

MATHEMATICS CLASS

November 16, 2020

Exercise 1. Determine the image set of each function, then draw its graph and determine its inverse function.

$$f(x) = \begin{cases} -(x-1)^2 & \text{if } x < 1 \\ \sqrt{x-1} & \text{if } x \geq 1 \end{cases} & g(x) = \begin{cases} x^2 - 1 & \text{if } x < 0 \\ -\sqrt{x} - 1 & \text{if } x \geq 0 \end{cases}$$

$$h(x) = \begin{cases} x^3 - 2 & \text{if } x < 0 \\ \sqrt{x} + 2 & \text{if } x \geq 0 \end{cases} & i(x) = \begin{cases} \frac{x}{x+1} & \text{if } x < -1 \\ 1 - (x+1)^2 & \text{if } x \geq -1 \end{cases}$$

Exercise 2. Determine the domain of the following function:

$$f(x) = \arcsin\left(\frac{1}{\sqrt{x+5} - \sqrt{x}}\right).$$

Exercise 3. Consider the following function, depending on the parameters a and b :

$$f: \mathbb{R} \rightarrow \mathbb{R}, \quad f(x) = \begin{cases} -x + 4 & \text{if } x < -2, \\ -2\sin(ax) + b & \text{if } -2 \leq x \leq 0, \\ x^a - 2x & \text{if } 0 < x. \end{cases}$$

- For which values of a and b the conditions $f(-1) = 2$ and $f(3) = -5$ are satisfied?
- Draw the graph of the function obtained.
- Is the function injective? Is it surjective? If not, determine a modification of f in the interval $[-2, 0[$ so that to obtain a monotone function.

Exercise 4. Recognise which graphs represent monotone functions and, for each of the remaining ones, determine the maximal intervals of monotonicity.

