$\eta \sim WN(0,1)$ White Noise $E(\eta)=0$

$\sigma_\eta^2 = 1$

$$v(t) = \eta(t) + 2\eta(t-1) + \eta(t-2)$$

$$\gamma_\eta(\tau) = \begin{cases} 1 & \tau = 0 \\ 0 & |\tau| \neq 0 \end{cases}$$

$E[v(t)] = ?$

$$E[v(t)] = E[\eta(t) + 2\eta(t-1) + \eta(t-2)]$$

x lineare

$$= \underbrace{E(\eta(t))}_{} + 2\,\overbrace{E[\eta(t-1)]}^{0} + \underbrace{E[\eta(t-2)]}_{0} = 0$$

$\eta \sim WN$ proc stocastico stazionario

$E[\eta] = 0 \quad \forall t$

$E[v(t)]$ costante

$$\gamma_v(t,t) = E\{v(t) \cdot v(t)\} = E\{[\eta(t) + 2\eta(t-1) + \eta(t-2)] \cdot [\eta(t) + 2\eta(t-1) + \eta(t-2)]\}$$

$E[v^2(t)]$

$$\gamma_v(t,t) = \boxed{E\left\{[\eta(t)]^2\right\}}_{\sigma_\eta^2} + 4E\left\{[\eta(t-1)]^2\right\} + E\left\{[\eta(t-2)]^2\right\} +$$

linearità di $E[\ ]$

$$+ 4\underbrace{\cancel{E\left\{\eta(t)\,\eta(t-1)\right\}}}_{\gamma(1)}{}^{0} + 2\underbrace{\cancel{E\left\{\eta(t)\,\eta(t-2)\right\}}}_{\gamma(2)}{}^{0} +$$

$$+ 4\underbrace{\cancel{E\left\{\eta(t-1)\cdot\eta(t-2)\right\}}}_{\gamma(1)}{}^{0} = 1 + 4\cdot 1 + 1 = 6$$

$\eta(\ )$ è processo WN stazionario $\quad \lambda^2 = 1 \quad E(\eta) = 0$

$$\sigma_\eta^2 = \gamma_\eta(0) = \underline{E[\eta^2(t)]} - [\underbrace{E(\eta(t))}_{0}]^2$$

$$\gamma(\tau) = 0 \qquad |\tau| \neq 0$$

$v(t) = \ldots$

$v(t\pm1) = \ldots$

$$\gamma(t, t\pm1) = E\left[v(t)\cdot v(t\pm1)\right] =$$

$$= E\left[\eta(t)\,\eta(t\pm1)\right] + 2E\left[\eta(t)\,\eta(t\pm1-1)\right] + E\left[\eta(t)\,\eta(t\pm1-2)\right] +$$

$$+ 2E\left[\eta(t-1)\cdot\eta(t\pm1)\right] + 4E\left[\eta(t-1)\cdot\eta(t\pm1-1)\right] +$$

$$+ 2E\left[\eta(t-1)\cdot\eta(t\pm1-2)\right] +$$

$$+ E\left[\eta(t-2)\,\eta(t\pm1)\right] + 2E\left[\eta(t-2)\,\eta(t\pm1-1)\right] +$$

$$+ E\left[\eta(t-2)\,\eta(t\pm1-2)\right]$$

$$\gamma(t, t\pm 1) = \begin{cases} t+1: & 2+2 = 4 \\ t-1: & 2+2 = 4 \end{cases}$$

$$\gamma(t, t\pm 2) = \begin{cases} t+2: & +1 \\ t-2: & +1 \end{cases}$$

$$\gamma(t, t\pm\tau) = 0 \qquad \tau > 2$$

$$\gamma(t, t+\tau) = \left.\begin{cases} 6 & \tau = 0 \\ 4 & \tau = \pm 1 \\ 1 & \tau = \pm 2 \\ 0 & |\tau| > 2 \end{cases}\right\} \gamma(\tau)$$

$E(\tau)$

processo stat in senso debole