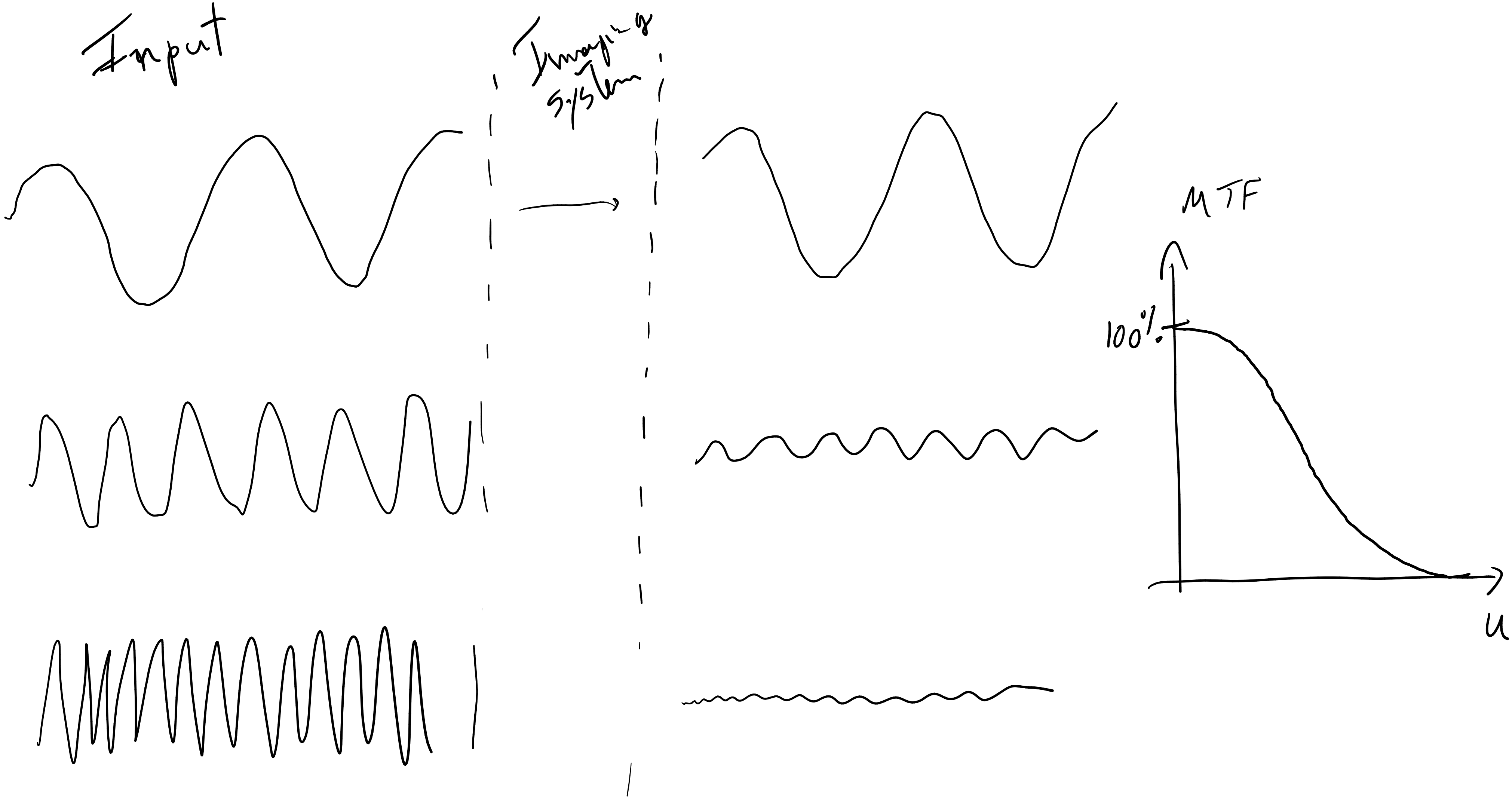
Amplitude: $|OTF(u)| = MTF$ "modulation transfer function"

Phase: $\arg\{OTF\} = PTF$ "phase transfer function"

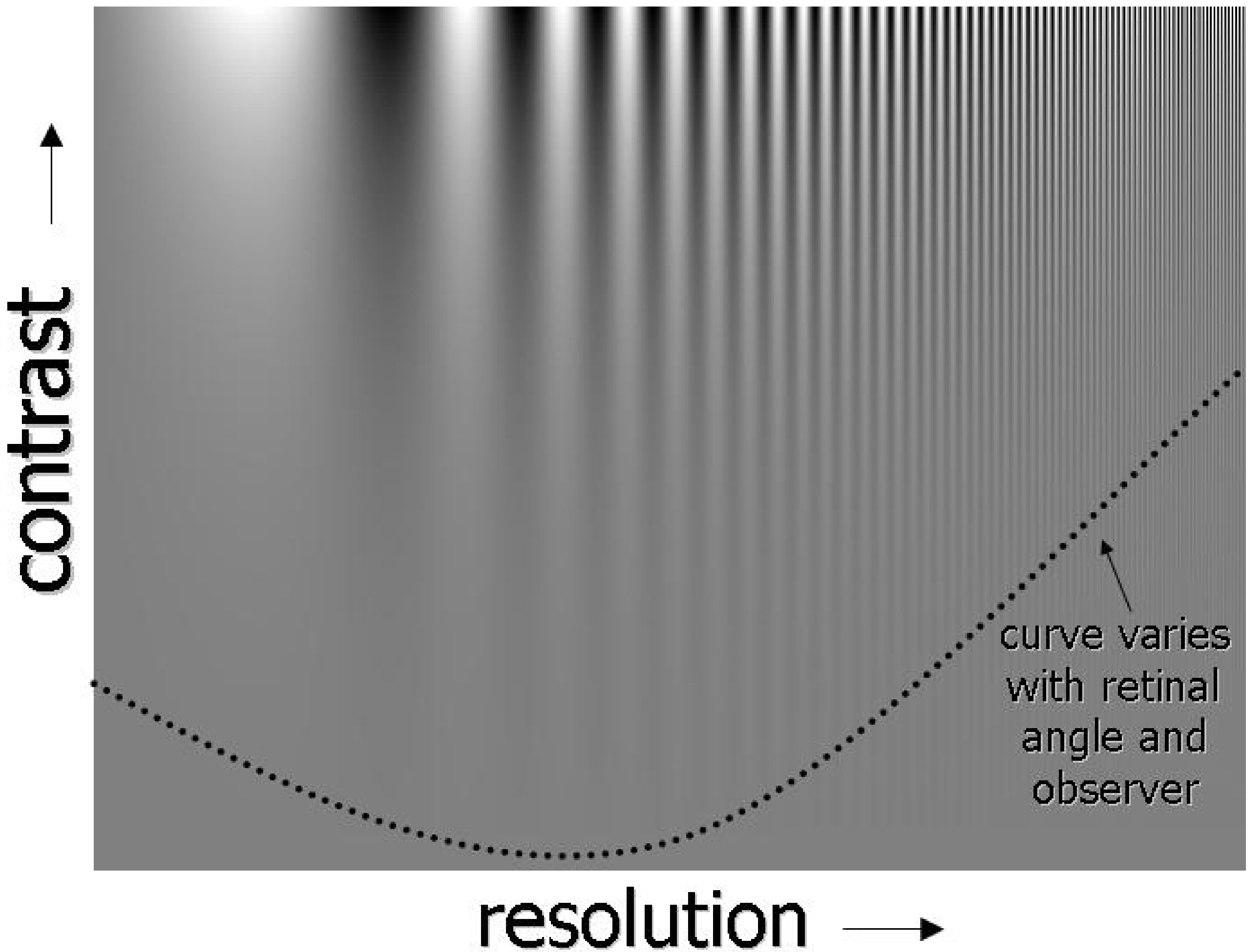
$$OTF = MTF \cdot e^{iPTF}$$

Modulation transfer function

Amplitude change of an oscillating signal for a given frequency



Eye MTF


contrast
resolution
curve varies
with retinal
angle and
observer

Campbell-Robson curve

(1968)

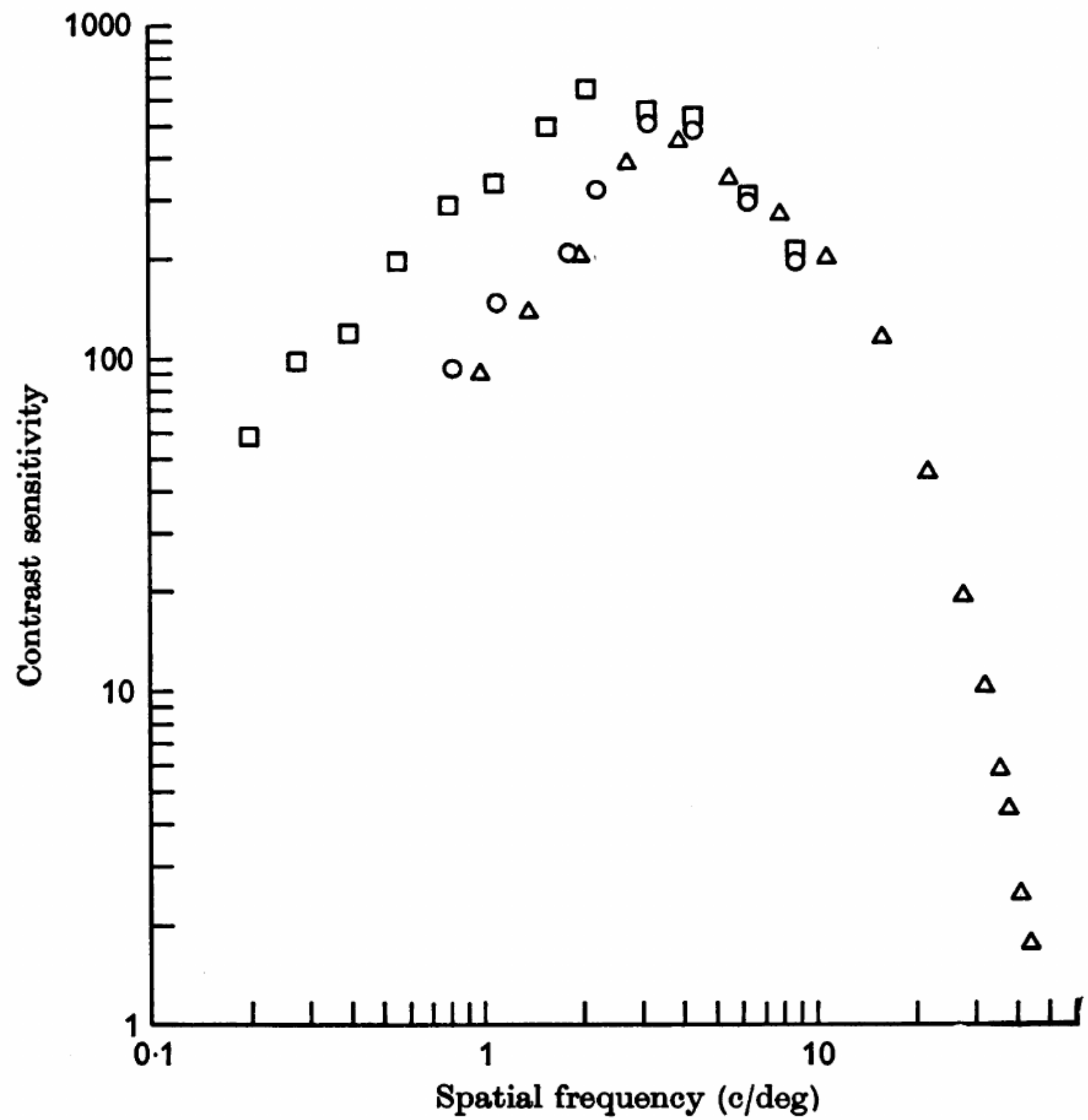
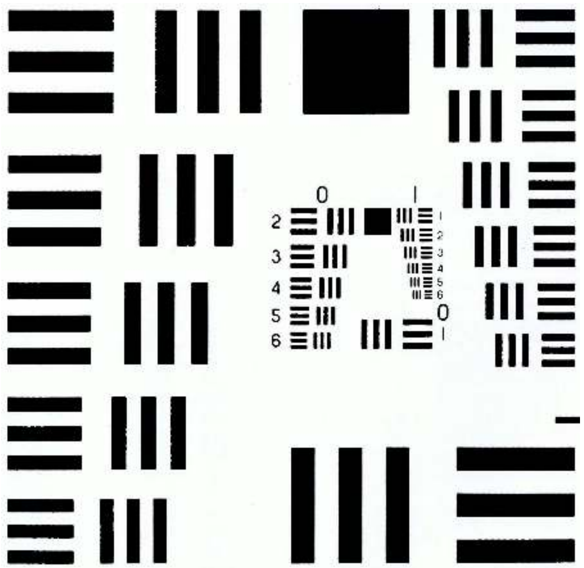
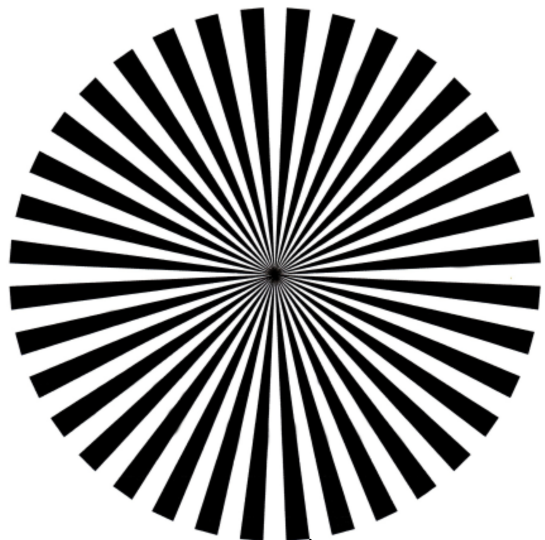


Fig. 2. Contrast sensitivity for sine-wave gratings. Subject F.W.C., luminance 500 cd/m². Viewing distance 285 cm and aperture 2° × 2°, △; viewing distance 57 cm, aperture 10° × 10°, □; viewing distance 57 cm, aperture 2° × 2°, ○.

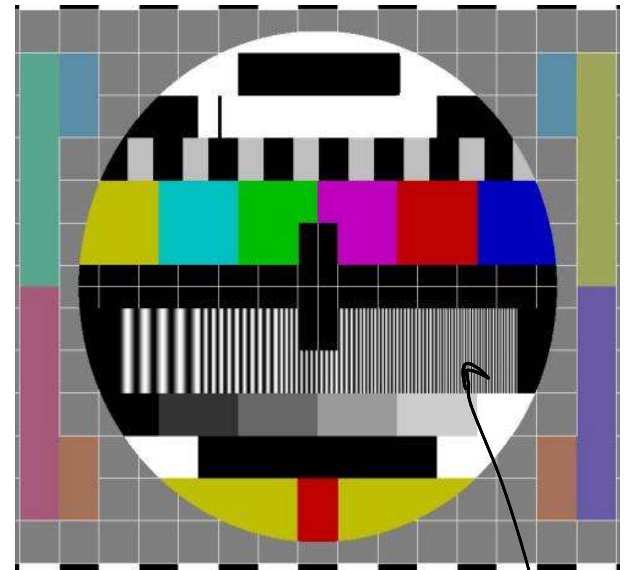
Measurement of MTF



US AF 1951

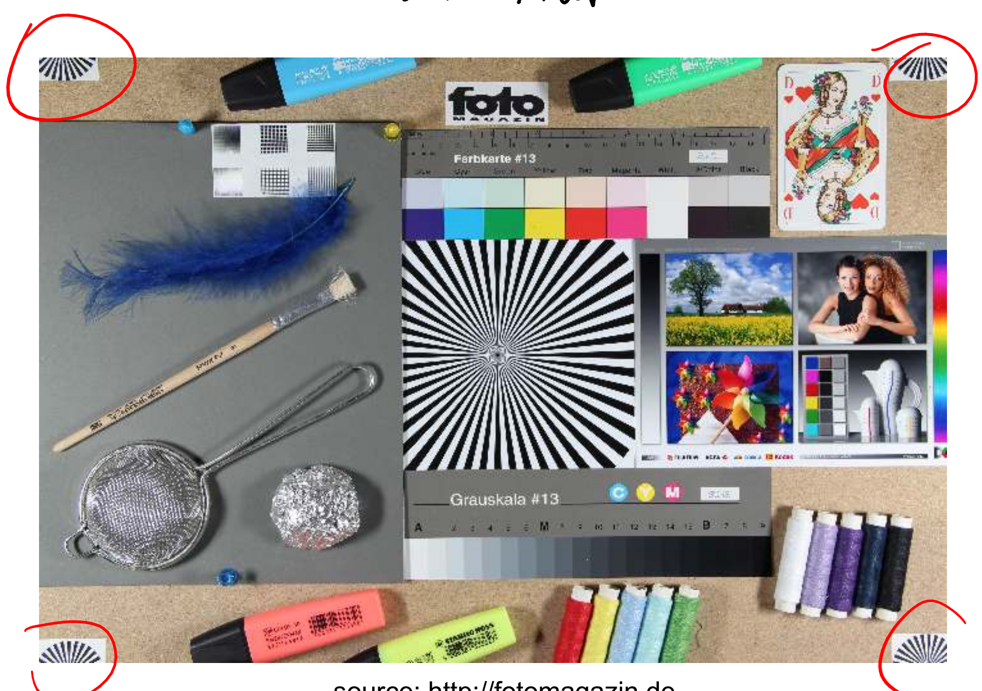


Siemens star



old TV

resolution test



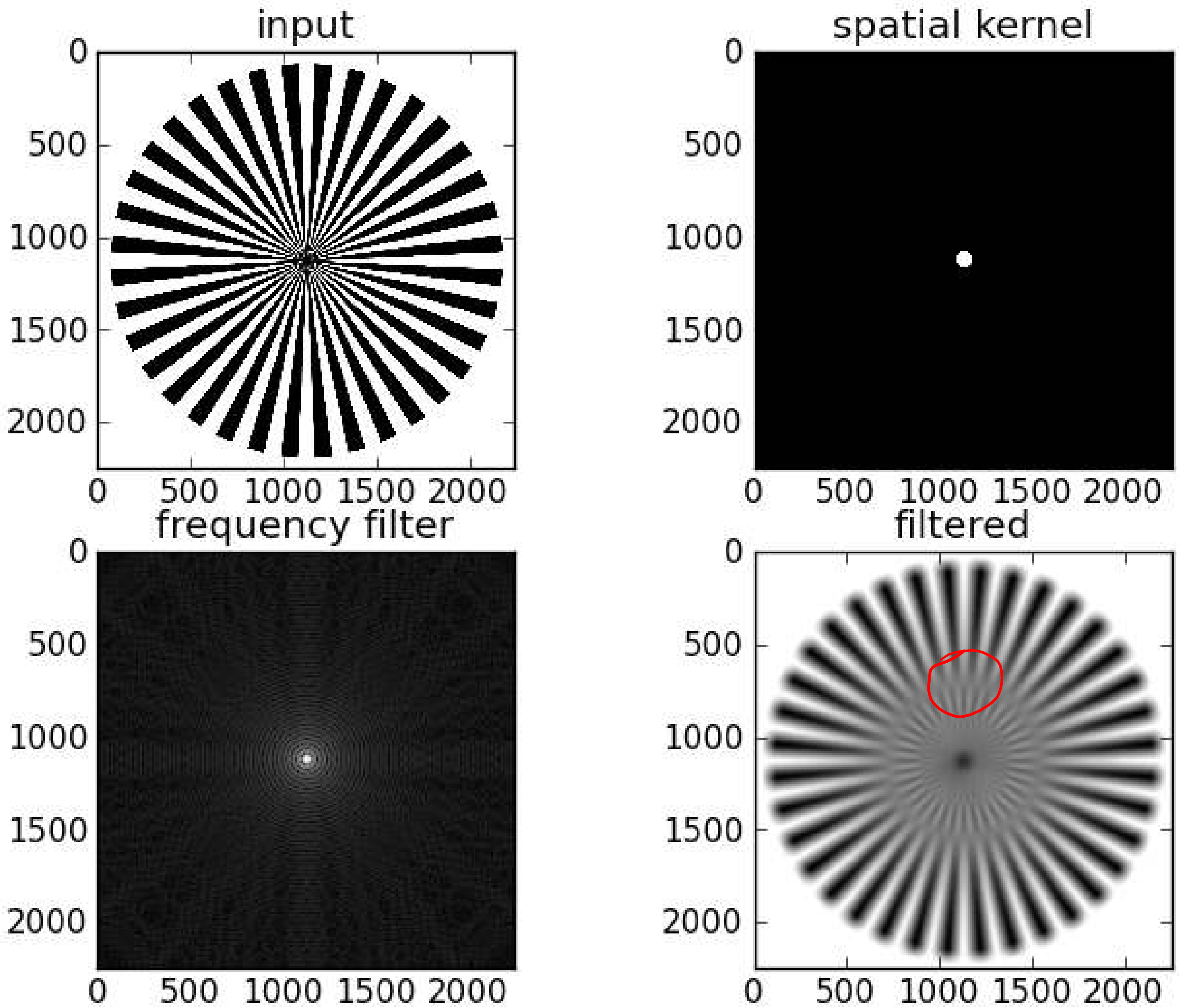
source: http://fotomagazin.de

Phase transfer function

describes how an oscillating signal changes in phase due to system

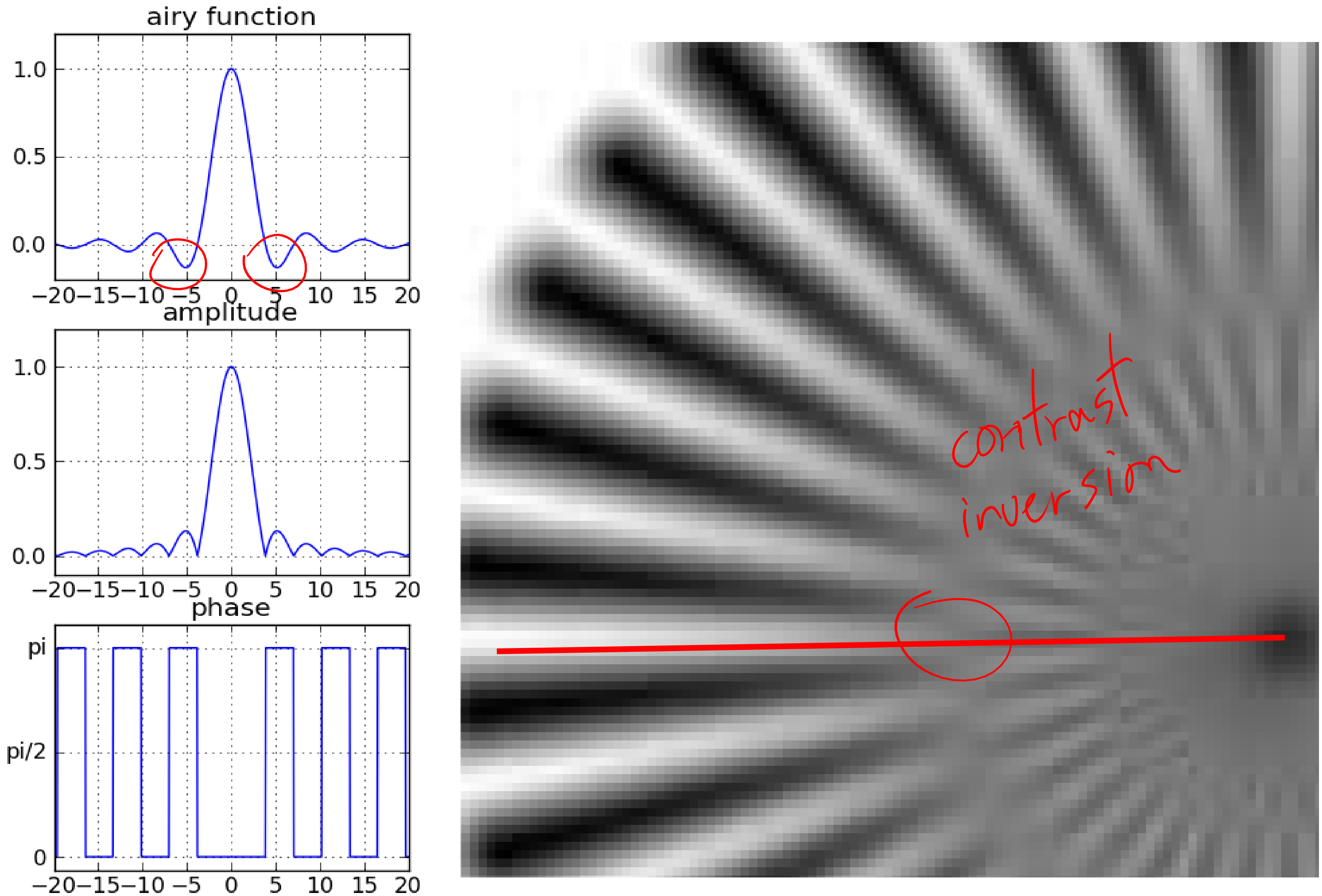
Airy function

F.T. of circular opening

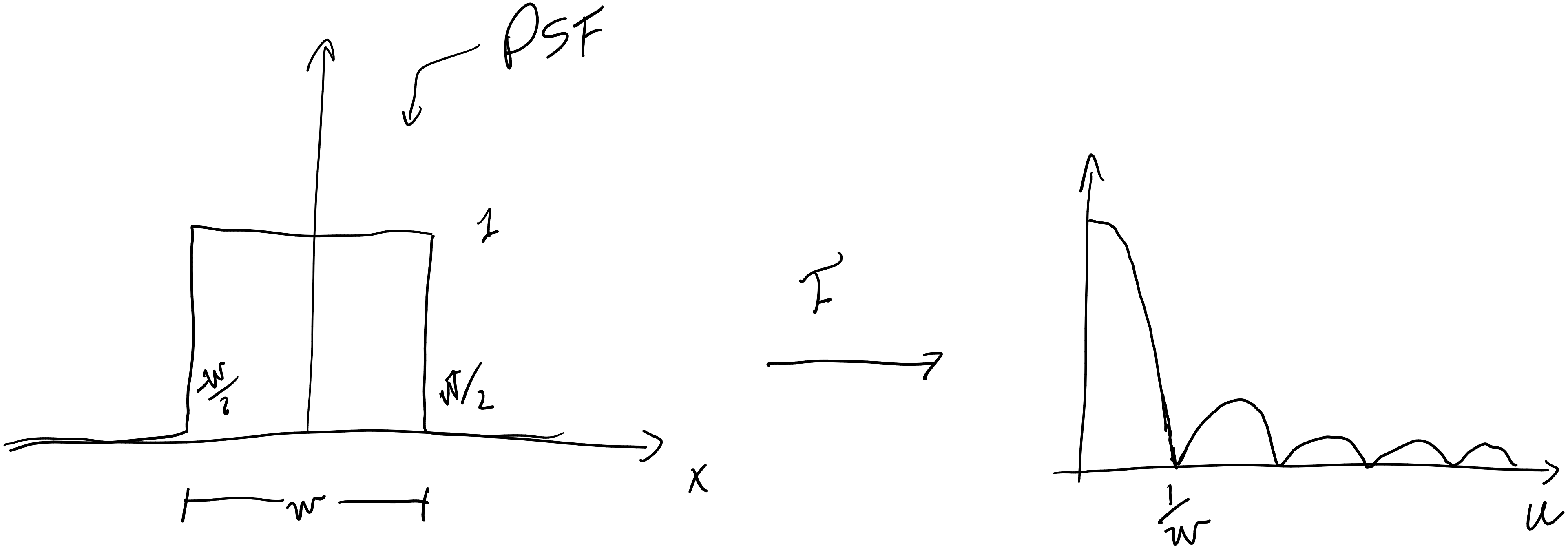


Phase transfer function

describes how an oscillating signal changes in phase due to system



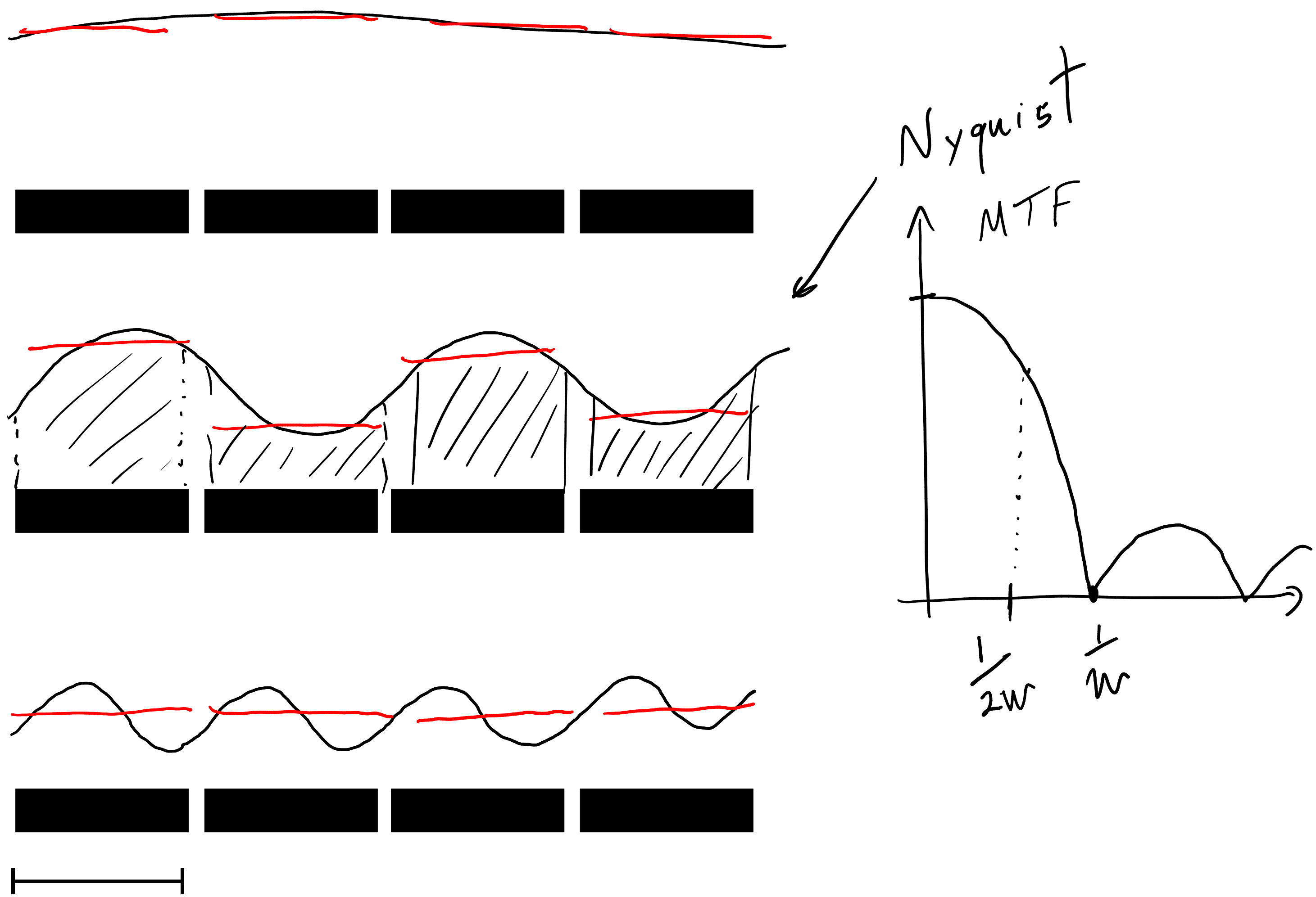
MTF of an "ideal pixel"



$$\text{"rect"}\left(\frac{x}{w}\right) \longrightarrow \operatorname{sinc}(wu)$$

Pixel MTF

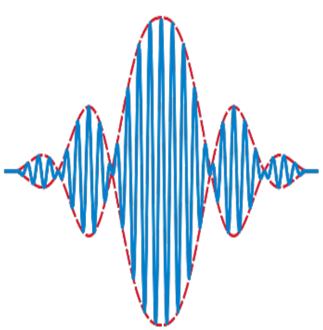
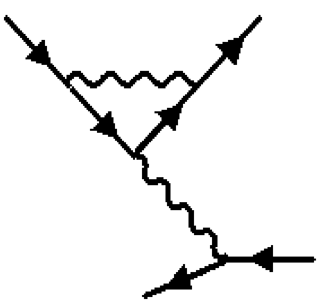
Modulation transfer function of a single detector pixel



Imaging as a linear filter

$$\text{output}(u) = \text{input}(u) \cdot MTF_{optic} \cdot MTF_{dector} \cdot MTF_{algorithm} \cdot \ldots$$

$$\text{imaging} = \text{linear filter}$$

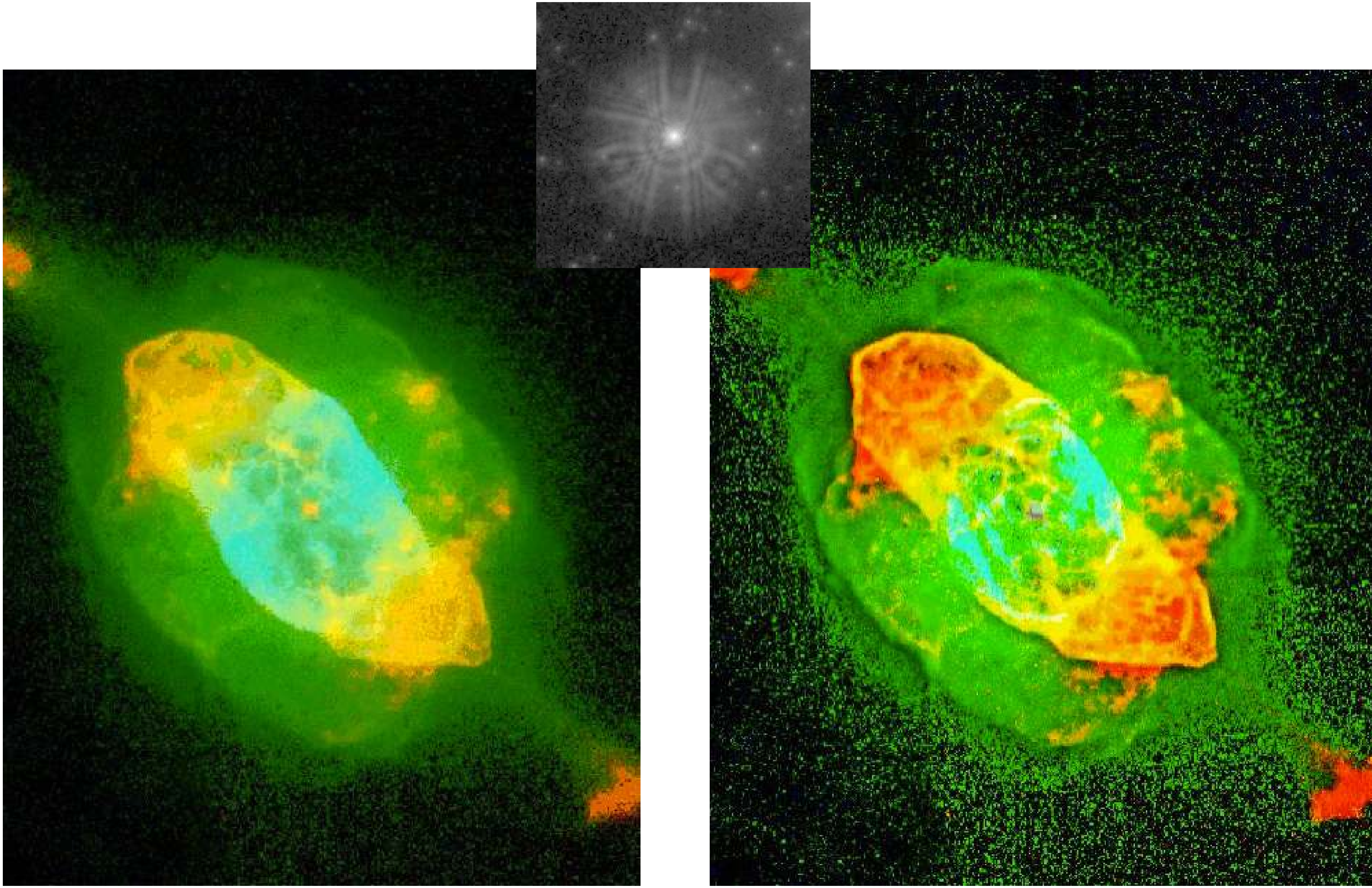

PSF examples

- isolated stars are essentially PSFs

source: www.apod.nasa.gov

PSF examples

Hubble flawed mirror deconvolution (correction for spherical aberration)

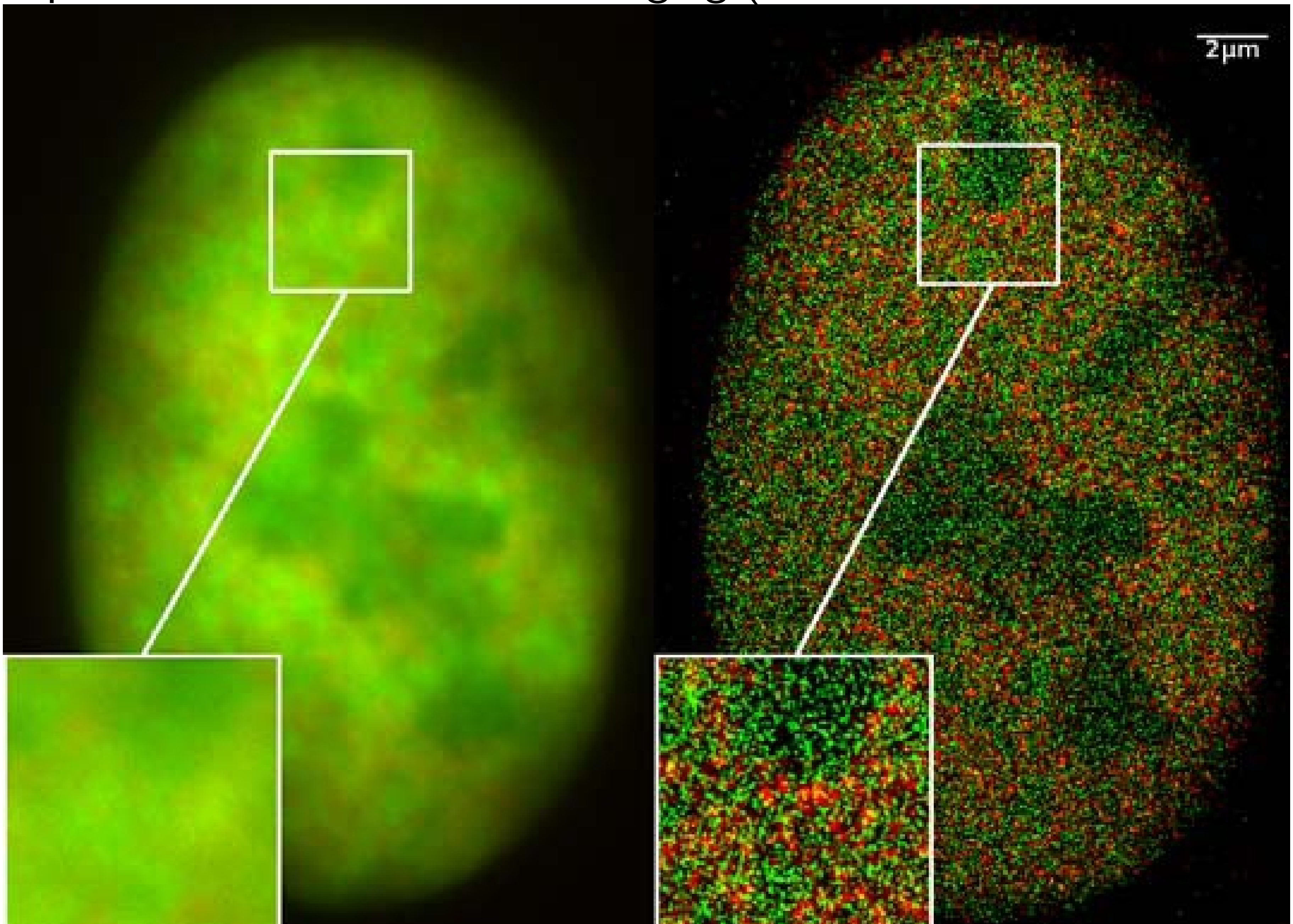


source: www.wikipedia.org

PSF examples

← one molecule $\xrightarrow{\text{system}}$ ← center

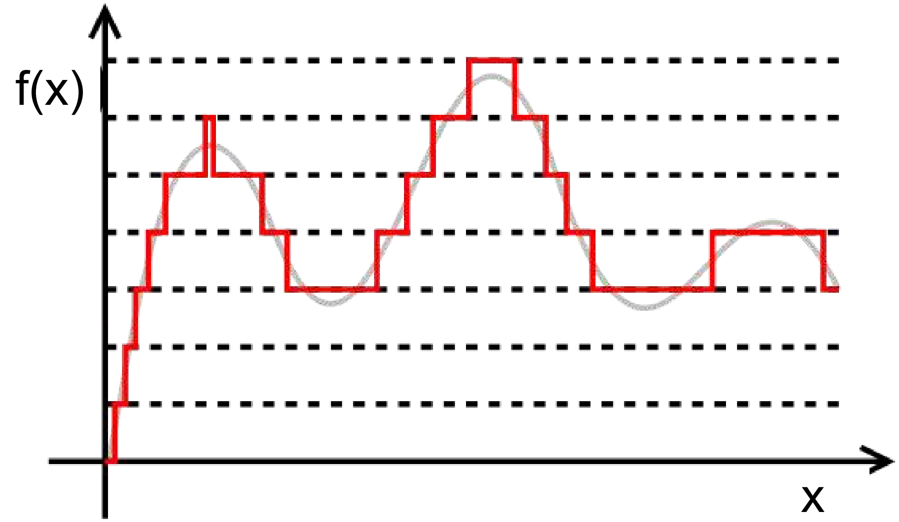
Super-resolution fluorescence imaging (STORM, STED, PALM, ...)



source: www.wikipedia.org

Signal and contrast

Signal S



Contrast

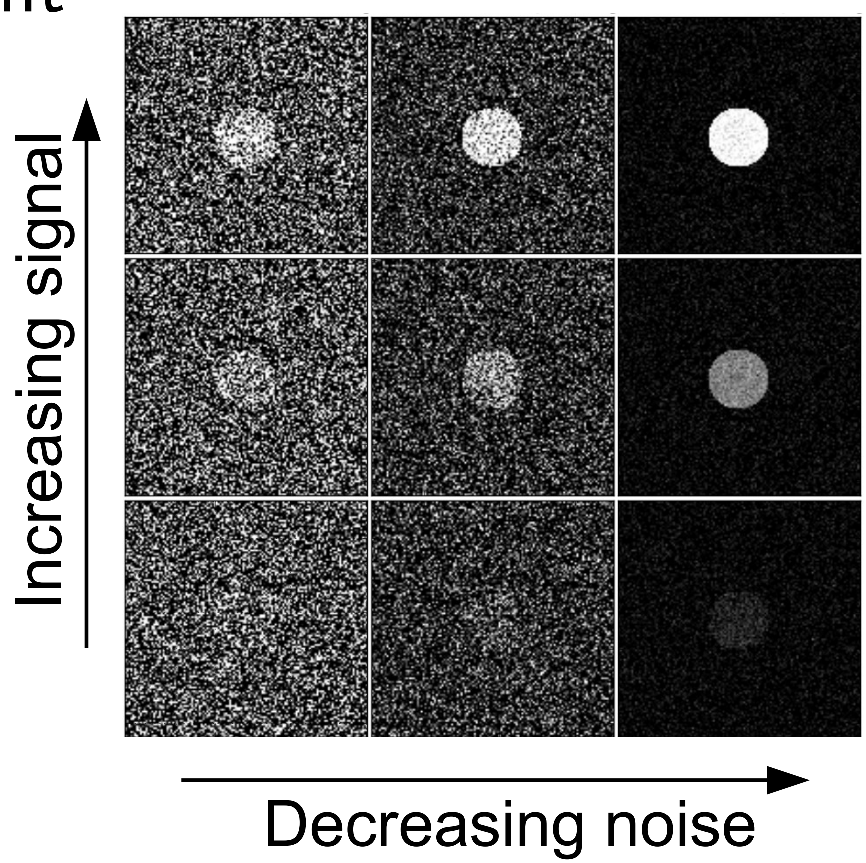
$C = (S_{max} - S_{min})$

(Michelson definition)

Visibility $\dfrac{C}{|S_{max} + S_{min}|}$

Contrast and noise

- Intensity operation: higher contrast, higher noise
- Contrast-to-noise remains constant



contrast
–
resolution

Random variables

- random variable, sample space

$$X \qquad \Omega$$

$$x \in \Omega$$

$$p(x) \geqslant 1$$

$$p(\Omega) = 1$$

- probability density function $\longrightarrow$ PDF

$$P(a < x < b) = \int_a^b \rho(x)\,dx \qquad \int_\Omega \rho(x)\,dx = 1$$

- expectation value

$$E[f(x)] = \int_\Omega f(x)\,\rho(x)\,dx$$

"$\langle x \rangle$"

$$E[x] = \int_\Omega x\,\rho(x)\,dx = \mu$$

"mean"

- variance

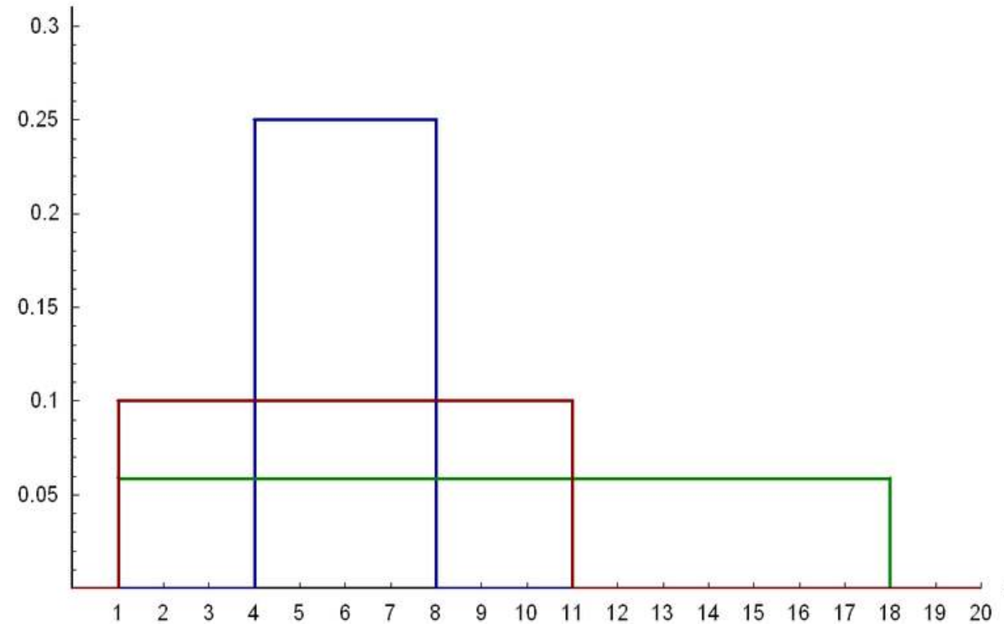
$$var(x),\ V[x] = E\left[(x - E[x])^2\right]$$

$$\langle (x - \langle x \rangle)^2 \rangle$$

Uniform distribution

- probability density function

$$\rho(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \end{cases}$$



- expectation value

mean : $\frac{1}{2}(a+b)$

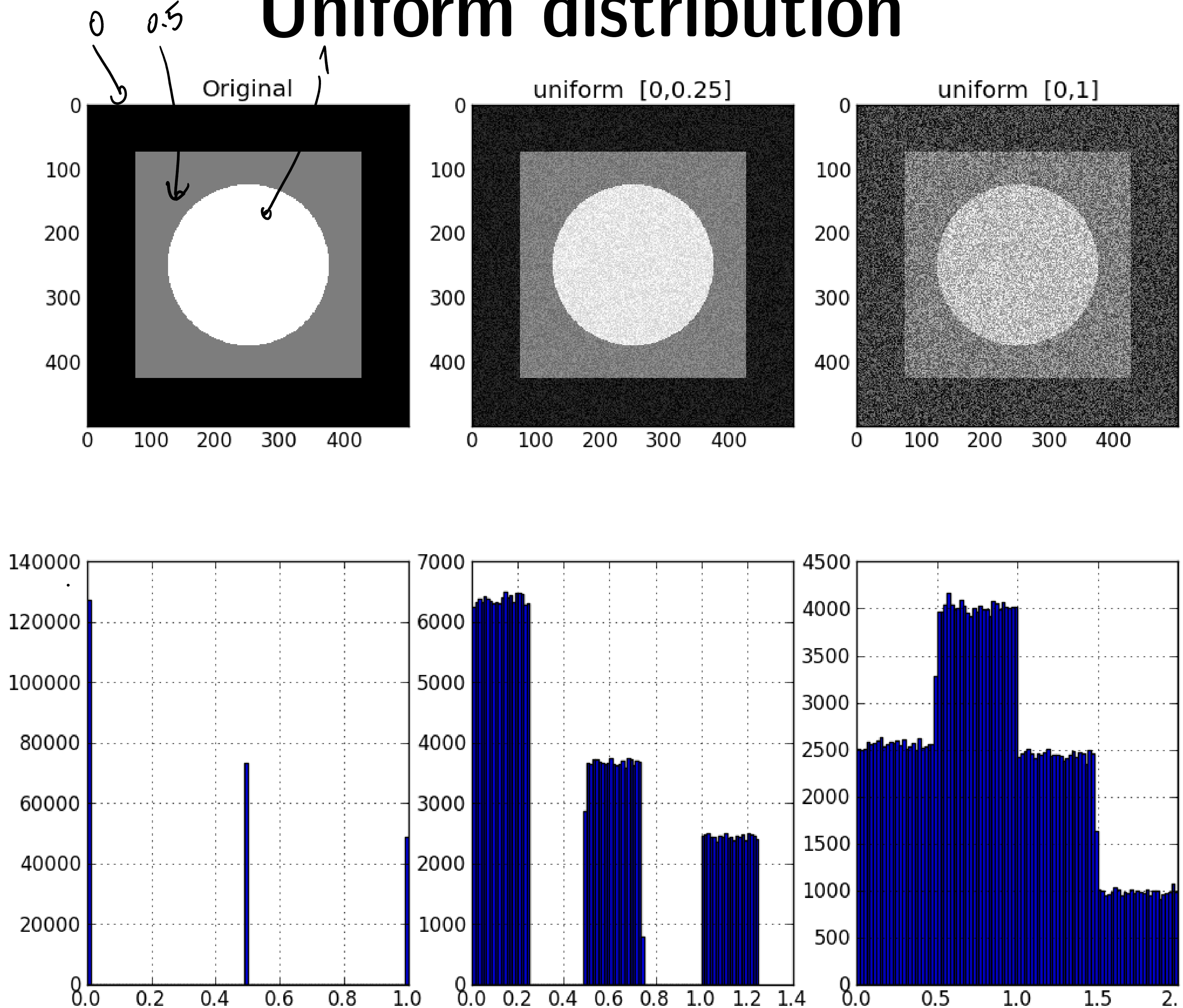
- variance

$$V(x) = \frac{(b-a)^2}{12}$$

- occurrence not very common in physics

useful to construct other PDFs

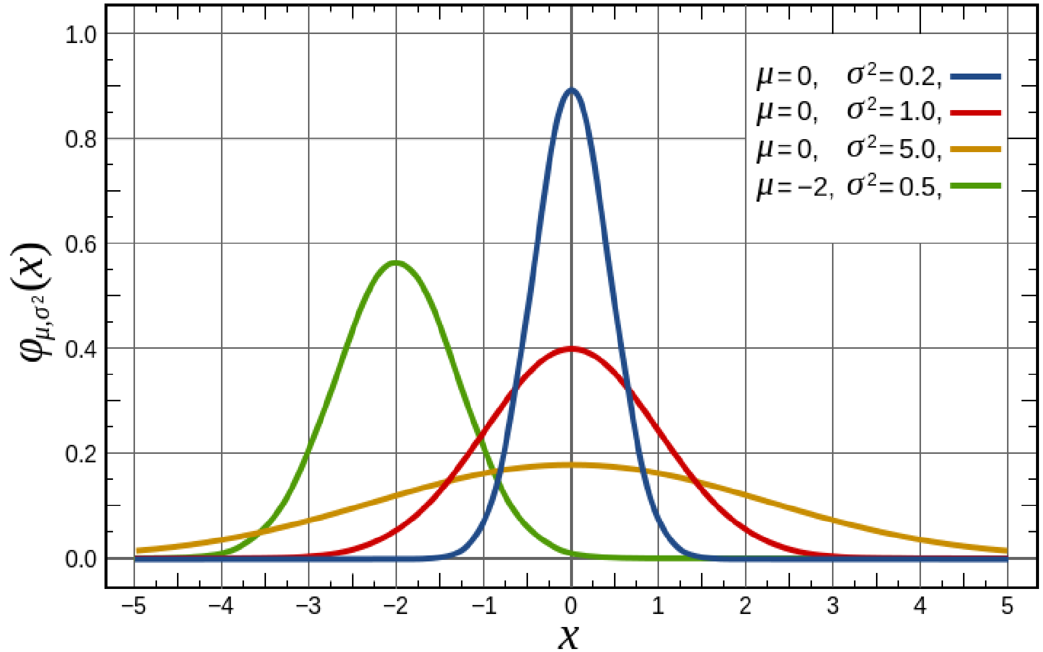
Uniform distribution



Gaussian distribution

- probability density function

$$\varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$



- ~~expectation value~~ mean

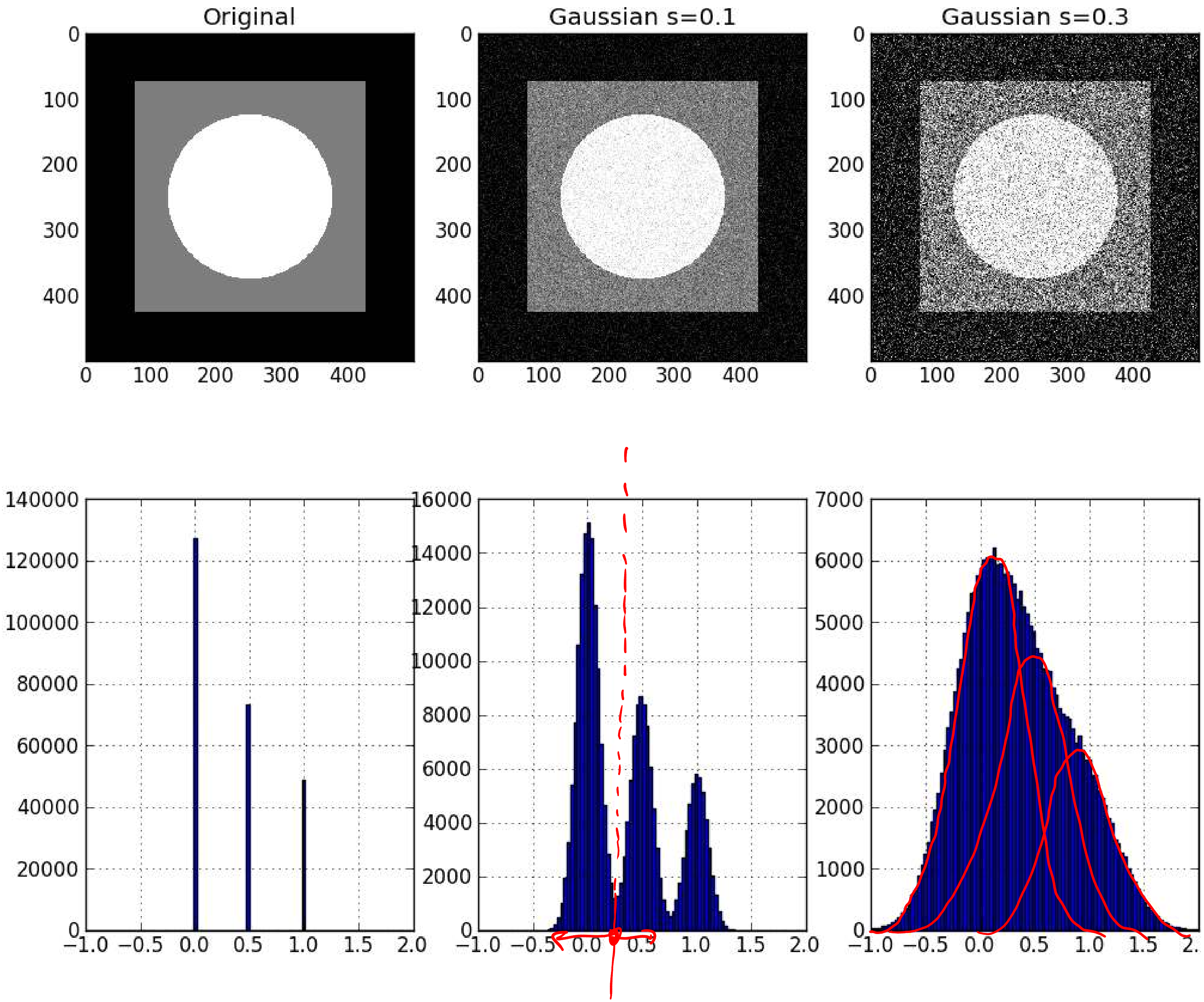
$$E(x) = \mu$$

- variance

$$V(x) = \sigma^2$$

- occurrence → very common (central limit theorem)

Gaussian distribution

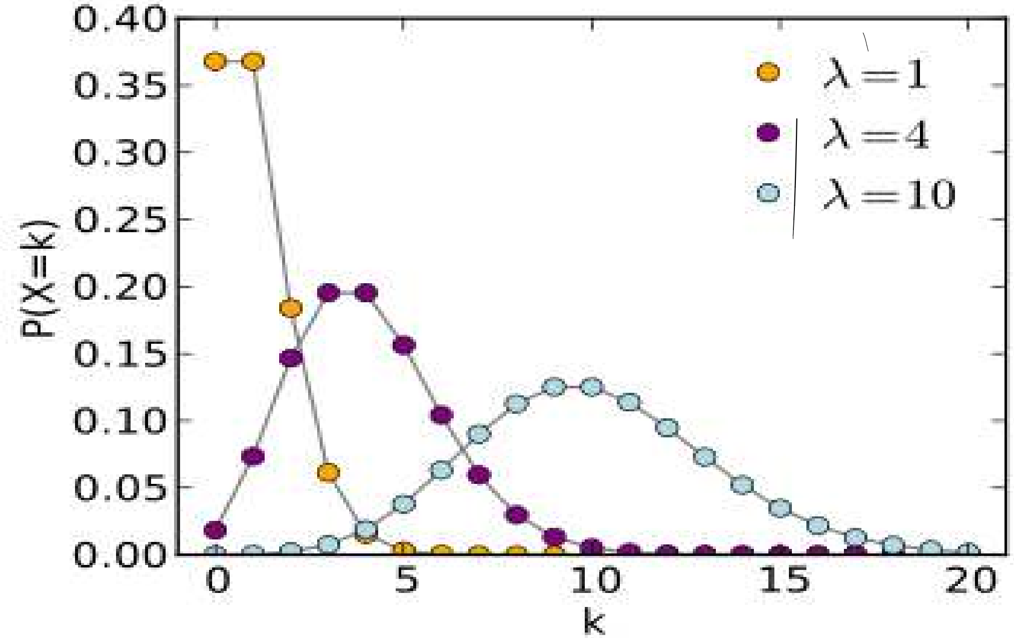


Poisson distribution

- probability mass function

$$p(n) = \frac{1}{n!} \lambda^{n} e^{-\lambda}$$

λ : parameter $\in \mathbb{R}$



- expectation value

$$E(n) = \lambda$$

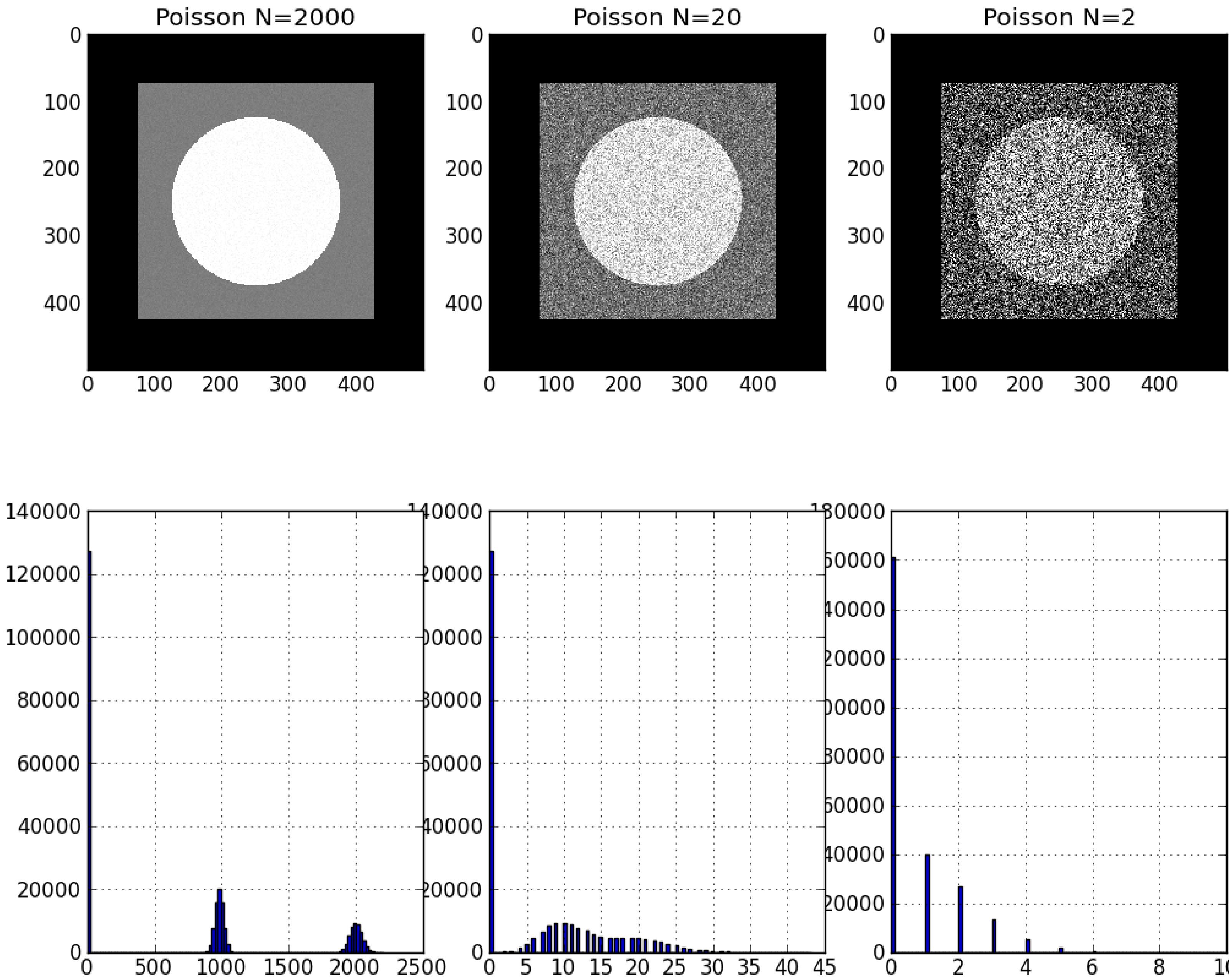
- variance

$$V(n) = \lambda$$

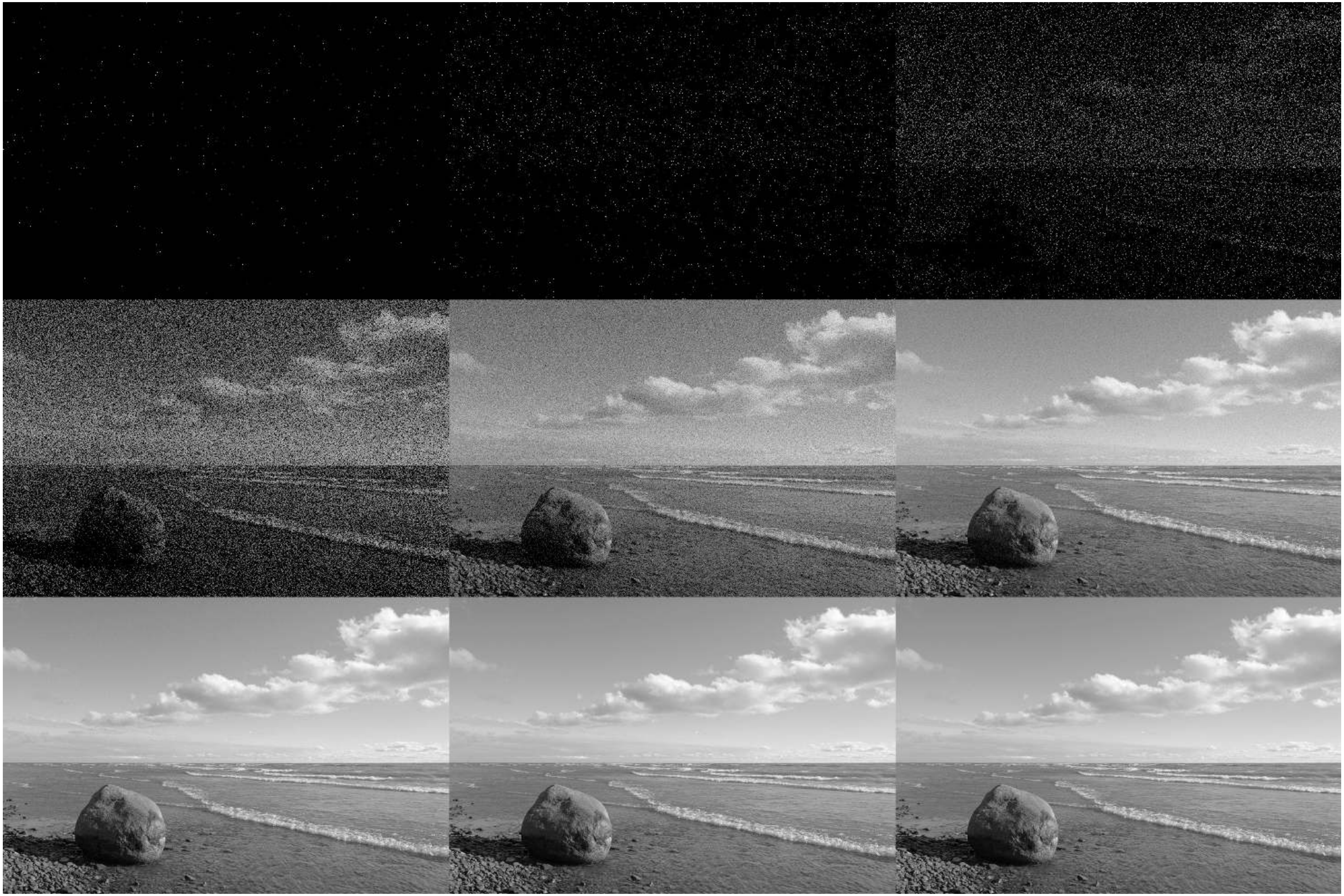
- occurrence counting processes (photons, electrons)

"shot noise" Schottky 1918

Poisson distribution

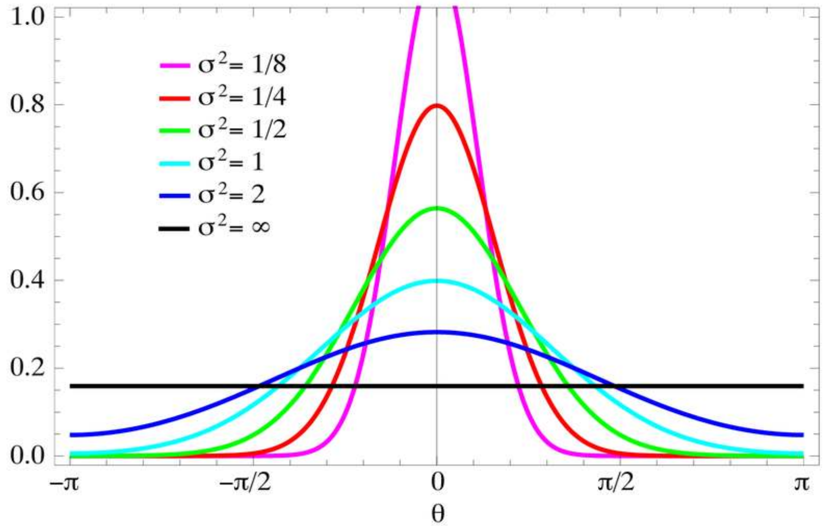


Poisson distribution

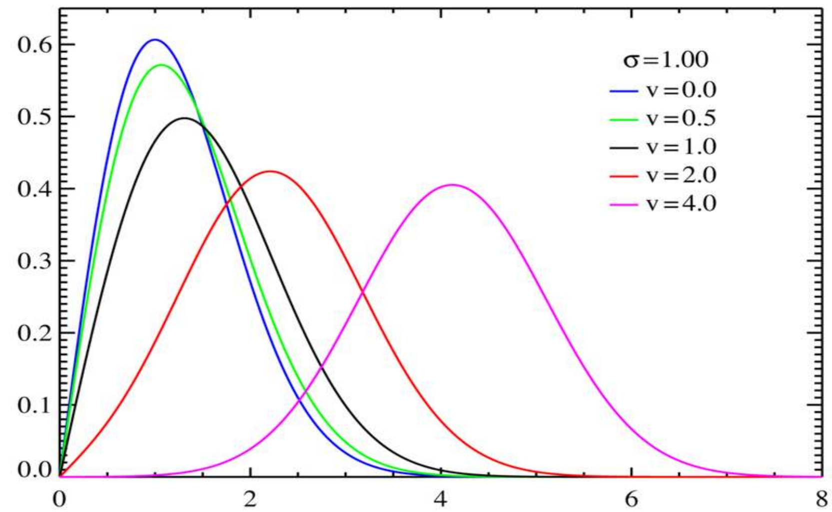


Many other distributions

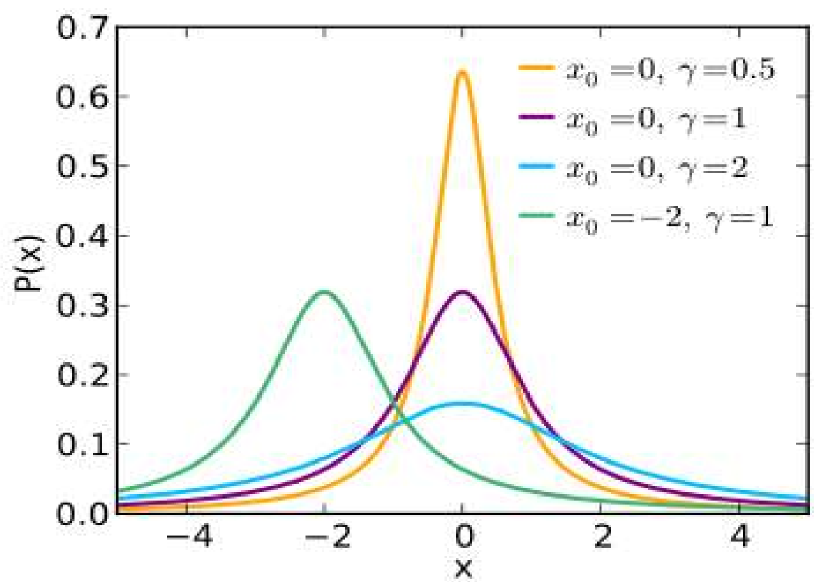
Wrapped normal distribution



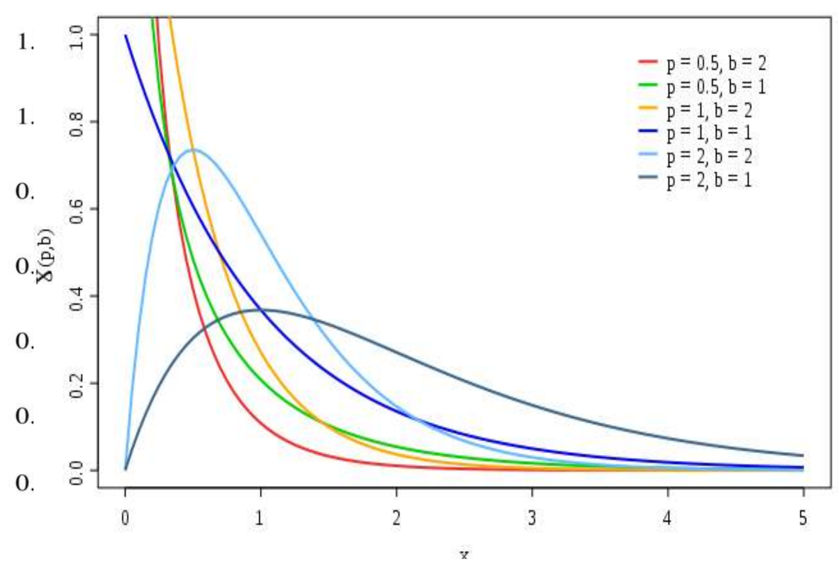
Rice distribution



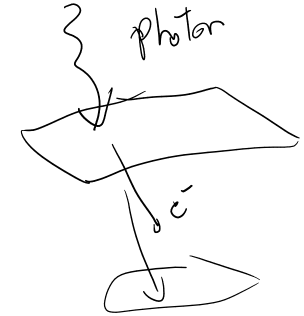
Lorentz distribution



Gamma distribution

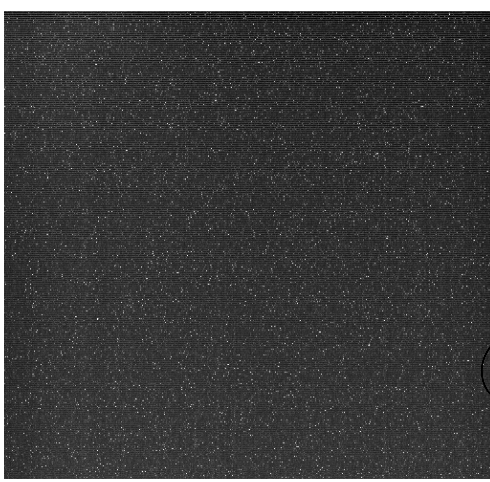


Detector noise (CCD)

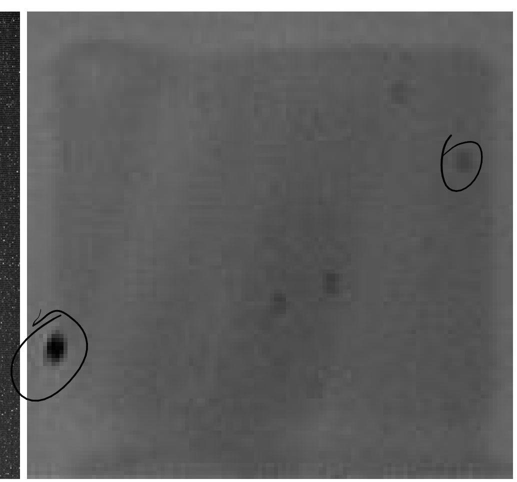


- Various sources:
  - shot noise (photon statistics, Poisson)
  - dark current (thermal electronic fluctuations in semiconductor, Poisson)
  - readout noise (fluctuations during amplification and digitization, Gauss)
  - many other imperfections ...
- dark frame measures detector noise, hot pixels, dead pixels
- bright frame measures gain differences and imperfections (dust, etc)

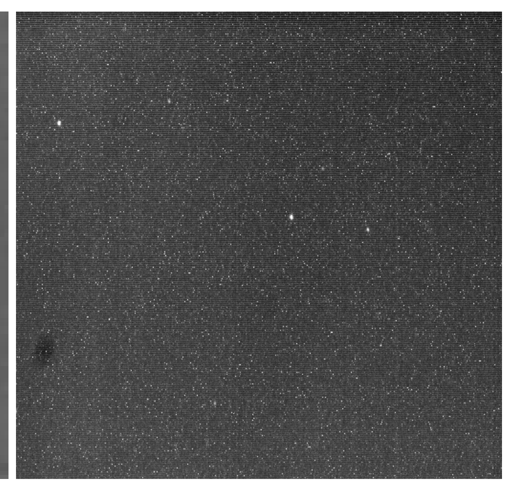
dark frame



bright frame



raw image



calibrated image

source: H. Raab, Johannes-Kepler-Observatory, Linz

Correlation & Convolution

Convution:

$$f * g = \int_{-\infty}^{\infty} f(x')\, g(x-x')\, dx'$$

Convolution theorem:

$$\mathcal{F}\{f * g\} = F \cdot G$$

Correlation:

$$f \circledast g = \int_{-\infty}^{\infty} f^*(-x')\, g(x-x')\, dx'$$

Fourier:

$$\mathcal{F}\{f \circledast g\} = F^* \cdot G$$

Noise power spectrum

- power spectrum of pure noise image

$$n(x,y) \xrightarrow{\mathcal{F}} N(u,v) \qquad NPS = E\left[|N(u,v)|^2\right]$$

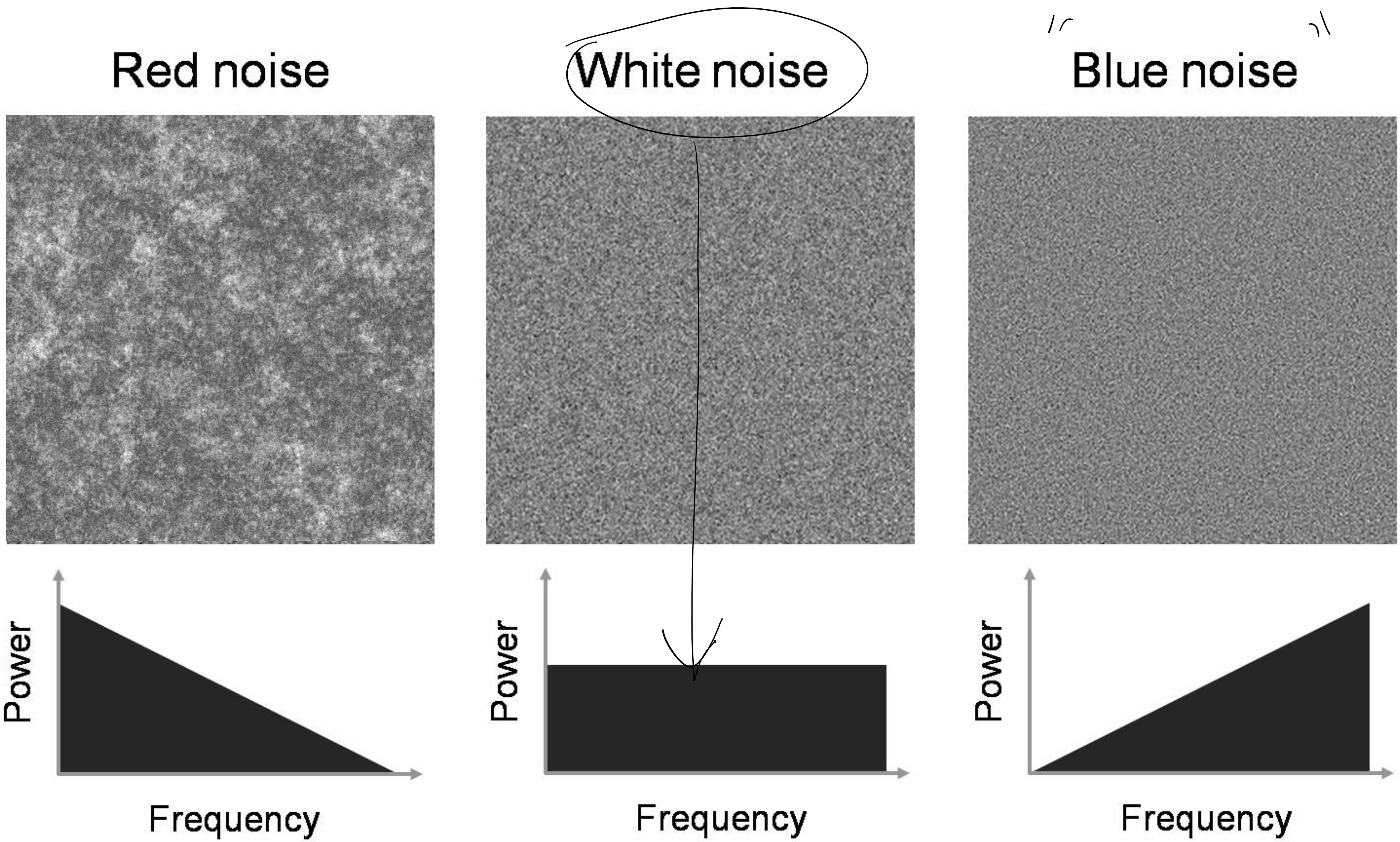
- connection to auto-correlation

$$|N(u,v)|^2 = N(u,v) \cdot N^*(u,v)$$

$$\Rightarrow \mathcal{F}^{-1}\{NPS\} = n(x,y) \circledast n(x,y)$$

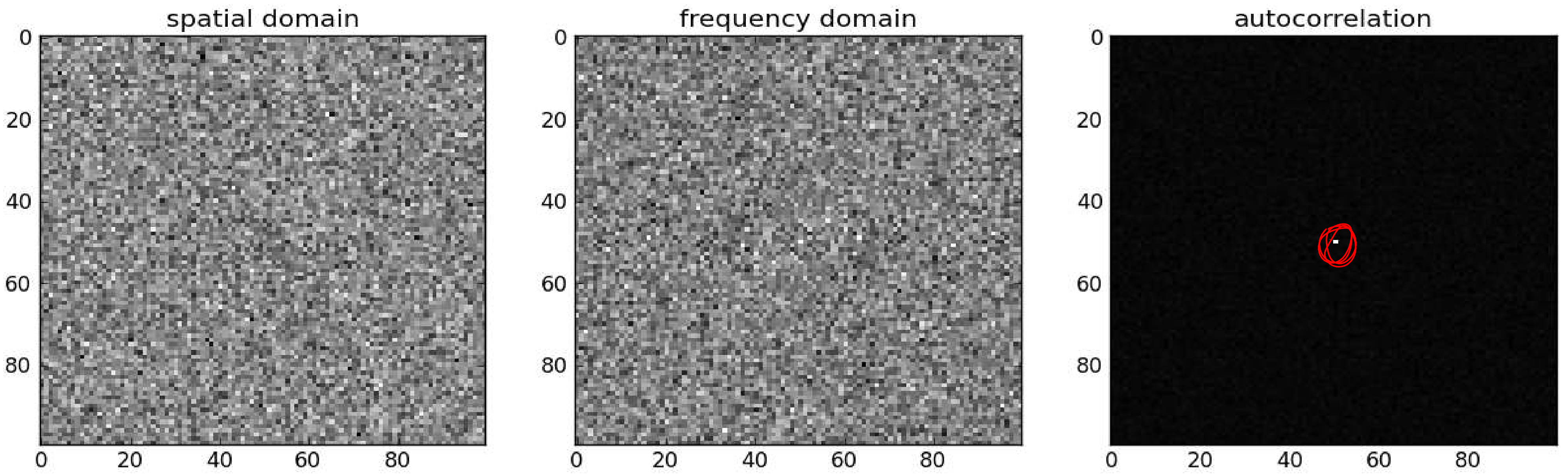
Wiener Chimchin theorem: autocorrelation $\updownarrow$ noise power spectrum

Noise power spectrum



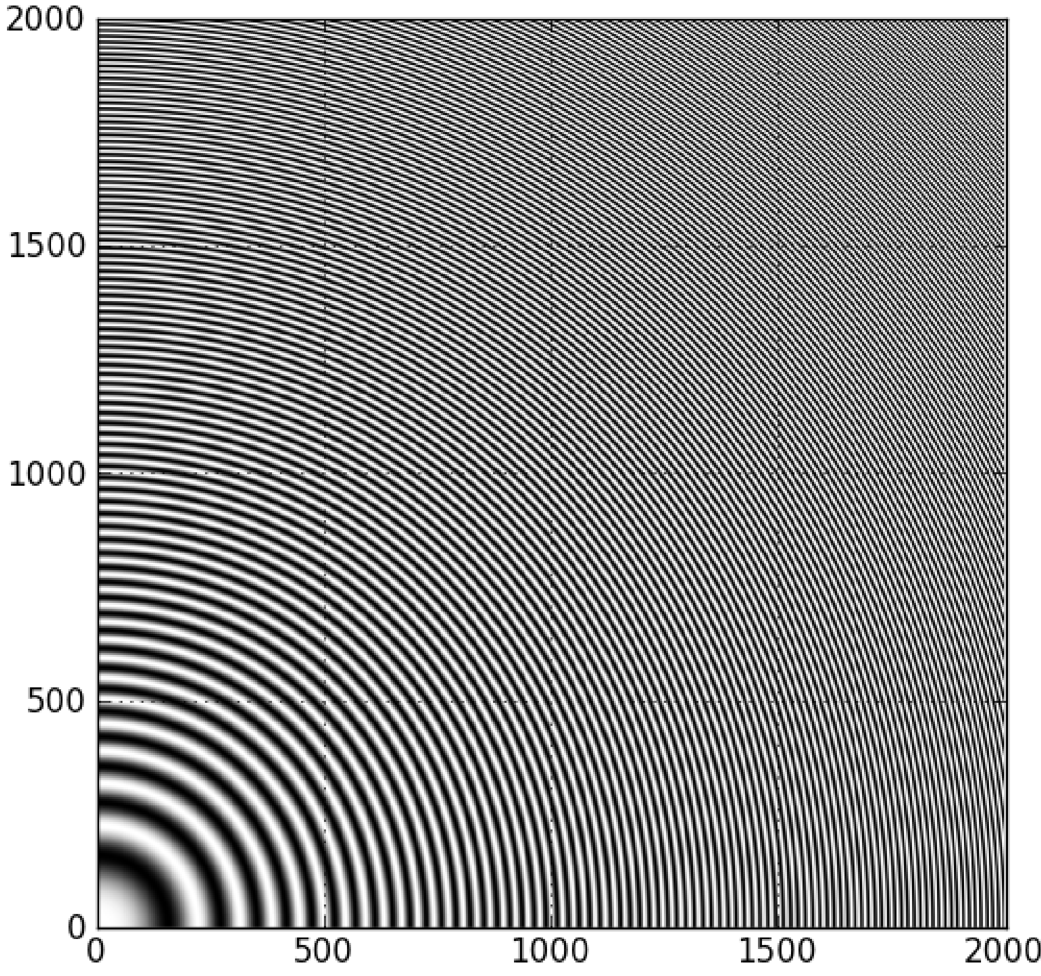
source: http://scien.stanford.edu/pages/labsite/2008/psych221/projects/08/AdamWang/project_report.htm

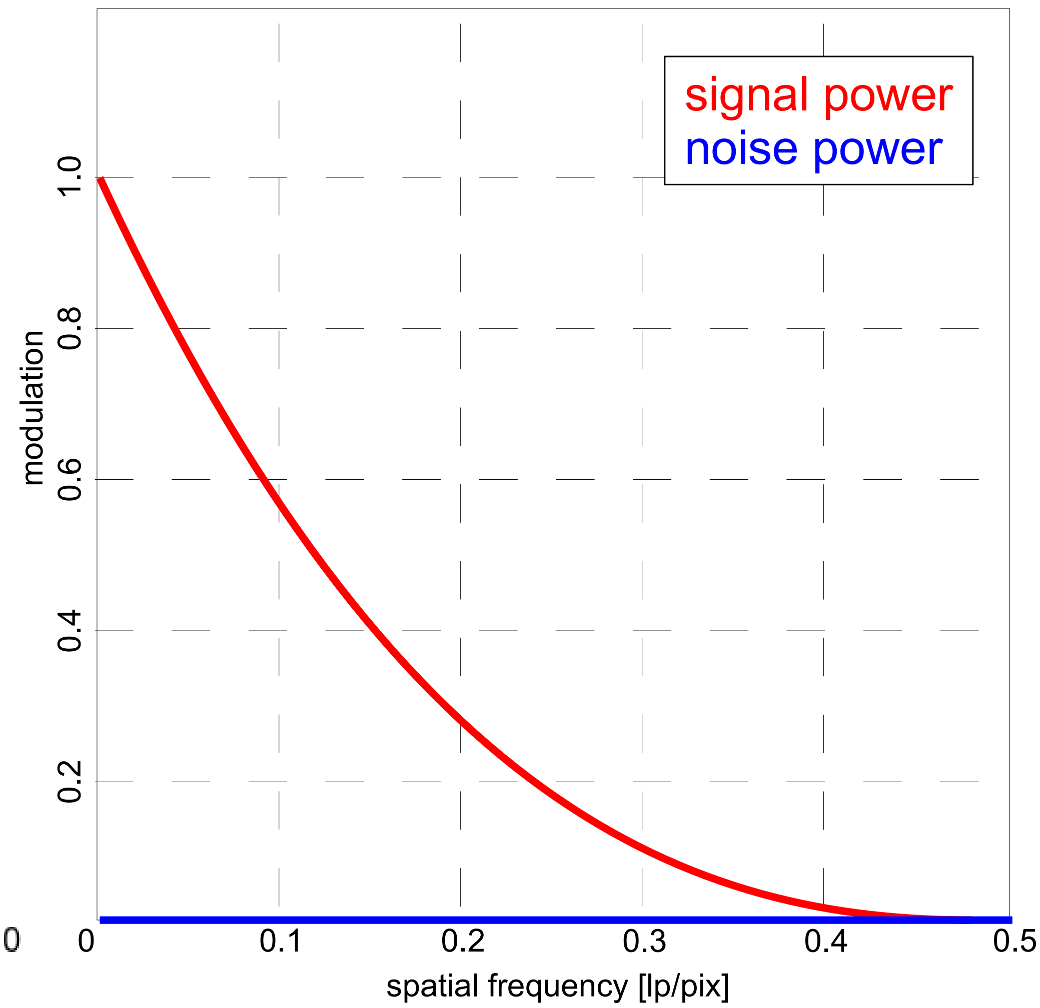
White noise



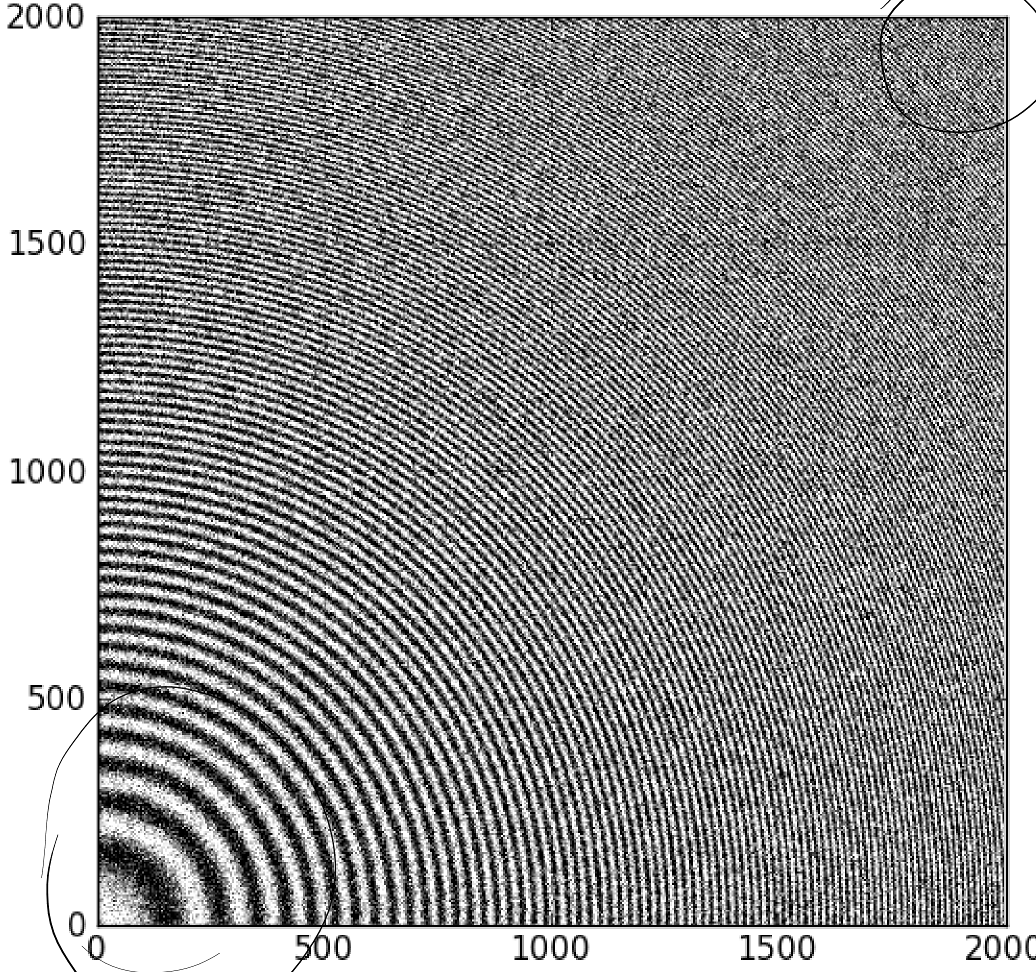
- white noise in spatial domain equals white noise in frequency domain
- white noise is perfectly uncorrelated
- all other types of noise are correlated to some degree
- white noise is an idealization

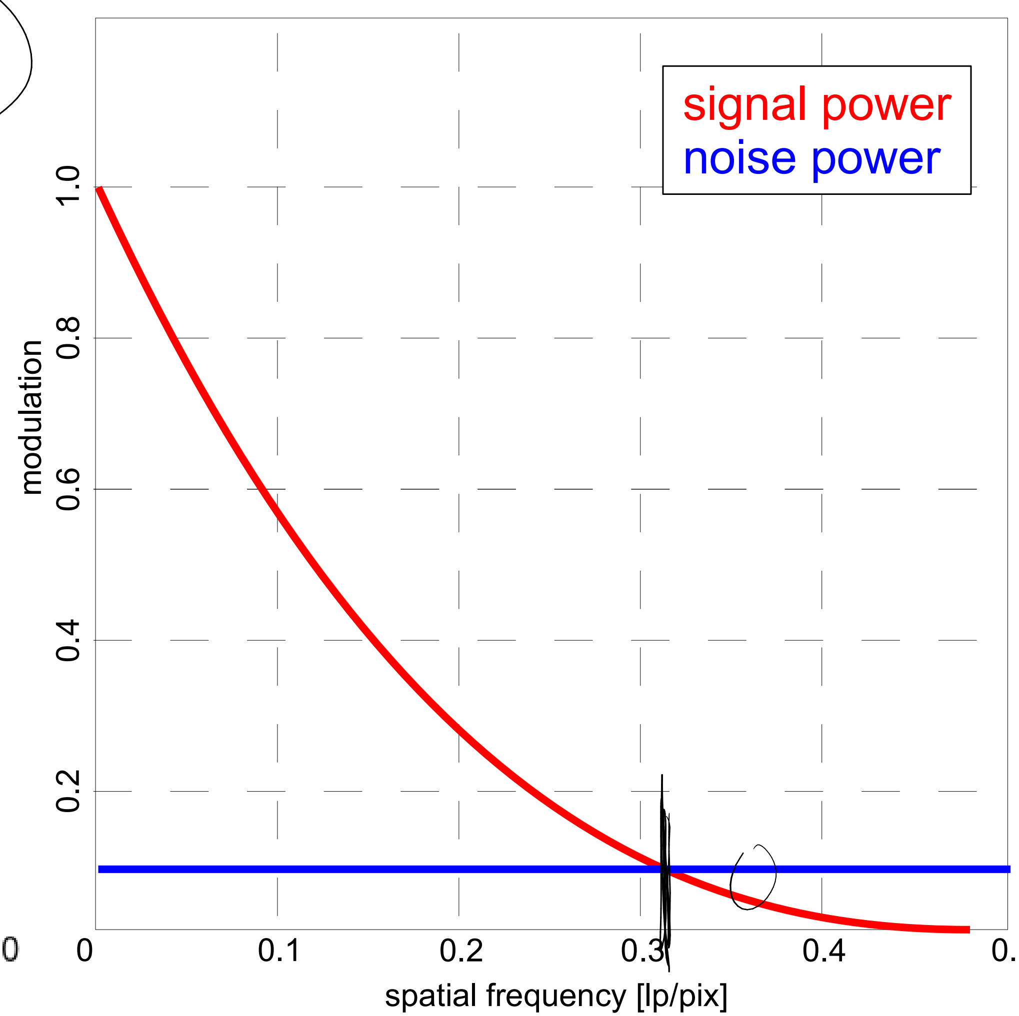
Signal power vs. noise power



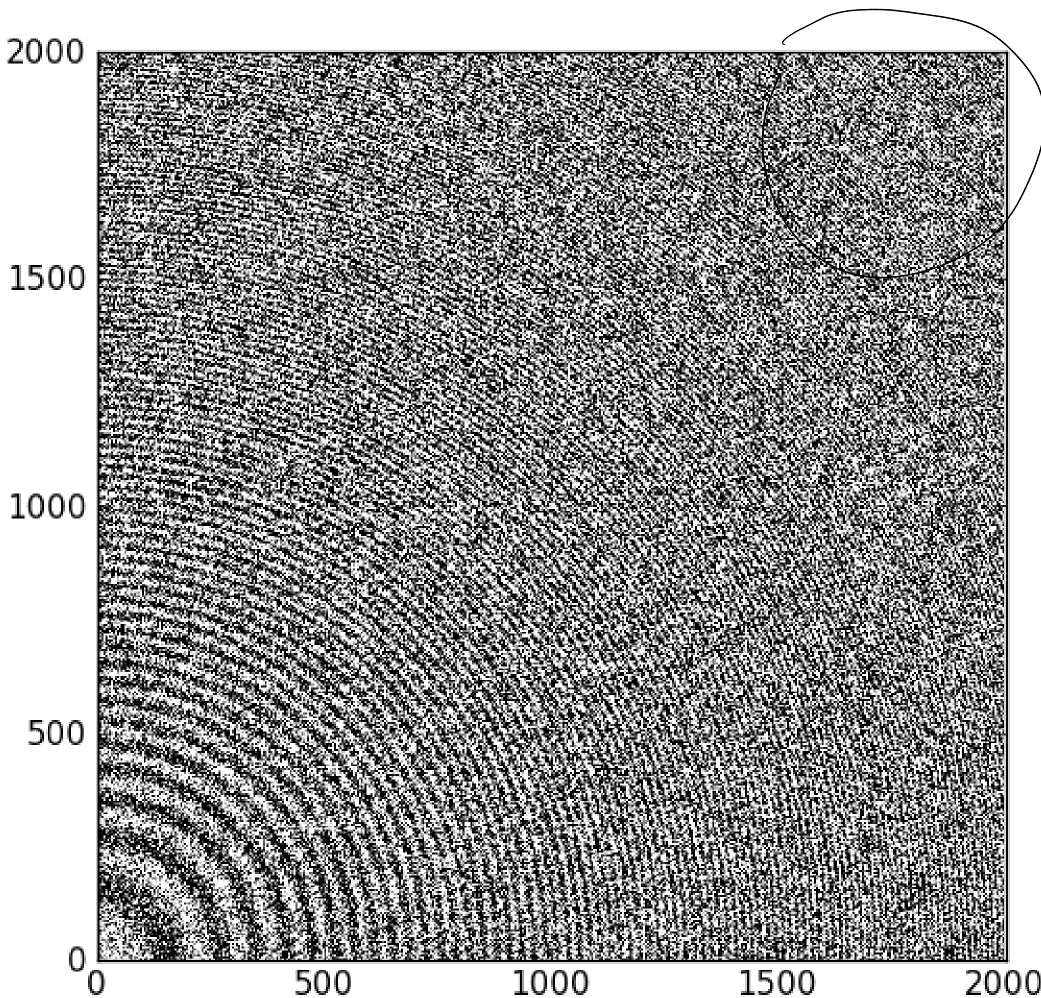


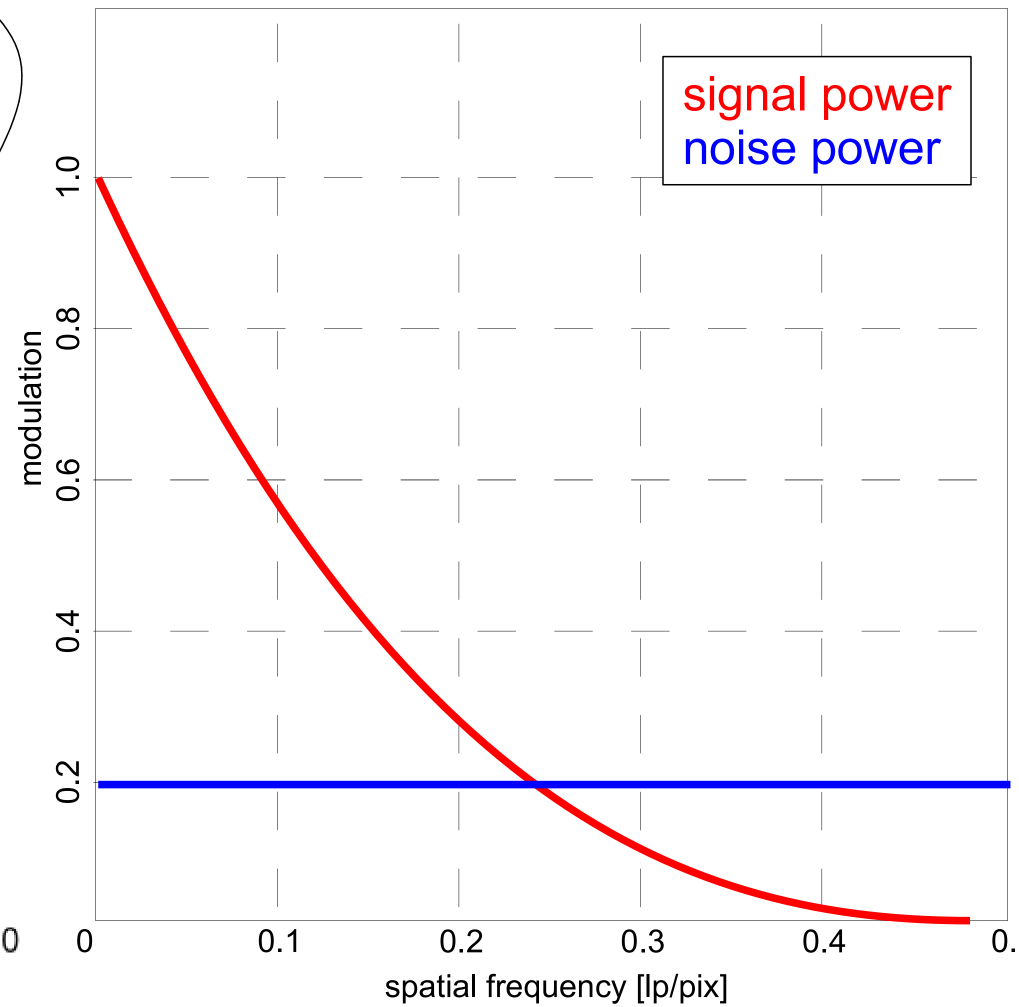
Signal power vs. noise power



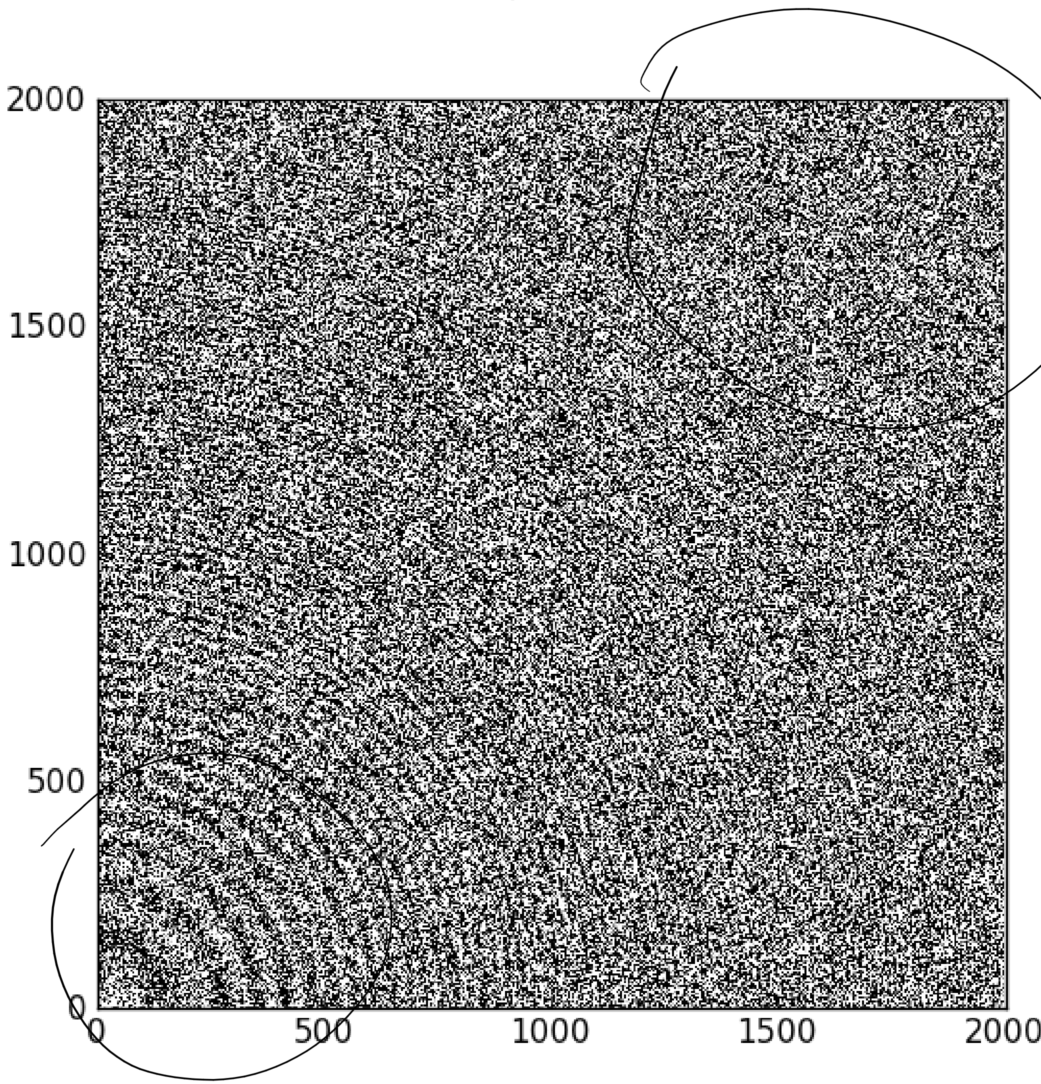


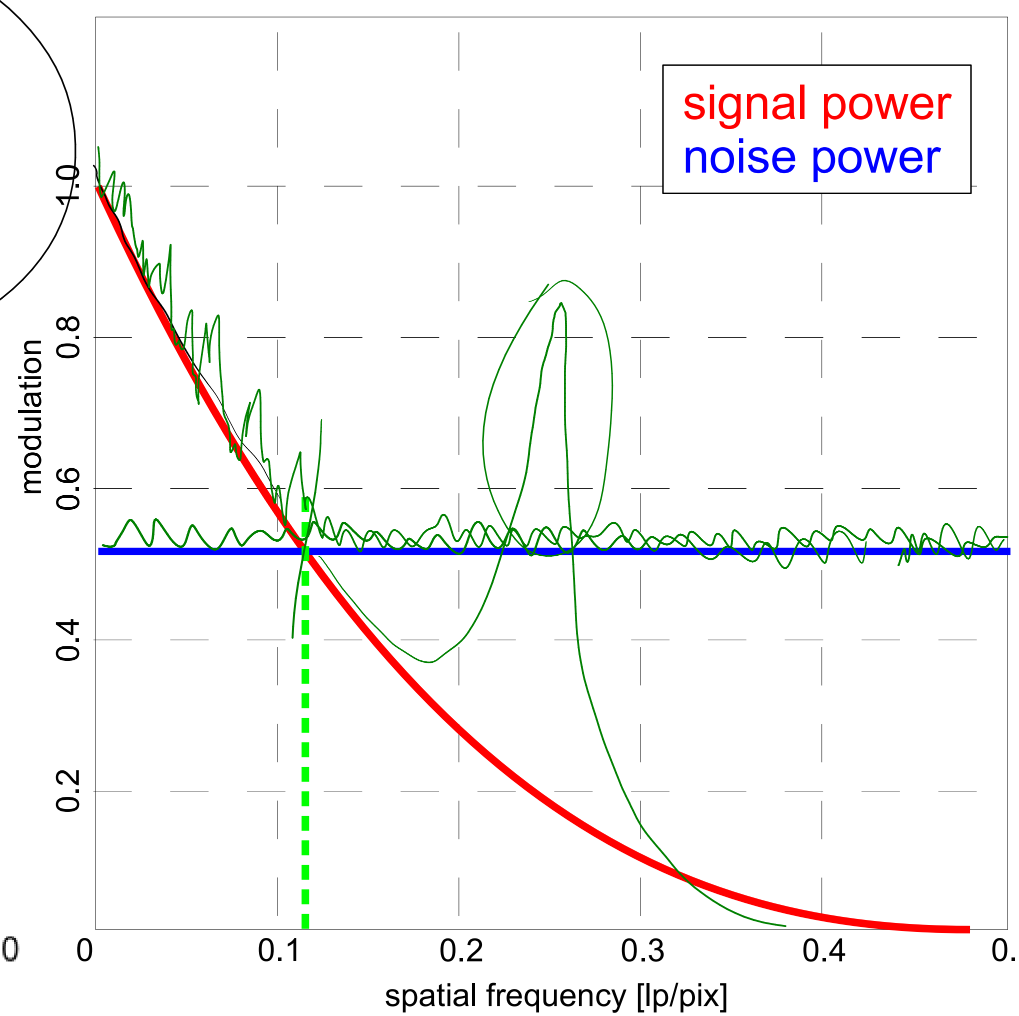
Signal power vs. noise power





Signal power vs. noise power

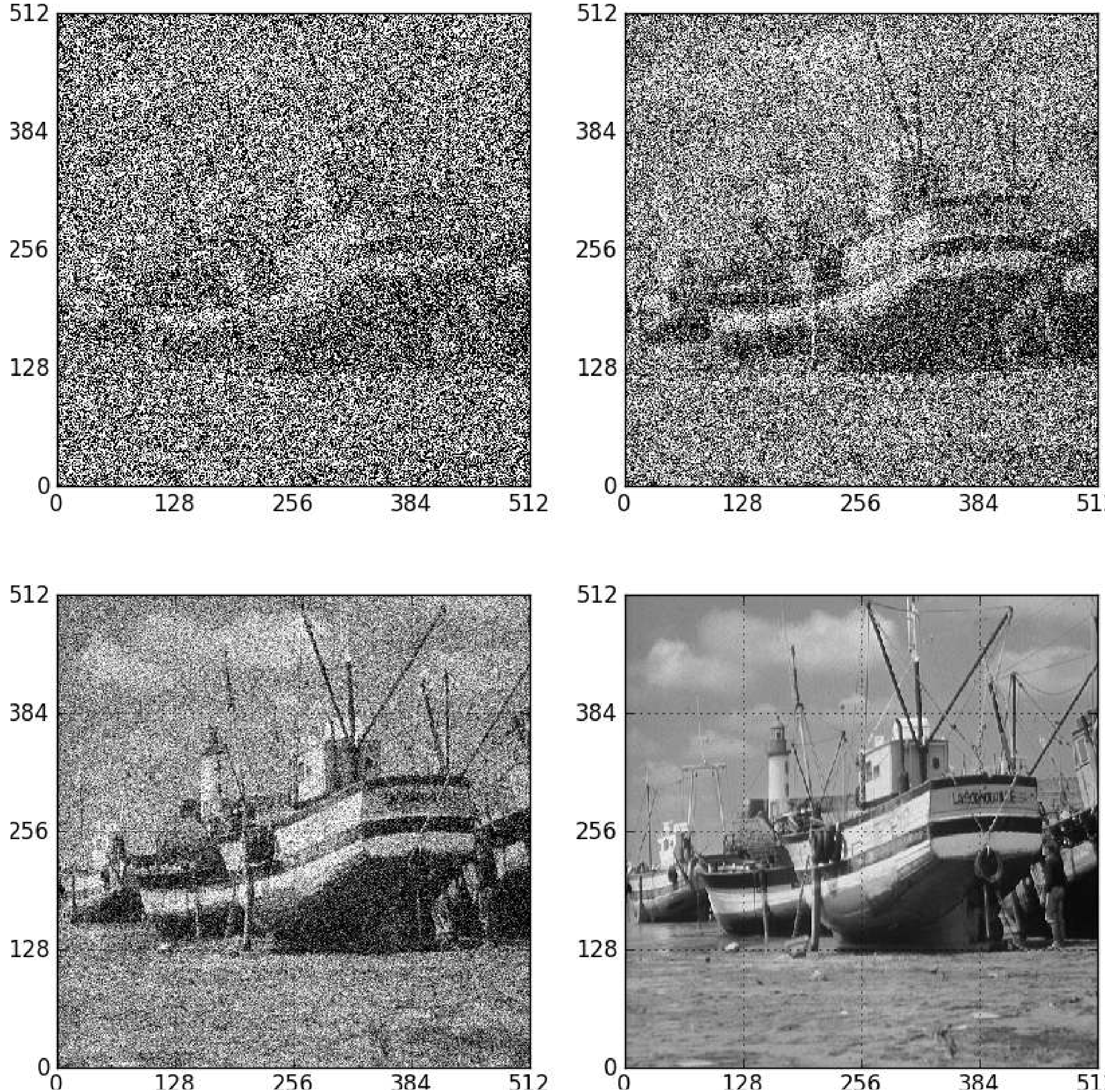




- Noise power exceeds signal power for high frequencies
- Small scale image details are lost in noise first

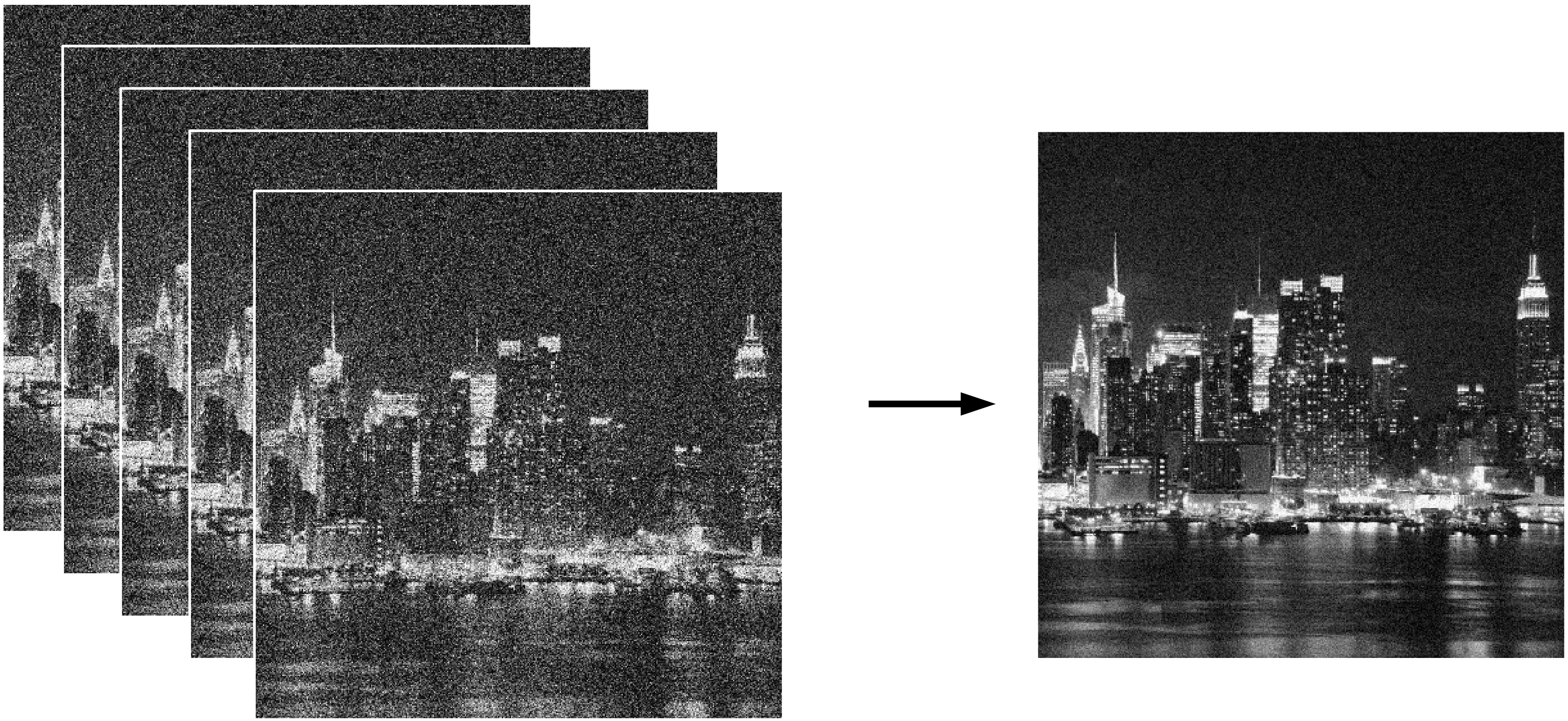
$$SNR(u) = \frac{S(u)}{NPS(u)}$$

Signal power vs. noise power



Noise reduction by averaging

- Average multiple images



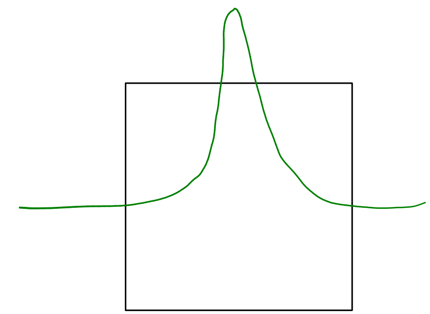
- requirement: additive noise, zero mean

Denoising by linear filtering

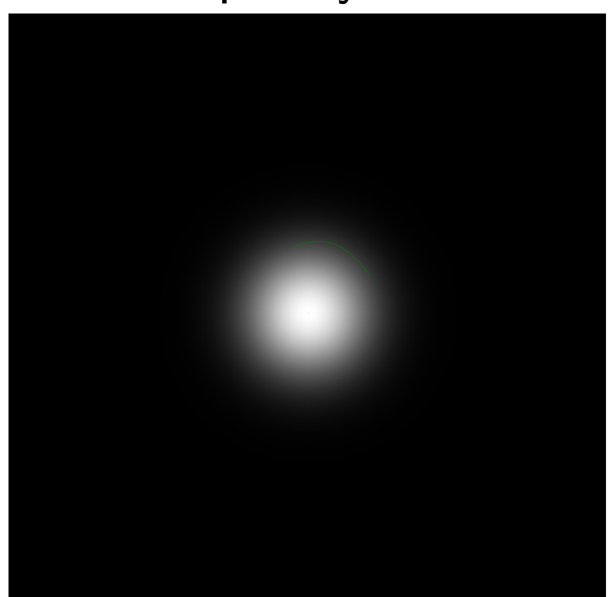
- use spatial convolution or frequency filtering to reduce noise
- noise reduction possible, but at cost of sharpness
- trade-off between noise reduction and resolution
- need fancier methods

original

convolution kernel

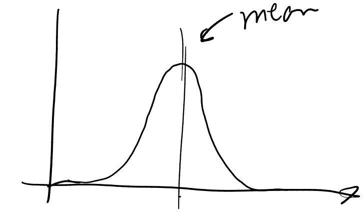


frequency filter

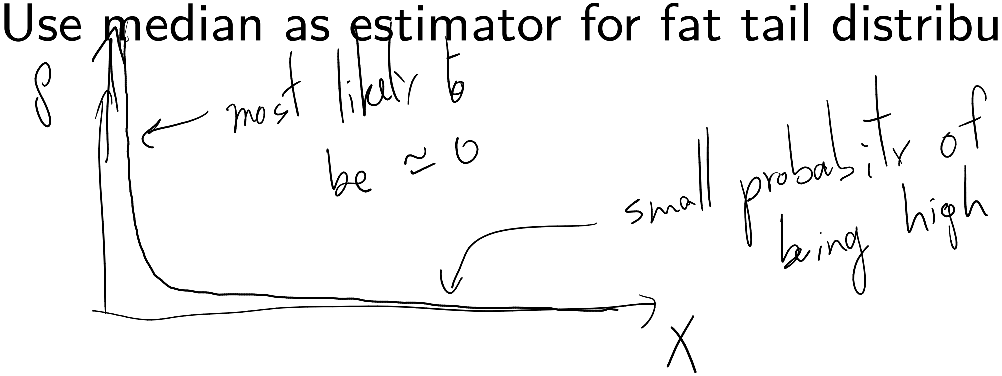


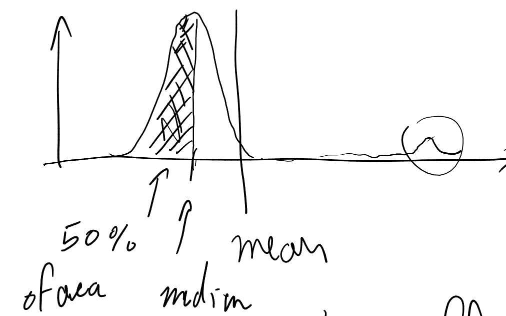
Resulting image

Median filtering



- Use median as estimator for fat tail distributions





- less sensitive to outliers in pixel ensemble, better edge preservation

↳ less affected by outliers

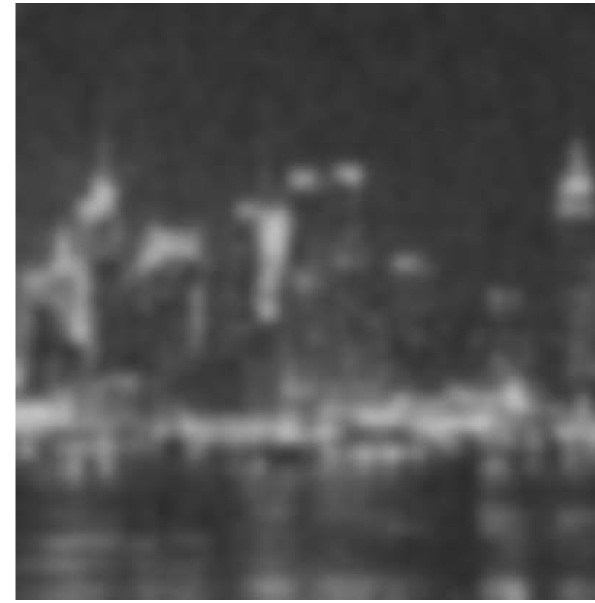
"Salt and pepper" noise | Gauss sigma=1 pixel | Median 1 pixel

Median filtering

1x Gauss

2x Gauss

5x Gauss



1x Median

2x Median

5x Median

Common abbreviations

Abbreviation	Name	Definition
IRF	Impulse response function	Linear operator map of delta function
PSF	Point spread function	Image of point object (optical IRF)
OTF	Optical transfer function	Fourier transform of PSF
PTF	Phase transfer function	Phase part of OTF
MTF	Modulation transfer function	Amplitude of OTF
CTF	Contrast transfer function	MTF for non-sinusoidal objects
PDF	Probability density function	Probability distribution for a given random variable
SPS	Signal power spectrum	Amplitude squared of signal F.T.
NPS	Noise power spectrum	Amplitude squared of noise F.T.
SNR	Signal to noise ratio	Mean signal / mean noise
CNR	Contrast to noise ratio	Mean contrast / mean noise