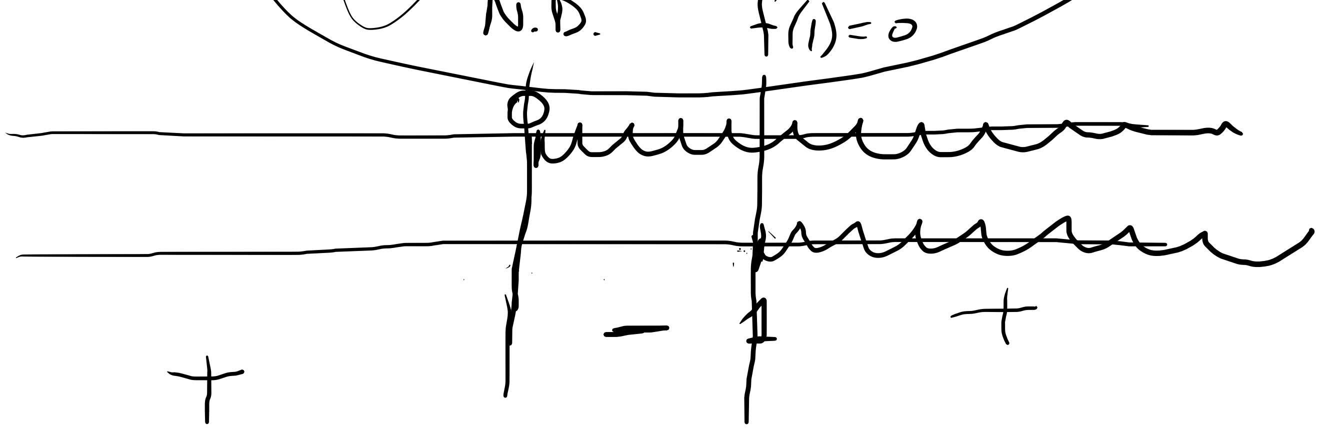
$$f(x) = \sqrt{\frac{x^3-1}{x}}$$

Dom f = ?

$$\frac{x^3-1}{x} \geq 0$$

$$\frac{1}{x} \cdot (x^3-1) \geq 0$$



$$\text{Dom} f = (-\infty, 0) \cup [1, +\infty) = \{x \in \mathbb{R} \quad x<0 \lor x \geq 1\}$$

$$\boxed{x \neq 0}$$

$$\frac{1}{x} > 0 \iff \boxed{x > 0}$$

$$x^3 - 1 \geq 0 \iff x^3 \geq 1$$

$$\boxed{x \geq 1}$$

f non è pari, né dispari, né periodica

$f(x) \geq 0 \quad \forall x \in \text{Dom} f$ $\qquad f(x) = \sqrt{\frac{x^3 - 1}{x}}$

(Infatti $\sqrt{s} \geq 0 \quad \forall s \geq 0$) $\qquad \sqrt{x^2} = |x|$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \sqrt{\frac{x^3 - 1}{x}} = +\infty$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \sqrt{\frac{x^3 - 1}{x}} =$$ $\qquad \lim_{x \to -\infty} f(x) = +\infty$

$$= \lim_{x \to +\infty} \sqrt{\frac{x^{\cancel{3}\,2}(1 - 1/x^3)}{\cancel{x}}} = \lim_{x \to +\infty} x \sqrt{1 - 1/x^3} = +\infty$$

L'asse delle ordinate (di eq. $x=0$) è un SEMI-asintoto verticale (sinistro)

Non vi sono asintoti orizzontali, in quanto

$$\lim_{x \to +\infty} f(x) = +\infty \quad e \quad \lim_{x \to -\infty} f(x) = +\infty$$

Tuttavia non è escluso che possano esistere <u>asintoti obliqui</u> (destro e/o sinistro).

$$\lim_{x \to +\infty} \frac{f(x)}{x} =$$

$$\lim_{x \to +\infty} \frac{\sqrt{\frac{x^3 - 1}{x}}}{x} = \lim_{x \to +\infty} \frac{\sqrt{\frac{\cancel{x^3}^{\,2}\left(1 - \frac{1}{x^3}\right)}{\cancel{x}}}}{x} =$$

$$= \lim_{x \to +\infty} \frac{\cancel{x} \cdot \sqrt{1 - \frac{1}{x^3}}}{\cancel{x}} = 1$$

$$\boxed{m = 1}$$

$$\lim_{x \to +\infty} \left(f(x) - mx \right) = \lim_{x \to +\infty} \left(\sqrt{\frac{x^3 - 1}{x}} - x \right)$$

$$m = 1$$

$$\boxed{\sqrt{\frac{x^3 - 1}{x}} = \frac{x^{\cancel{3}\,2}\left(1 - \frac{1}{x^3}\right)}{\cancel{x}}}$$

$$= \lim_{x \to +\infty} \left(\frac{\cancel{x} \cdot \sqrt{\cancel{x}\left(1 - \frac{1}{x^3}\right)}}{\cancel{x}} - x \right) =$$

$$= \lim_{x \to +\infty} \left[x\sqrt{\cancel{x}\left(1 - \frac{1}{x^3}\right)} - x \right] = \lim_{x \to +\infty} x\sqrt{\cancel{x}\left(1 - \frac{1}{x^3}\right)} - x \cdot \frac{x\sqrt{\cancel{x}\left(1 - \frac{1}{x^3}\right)} + x}{x\sqrt{\cancel{x}\left(1 - \frac{1}{x^3}\right)} + x}$$

$$= \lim_{x \to +\infty} \frac{x^2\left(1-\frac{1}{x^3}\right) - x^2}{x\sqrt{\cancel{x}\left(1-\frac{1}{x^3}\right)} + x} = \lim_{x \to +\infty} \frac{\cancel{x^2} - \frac{1}{x} - \cancel{x^2}}{x \cdot \left[\sqrt{1-\frac{1}{x^3}} + 1\right]} = 0.$$

$$\cancel{= \lim_{x \to +\infty} \frac{x^2\left(\frac{x^3-1}{x^3}\right) - x^2}{x\sqrt{x\left(1-\frac{1}{x^3}\right)} + x}}$$

$q = 0$

$y = mx + q \qquad m = 1$

$y = x$ asintoto obliquo destro

$$\lim_{x \to -\infty} \frac{f(x)}{x} =$$

$$= \lim_{x \to -\infty} \frac{\sqrt{\frac{x^3-1}{x}}}{x} = \lim_{x \to -\infty} \frac{\sqrt{\frac{x^3(1-1/x^3)}{x}}}{x} =$$

$$\sqrt{x^2} = |x|$$

$$= \lim_{x \to +\infty} \frac{-x\sqrt{1-1/x^3}}{x} = -1$$

$$m = -1$$

$$\lim_{x \to -\infty} \left(f(x) - mx \right) = \lim_{x \to -\infty} \left(\sqrt{\frac{x^3-1}{x}} + x \right) =$$

$$\lim_{x\to-\infty}\left(\sqrt{\frac{x^3-1}{x}}+x\right)=$$

$$=\lim_{x\to-\infty}\left((-x)\cdot\sqrt{1-1/x^3}+x\right)=$$

$$=\lim_{x\to+\infty}\left[(-x)\cdot\sqrt{1-1/x^3}+x\right]\cdot\frac{(-x)\cdot\sqrt{1-1/x^3}-x}{(-x)\sqrt{1-\frac{1}{x^3}}-x}=$$

$$=\lim_{x\to+\infty}\frac{x^2\left(\sqrt{1-\frac{1}{x^3}}\right)^2-x^2}{-x\sqrt{1-1/x^3}-x}=\lim_{x\to-\infty}\frac{\cancel{x^2}-\frac{1}{x}-\cancel{x^2}}{-x\sqrt{1-1/x^3}-x}=0$$

$$f(x) = (x-1) \cdot e^{\frac{x-1}{x}}$$

$$\mathrm{Dom}\, f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, +\infty)$$

$$f(-x) = (-x-1) \cdot e^{\frac{-x-1}{-x}} \neq f(x)$$

f non ha simmetria o periodicità.

$$f(x) = 0 \iff x = 1$$

$f(x) > 0 \iff$ $x-1$ e $e^{\frac{x-1}{x}}$ sono concordi $\iff x > 1$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \underbrace{(x-1)}_{\nearrow} \cdot \underbrace{e^{\frac{x-1}{x}}}_{\nearrow} = +\infty$$

Osserviamo

$$\lim_{x\to+\infty} \frac{x-1}{x} = 1$$

$$\lim_{x\to+\infty} (x-1) = +\infty$$

e $\boxed{\lim_{x\to+\infty} e^{\frac{x-1}{x}} = e > 0}$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \underbrace{(x-1)}_{\swarrow} \cdot e^{\frac{x-1}{x}} = -\infty$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \underbrace{(x-1)}\cdot \underbrace{e^{\frac{x-1}{x}}} = -\infty$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x-1)\cdot e^{\frac{x-1}{x}} = 0$$

$$\boxed{\frac{x-1}{x} = 1 - \frac{1}{x}}$$

$$\lim_{x\to 0^-} \frac{x-1}{x} = \lim_{x\to 0^-} \left(1-\frac{1}{x}\right) = +\infty$$

$$\lim_{x\to 0^+} \left(1-\frac{1}{x}\right) = -\infty$$

$$\boxed{\lim_{s\to -\infty} e^{s} = 0}$$

$$\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \left(1 - \frac{1}{x}\right) \cdot e^{\frac{x+1}{x}} = e \qquad (m = e)$$

$$\lim_{x \to +\infty} \left(f(x) - ex \right) =$$

$$= \lim_{x \to +\infty} \left((x-1) \cdot e^{\frac{x+1}{x}} - e \cdot x \right) =$$

$$= \lim_{x \to +\infty} \left\{ \underbrace{x \cdot \left[e^{\frac{x+1}{x}} - e \right]} - e^{\frac{x+1}{x}} \right\}$$

$$\lim_{x \to +\infty} x \cdot \left[e^{\frac{x+1}{x}} - e \right] = ?$$

$$e^{\frac{x+1}{x}} - e = \left(e^{\frac{x+1}{x}} - e \right) \cdot \frac{e^{\frac{x+1}{x}} + e}{e^{\frac{x+1}{x}} + e} =$$

$$= \frac{e^{\left(\frac{x+1}{x}\right)^2} - e^2}{e^{\frac{x+1}{x}} + e}$$

$$x \cdot \left[e^{\frac{x+1}{x}} - e \right] =$$

$$x \cdot \left[e^{1+\frac{1}{x}} - e \right] = x \cdot \left[\underline{\underline{e}} \cdot e^{1/x} - \underline{\underline{e}} \right] =$$

$$= e \cdot x \cdot \left[e^{1/x} - 1 \right] = e \cdot \left[\frac{e^{1/x} - 1}{1/x} \right]$$

$$e \;\cancel{:}\; \frac{e^{1/x}-1}{1/x} \xrightarrow[x\to+\infty]{} e$$

$$1/x \to 0 \quad \text{quando } x \to +\infty$$

$$\boxed{\lim_{t\to 0} \frac{e^t-1}{t} = 1}$$

$y = ex + e$ asintoto obliquo destro

Per esercizio

Trovare, se esiste, l'equazione dell'asintoto obliquo sinistro di f.

Def Sia $f : \mathcal{D}om f \rightarrow \mathbb{R}$ una funzione reale di variabile reale

x_0 punto INTERNO del dominio

di f $\mathcal{D}om f$.

Diremo che f è <u>continua</u> in x_0, se

$$\boxed{\lim_{x \to x_0} f(x) = f(x_0)}$$

.

$y = -x$ è l'eq. dell'asintoto obliquo sinistro