Image Processing for Physicists

Prof. Pierre Thibault

pthibault@units.it

Overview

- The phase problem
- In-line holography
- Off-axis holography
- Other interferometric imaging methods
- Far-field amplitude measurements

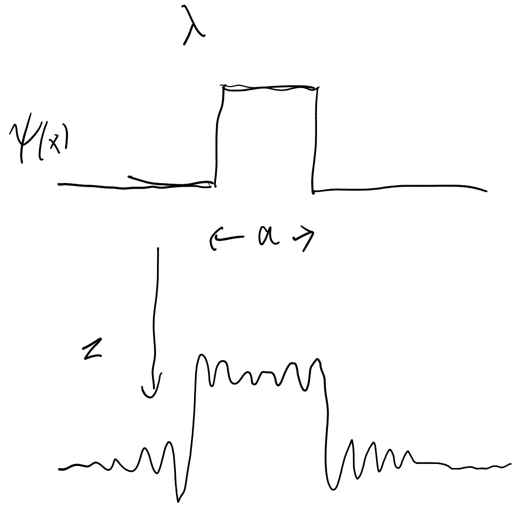
Wave propagation

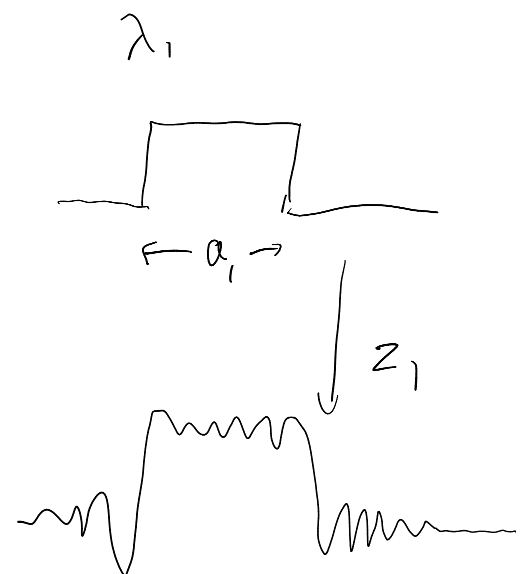
Near-field

$$\psi(\vec{r}; z) = \mathcal{F}\{\mathcal{F}\{\psi(r; z=0)\}\exp(-i\pi \underbrace{u^2 \lambda z})\}$$

2D coordinate

$u\sqrt{\lambda z}$ is unitless





$$\frac{a^2}{\lambda z} = f$$

"Fresnel number"

mathematically identical as long as

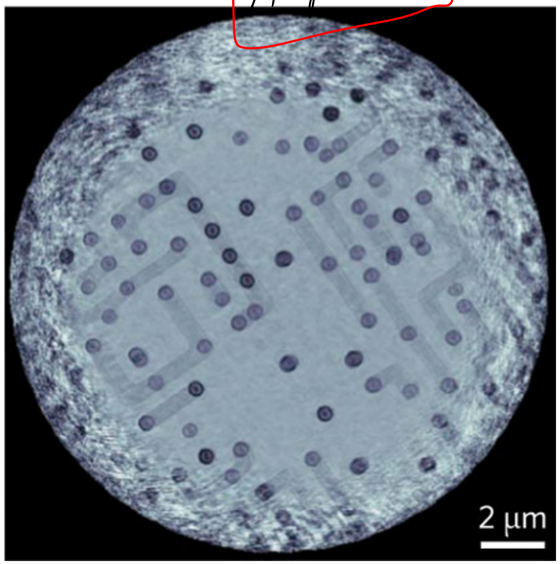
$$\frac{1}{a}\sqrt{\lambda z} = \frac{1}{a_1}\sqrt{\lambda_1 z_1}$$

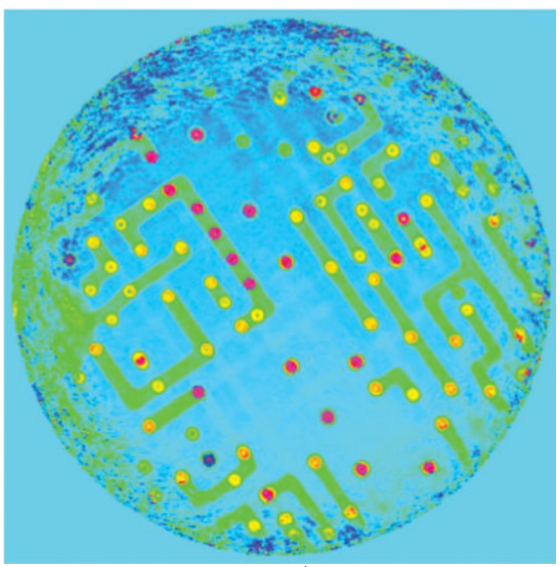
$f \ll 1$: Far-field

$f \gg 1$: Near-field

Complex-valued images

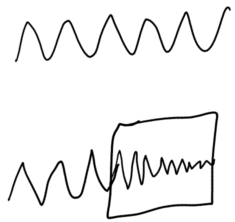
Amplitude





Phase

X-ray transmission function



wave travelling trough material

thickness

$$\psi_{out} = \psi_{in} \exp\left(ik(n-1)t\right)$$

attenuation

$$n = n_r + i n_i$$

usual index of refraction

$$\psi_{out} = \psi_{in}\, e^{\boxed{i k n_r t}} \boxed{e^{-k n_i t}}$$

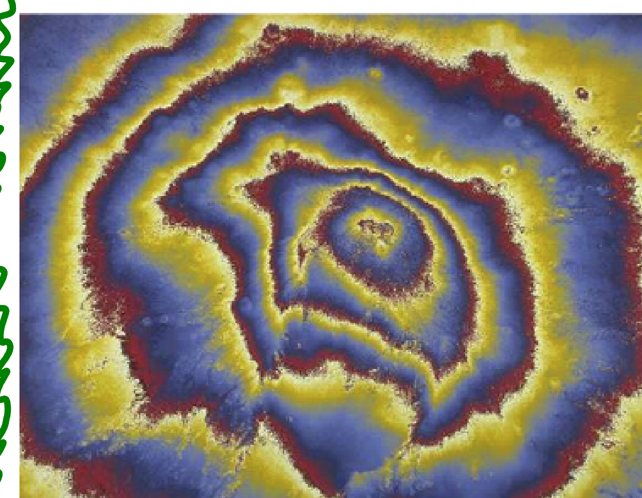
Beer - Lambert law

Synthetic aperture radar (SAR)

interferogram

phase ↔ distance

amplitude ↔ dispersion absorption



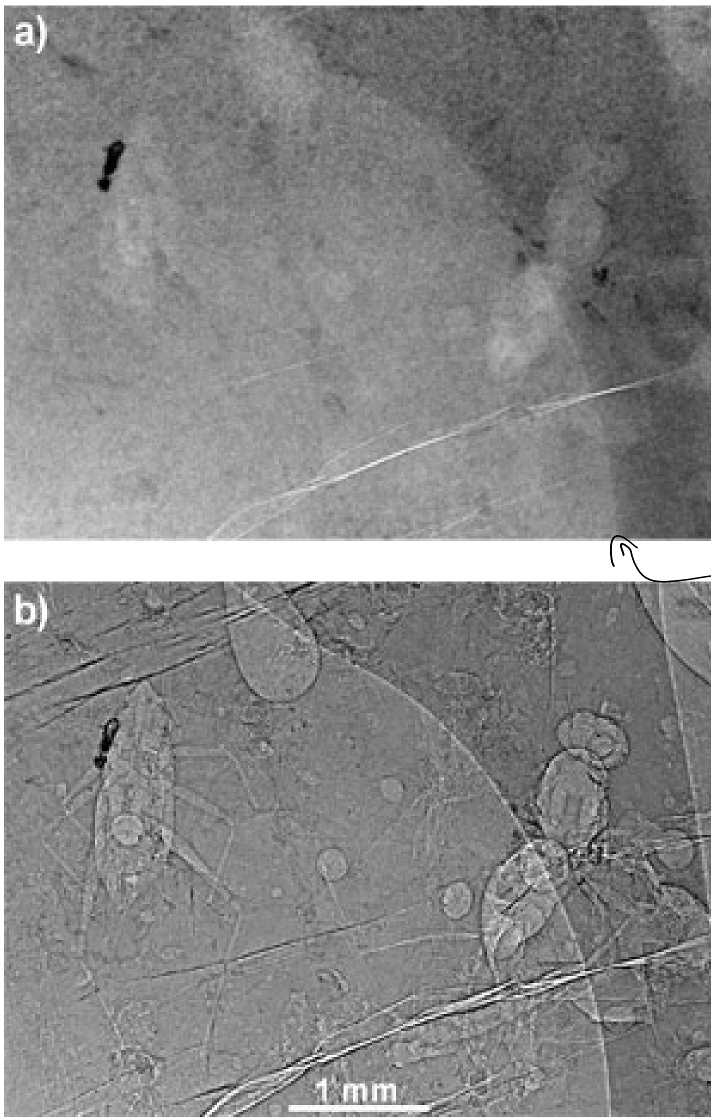
(Etna)

Interferometric imaging

Phase-contrast

Zernike Nobel for phase contrast

Hard X-ray propagation-based phase contrast



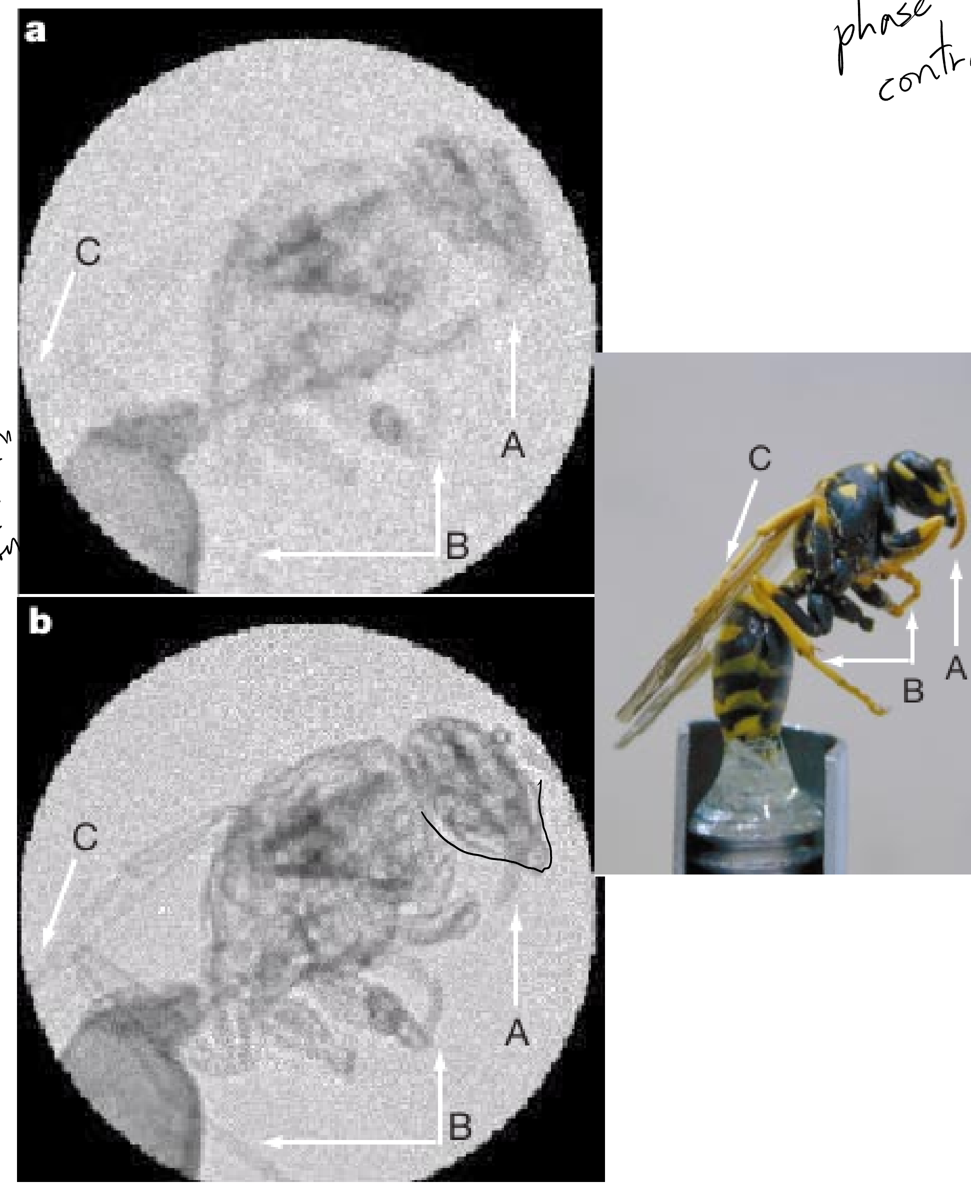
Large Fresnel number

"contact measurement" no propagation

after some propagation

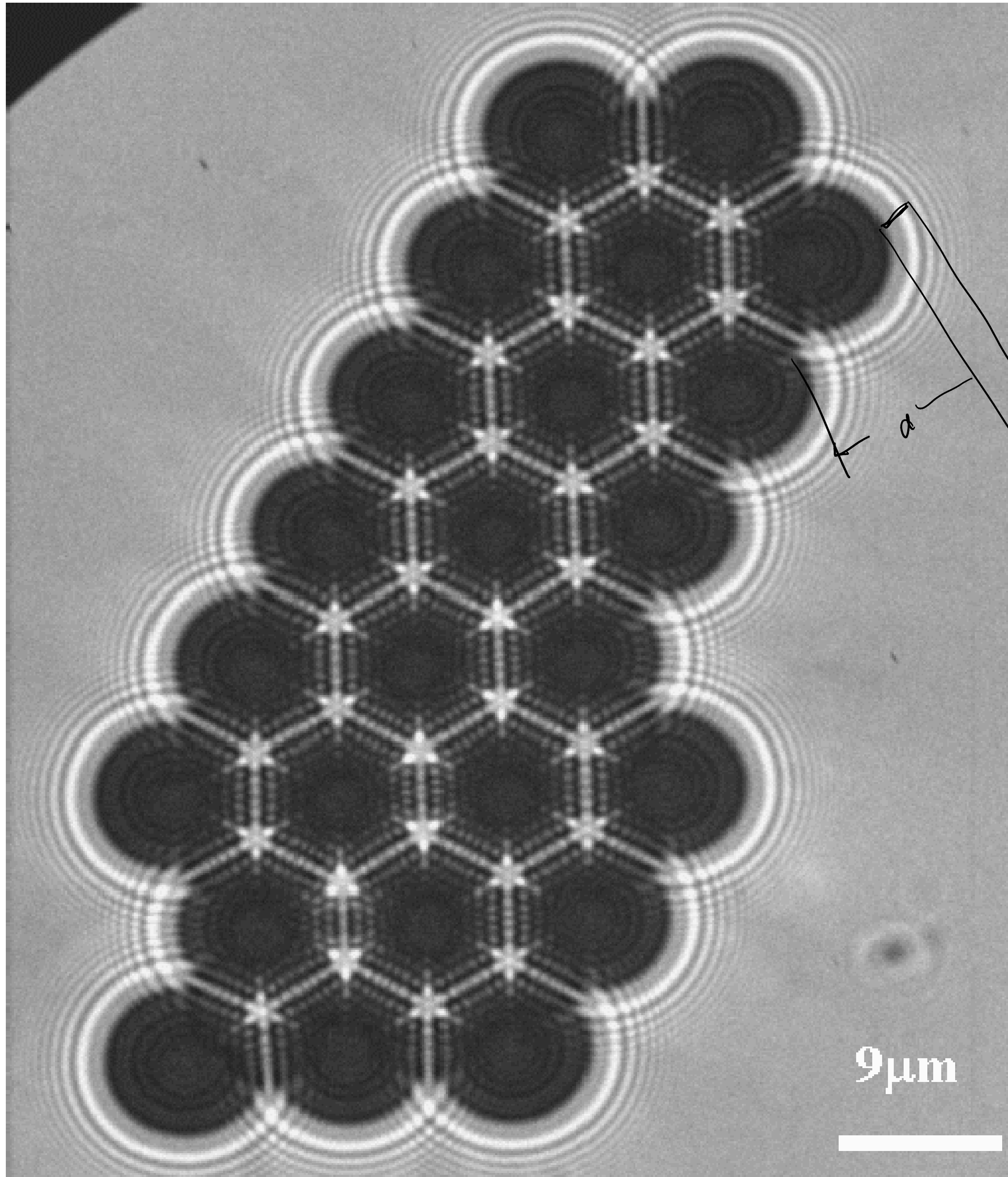
Source: www.esrf.eu/news/general/amber/amber/

Neutron phase contrast



Source: Allman et al. Nature **408** (2000).

In-line holography



Source: Mayo et al. Opt Express **11** (2003).

Towards smaller Fresnel number

a

$\sim \sqrt{\lambda z}$ dimension of the "first Fresnel zone"

if a = diameter

$$\frac{a^2}{\lambda z} = \left(\frac{a}{\sqrt{\lambda z}}\right)^2 \simeq 10$$

In-line holography

Measured: $I(\vec{r}) = |\psi(\vec{r};z)|^2$

weak phase object

common model: $\psi(r;z=0) = A(1+\varepsilon(r))$

$$\psi(r;z) = A(1+\varepsilon(r;z))$$

$$I(\vec{r}) = |A|^2\Big(1 + \varepsilon(r;z) + \underbrace{\varepsilon^*(r;z)} + \underbrace{O(\varepsilon^2)}_{\text{negligible}}\Big)$$

ε propagated by distance z

ε propagated by distance $-z$

twin image problem

The phase problem

The problem: we can measure only the squared amplitude of a wave.

Remember Q.M.: $\psi \leftarrow$ probability amplitude (complex-valued)

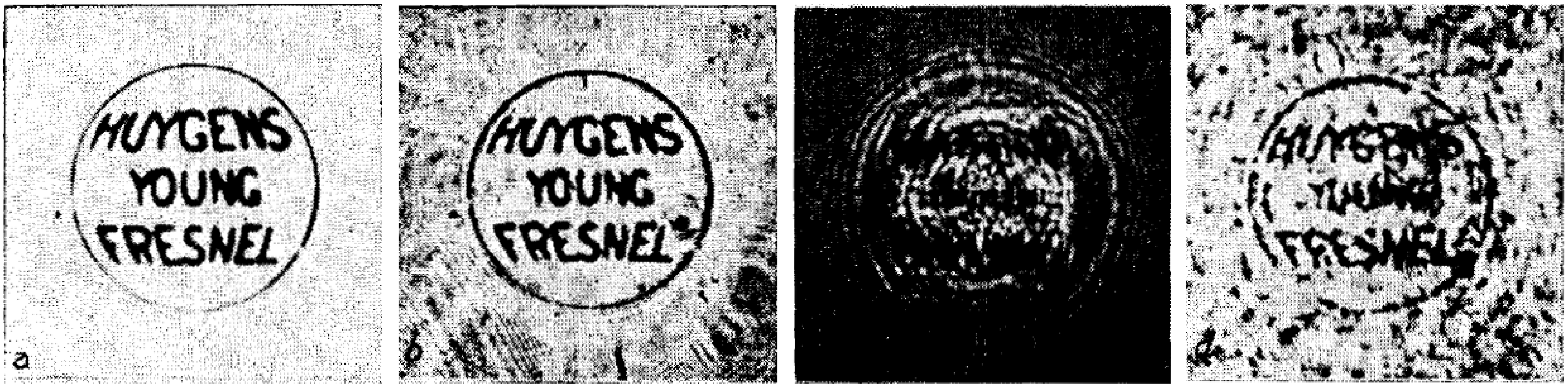
measurement: $I = |\psi|^2 \leftarrow$ probability distribution

$$|\psi| = \sqrt{I}$$

Sometimes: - phase is the interesting quantity $\exp(ik(n-1)t)$

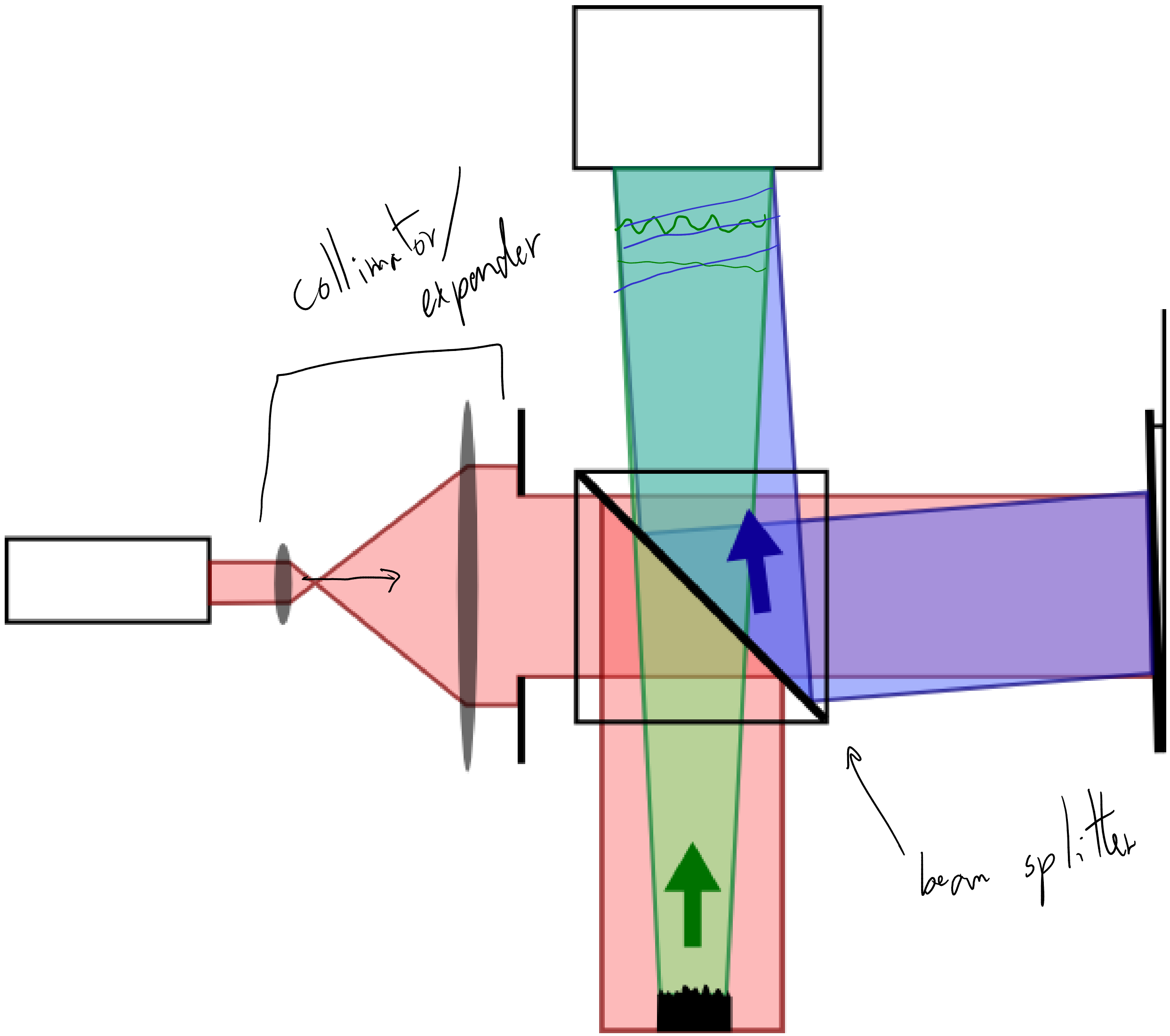
- phase is just a way to propagate back in the sample plane

In-line holography



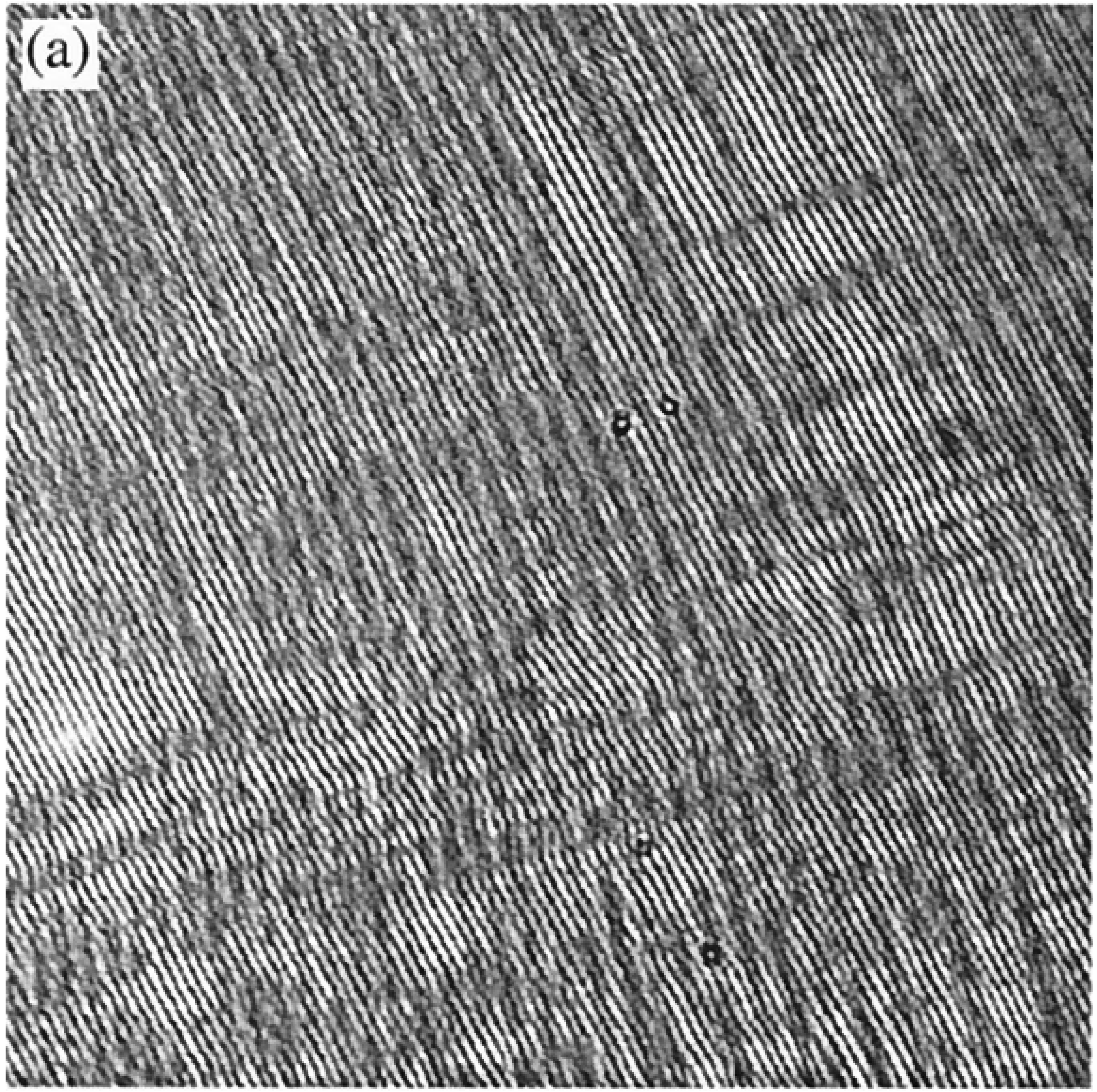
D. Gabor, Nature **161**, 777-778 (1948).

Fringe interferometry



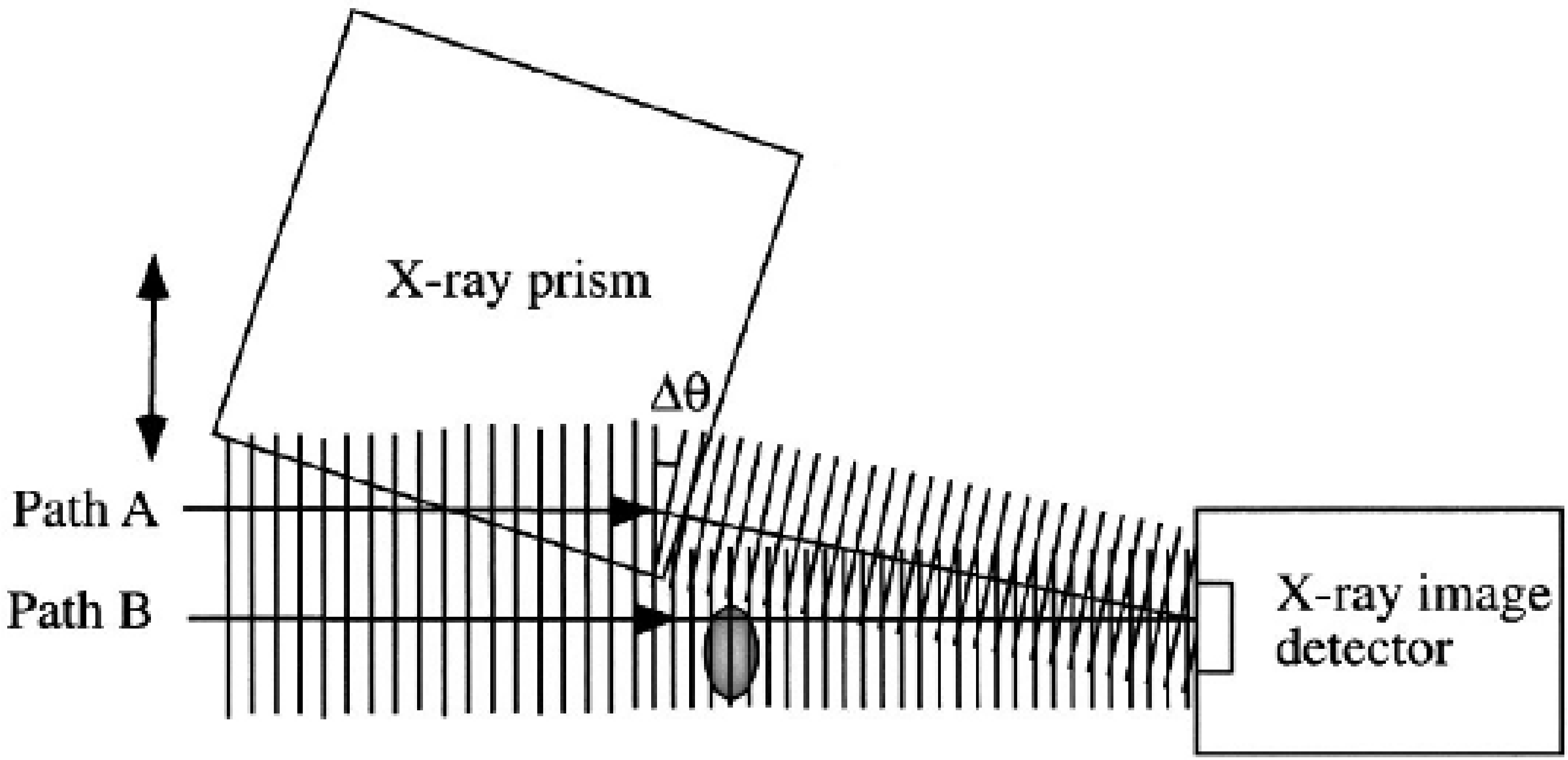
Twyman-Green interferometer

Fringe interferometry



Source: Cuche et al. Appl. Opt. **39**, 4070 (2000)

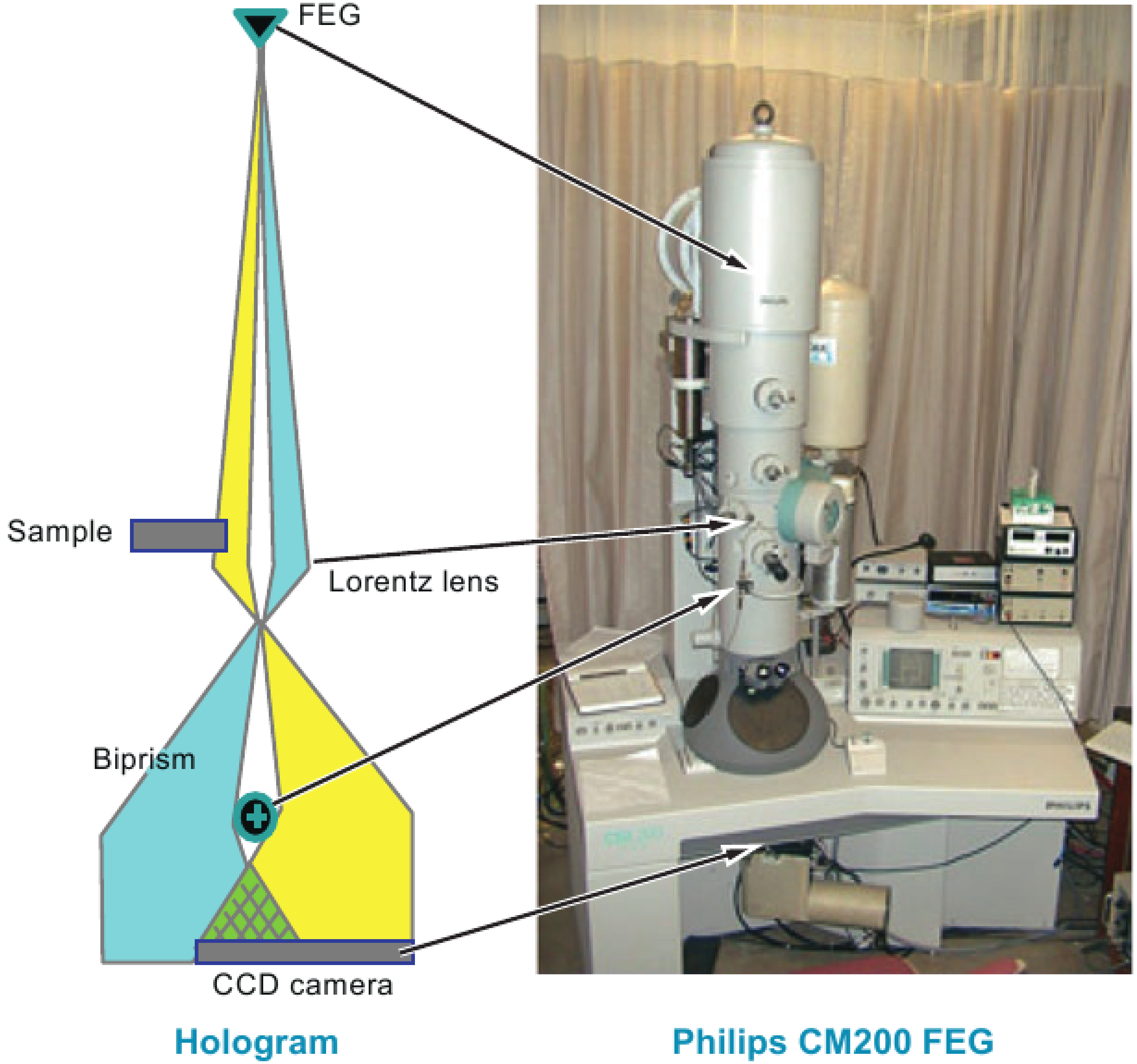
Off-axis X-ray holography



Source: Y. Kohmura, J. Appl. Phys. **96**, 1781-1784 (2004)

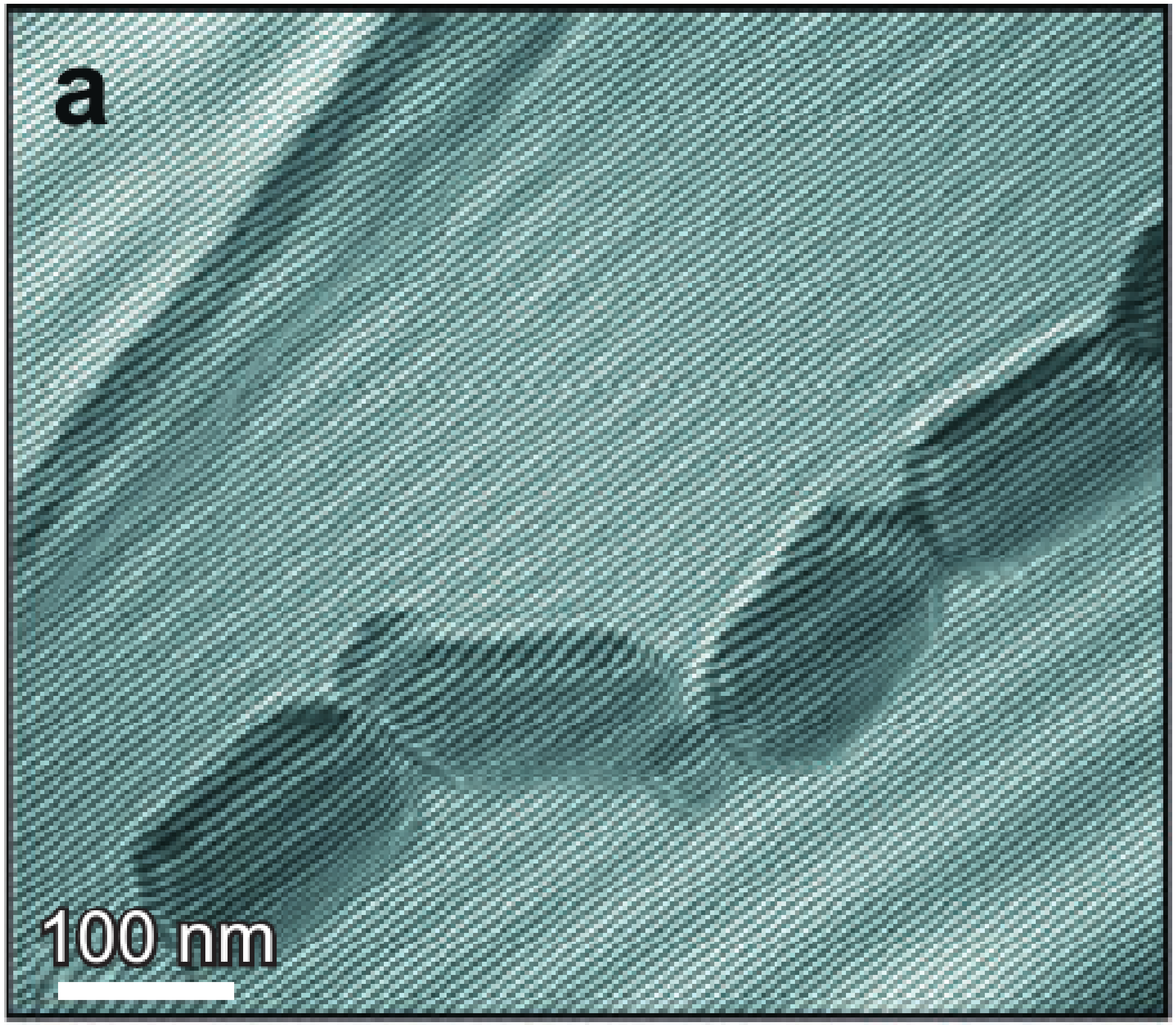
Off-axis electron holography

Electron microscopy



Source: M. R. McCartney, Ann. Rev. Mat. Sci. **37** 729-767 (2007)

Off-axis electron holography



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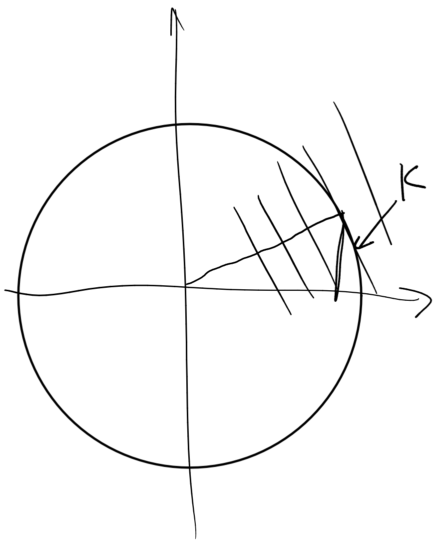
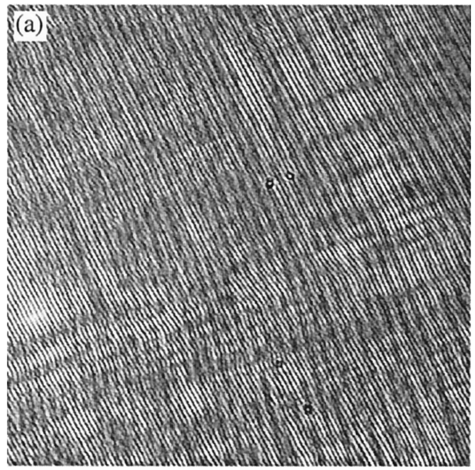
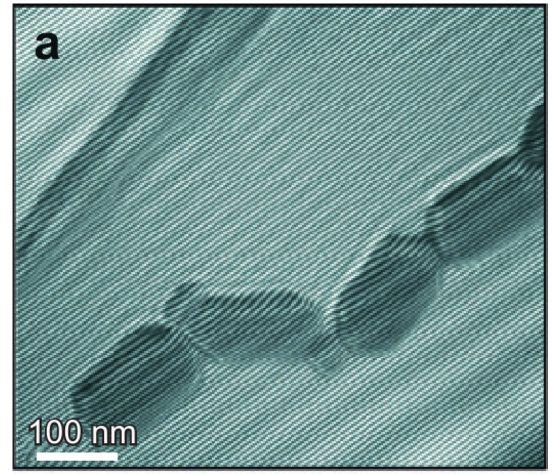
Fringe interferometry

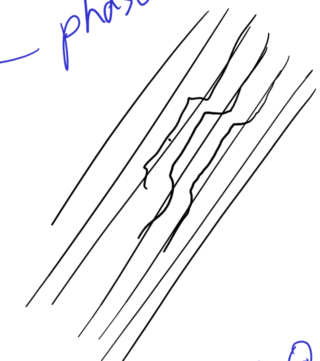






$\vec{K}$: perpendicular component of the wave vector

phase stepping



$$\psi = \psi_0 + \psi_r$$

object reference

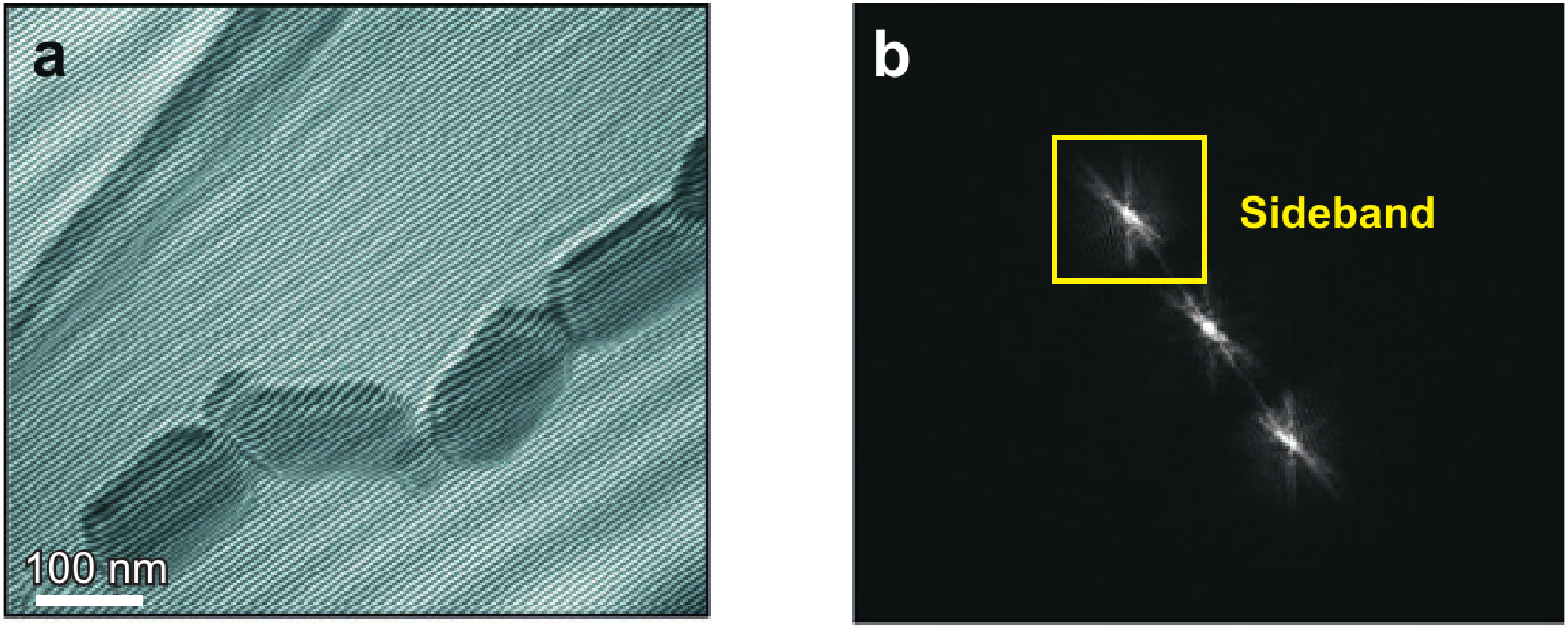
$$\psi_0(r) = a(r)\, e^{i(\varphi(r) + \theta)}$$

$$\psi_r(r) = A\, e^{i\vec{K}\cdot\vec{r}}$$

Measured $|\psi(r)|^2 = \underbrace{|a(r)|^2 + |A|^2}_{\text{smooth}} + \underbrace{2\,a(r)\, A \cos(\vec{K}\cdot\vec{r} - \varphi(\vec{r}) + \theta)}_{\text{fringes}}$

phase "encoding"

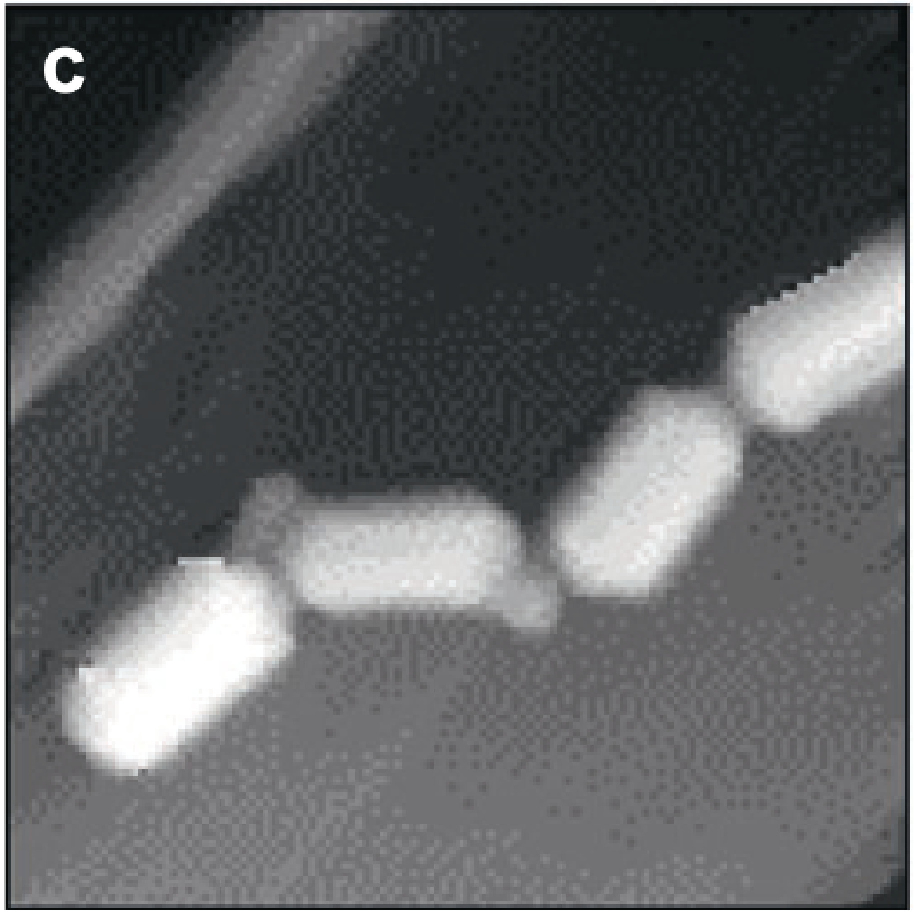
Off-axis holography

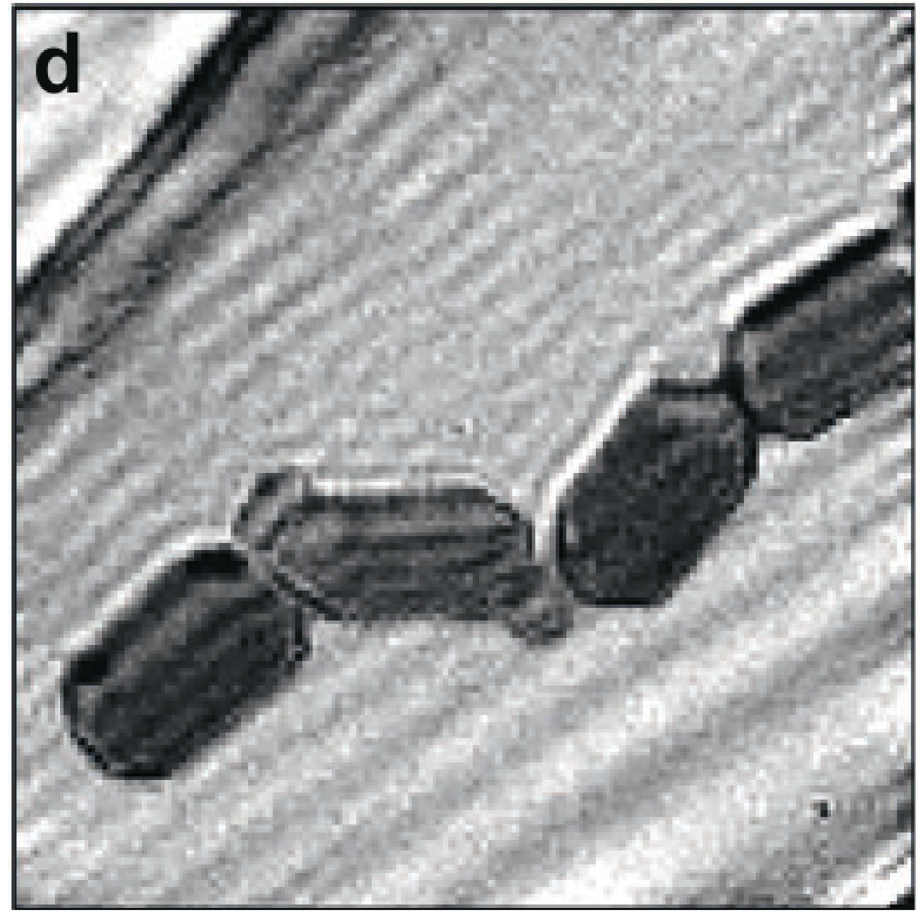


Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

Off-axis holography

Phase $\varphi(r)$ Amplitude $a(r)$





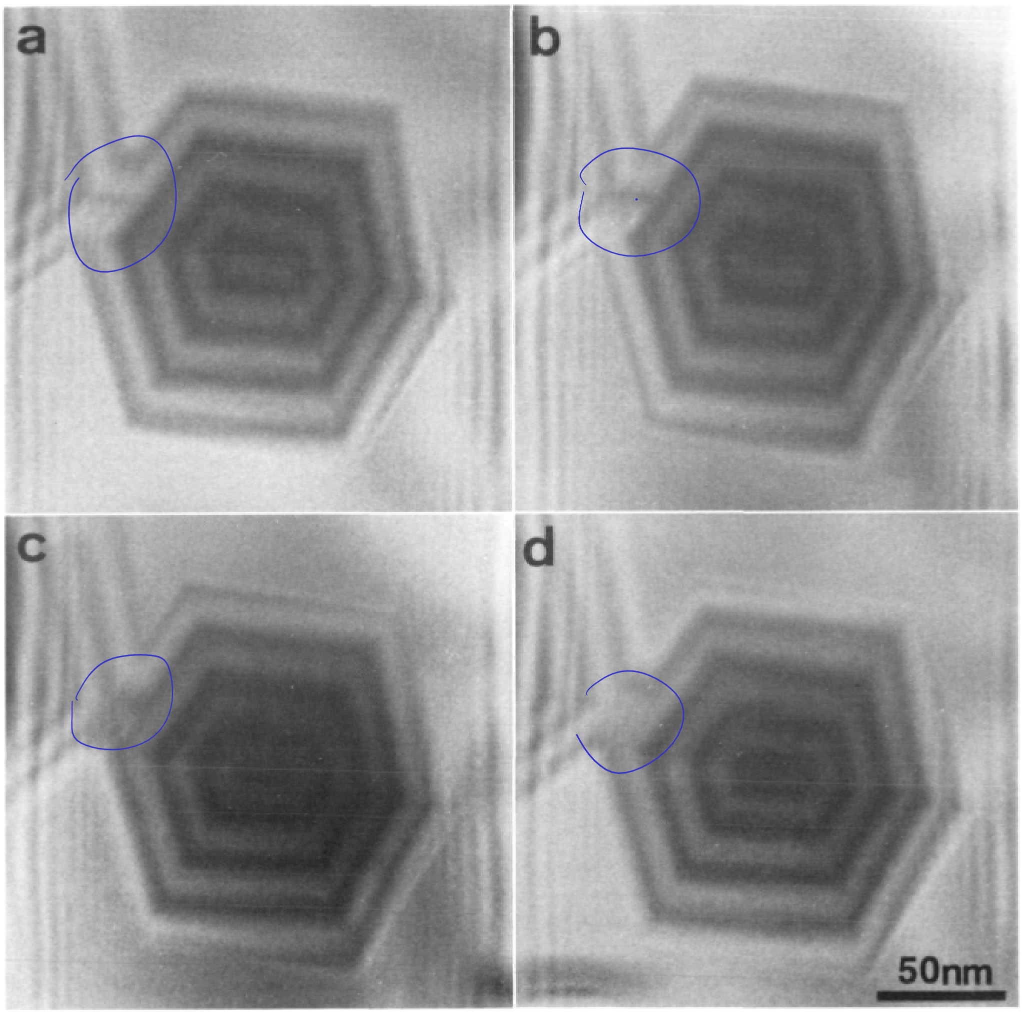
Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

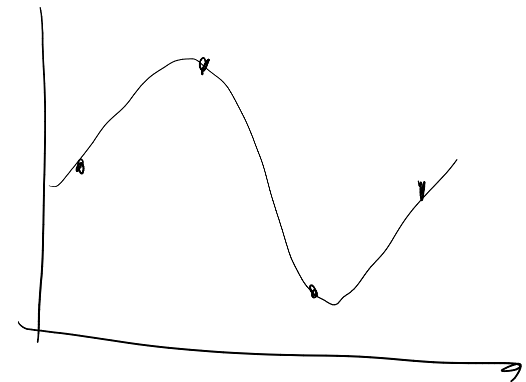
Phase stepping

- Encoding phase **and** amplitude in a single image has a price: resolution

  $\rightarrow$ Take more than one image, changing the reference in each.

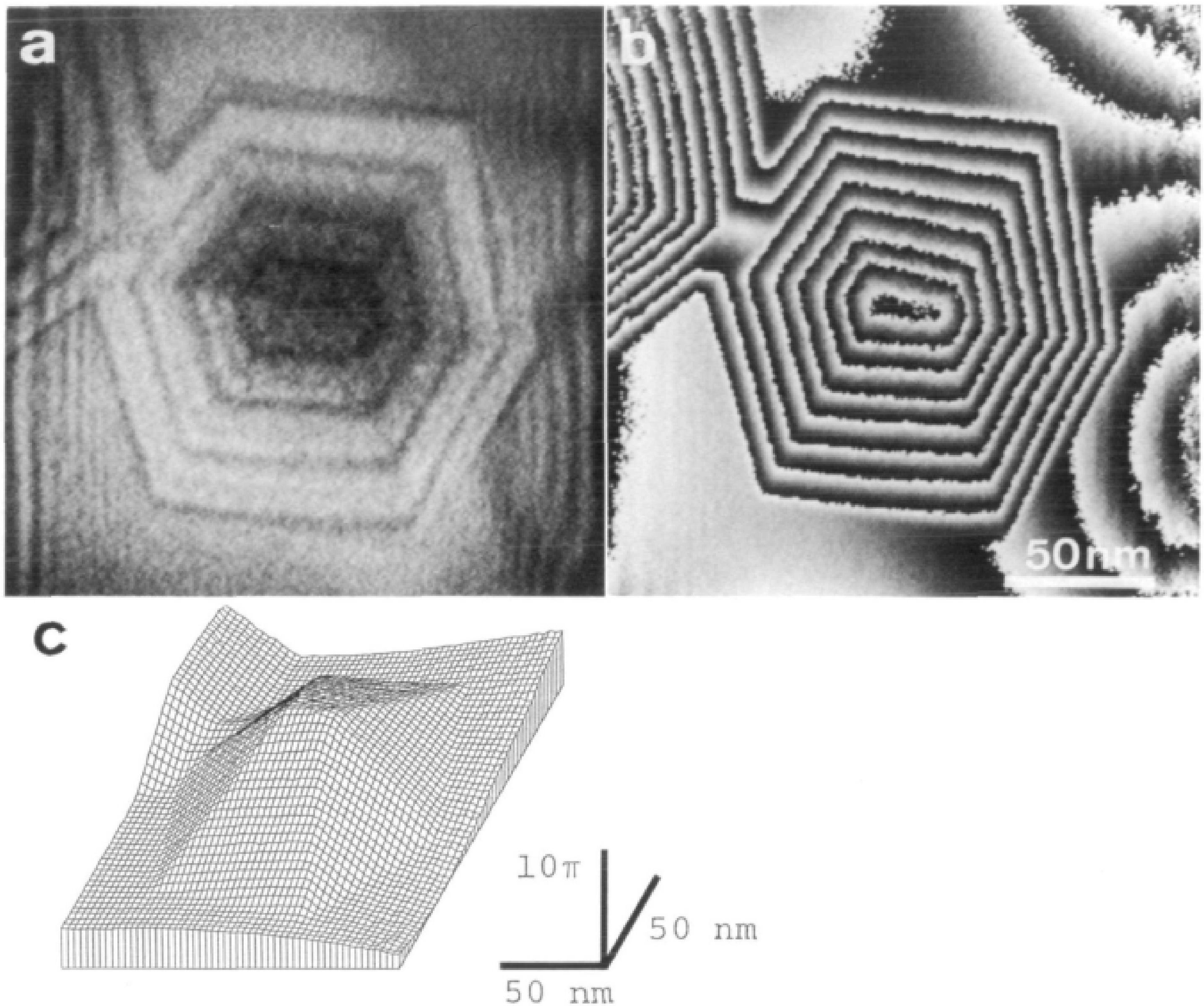
Fringe scanning





Source: K. Harada, J. Electron Microsc. **39** 470-476 (1990)

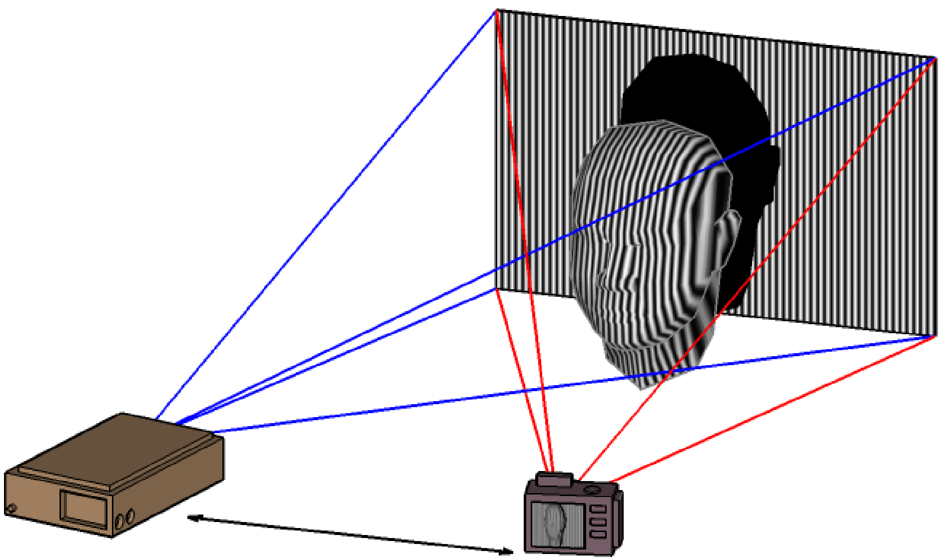
Fringe scanning

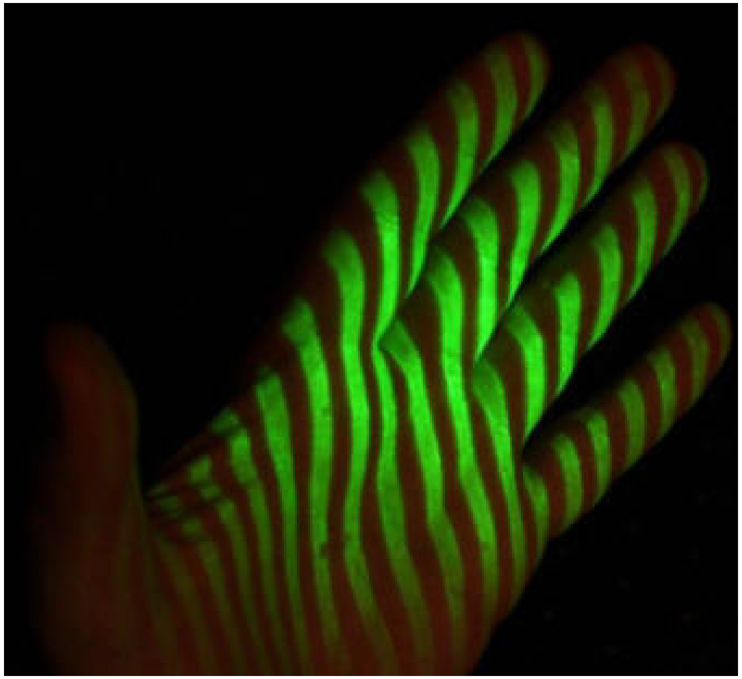


Source: K. Harada, J. Electron Microsc. **39** 470-476 (1990)

Structured light sensing

- Project a structured light pattern onto sample
- Distortions of light pattern allow reconstruction of sample shape





Phase unwrapping

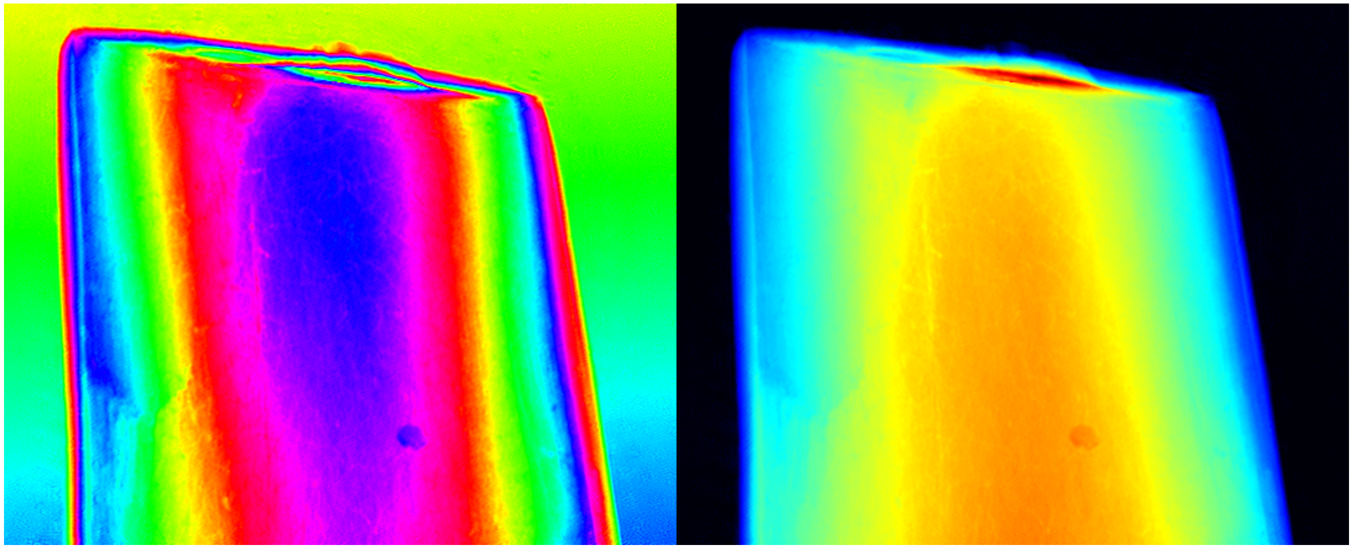
- Phase is measured only in the interval $[0, 2\pi)$
- Physical phase shifts (which can be larger) are wrapped on this interval

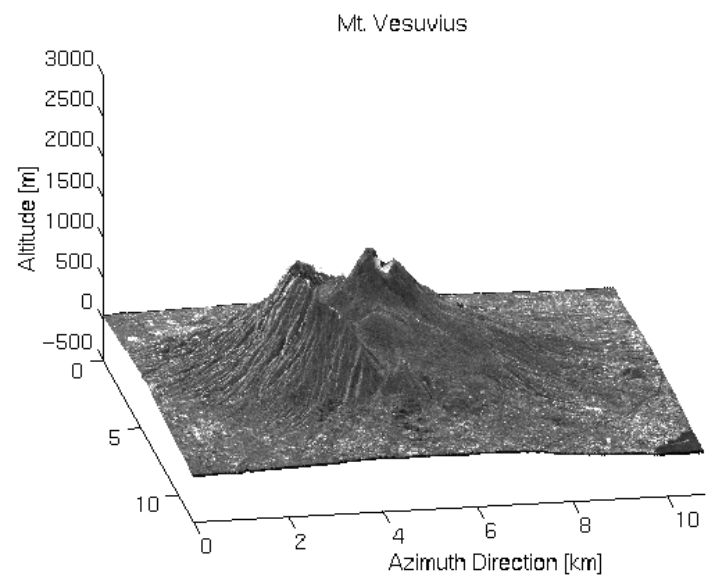
  $\rightarrow$ Any multiple of 2π is possible

- Unwrapping: use correlations in the image to guess the total phase shift.
- Main difficulties:
  - aliasing: phase shifts are too rapid for the image sampling
  - noise: produces local singularities (vortices)
- Many strategies exist

Complex-valued images

Phase unwrapping

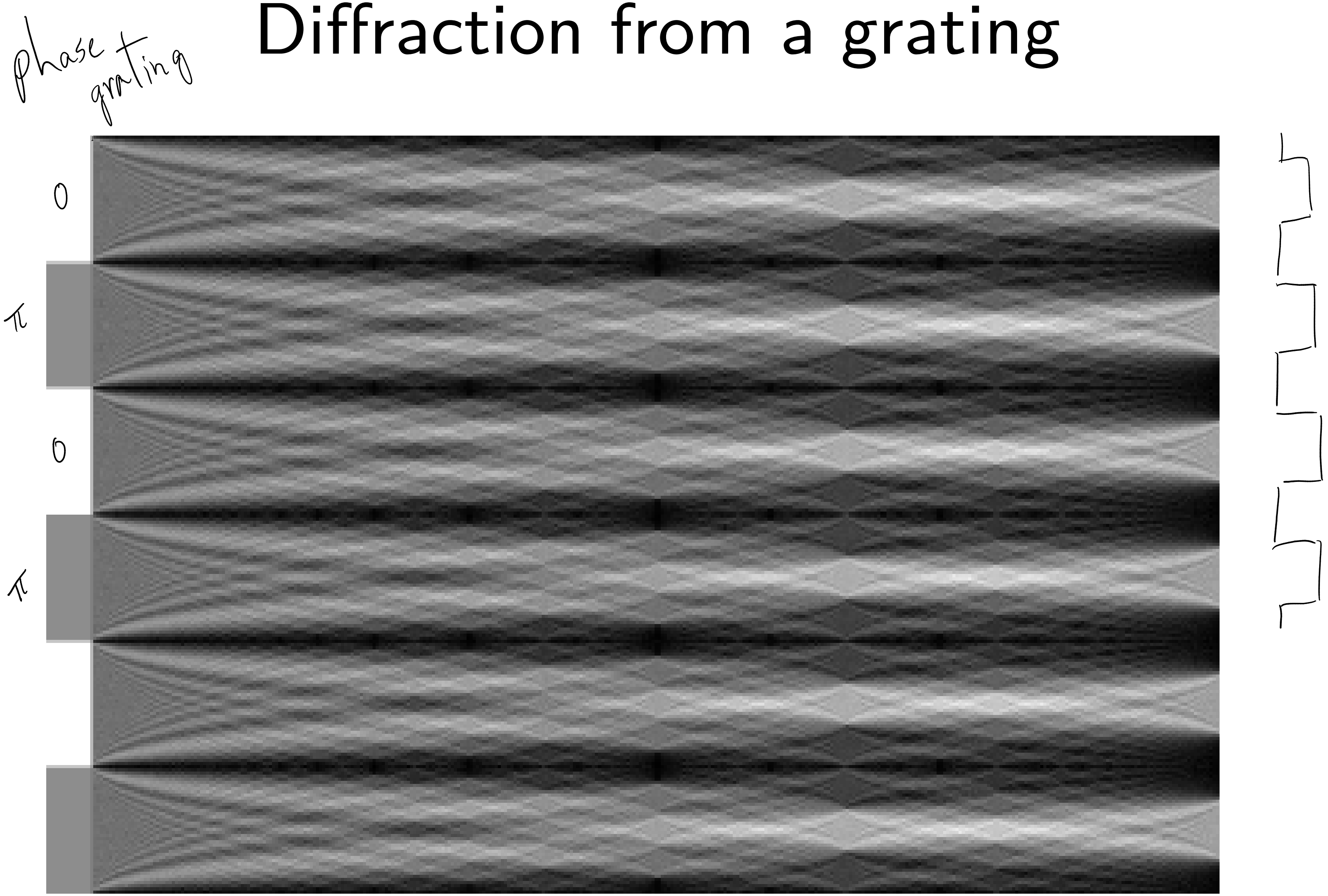




Source: http://earth.esa.int/workshops/ers97/program-details/speeches/rocca-et-al/

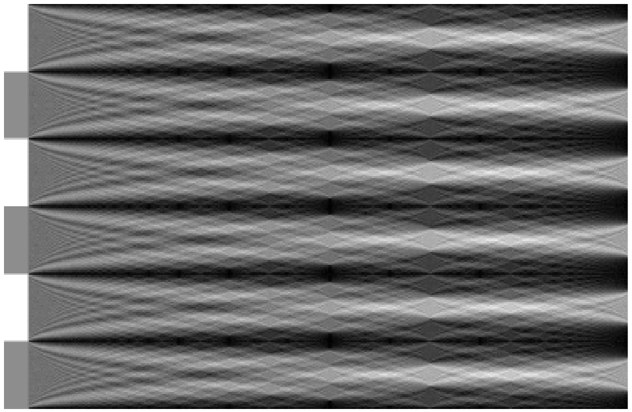
Grating interferometry

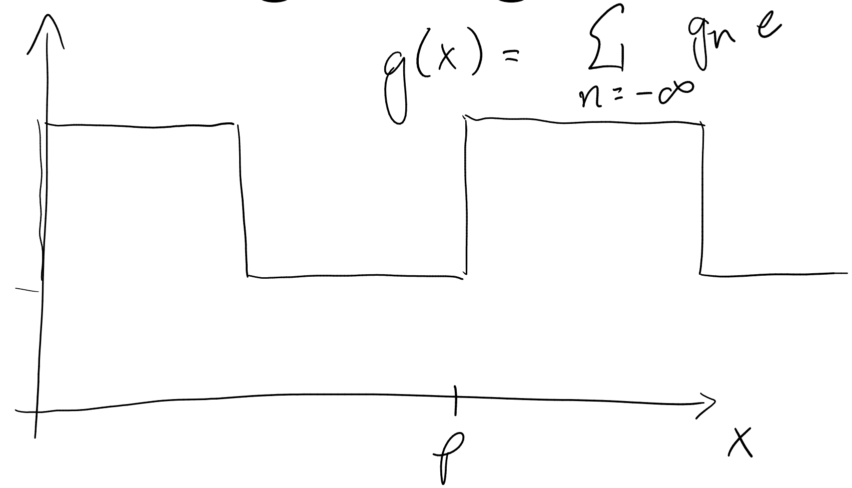
Diffraction from a grating



Grating interferometry

Diffraction from a grating

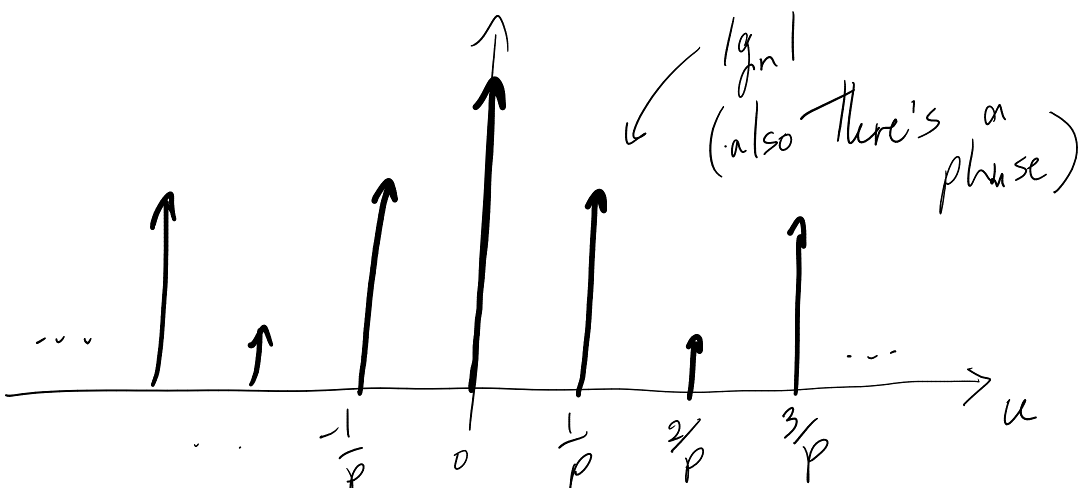




$$g(x) = \sum_{n=-\infty}^{\infty} g_n e^{2\pi i x n/p}$$

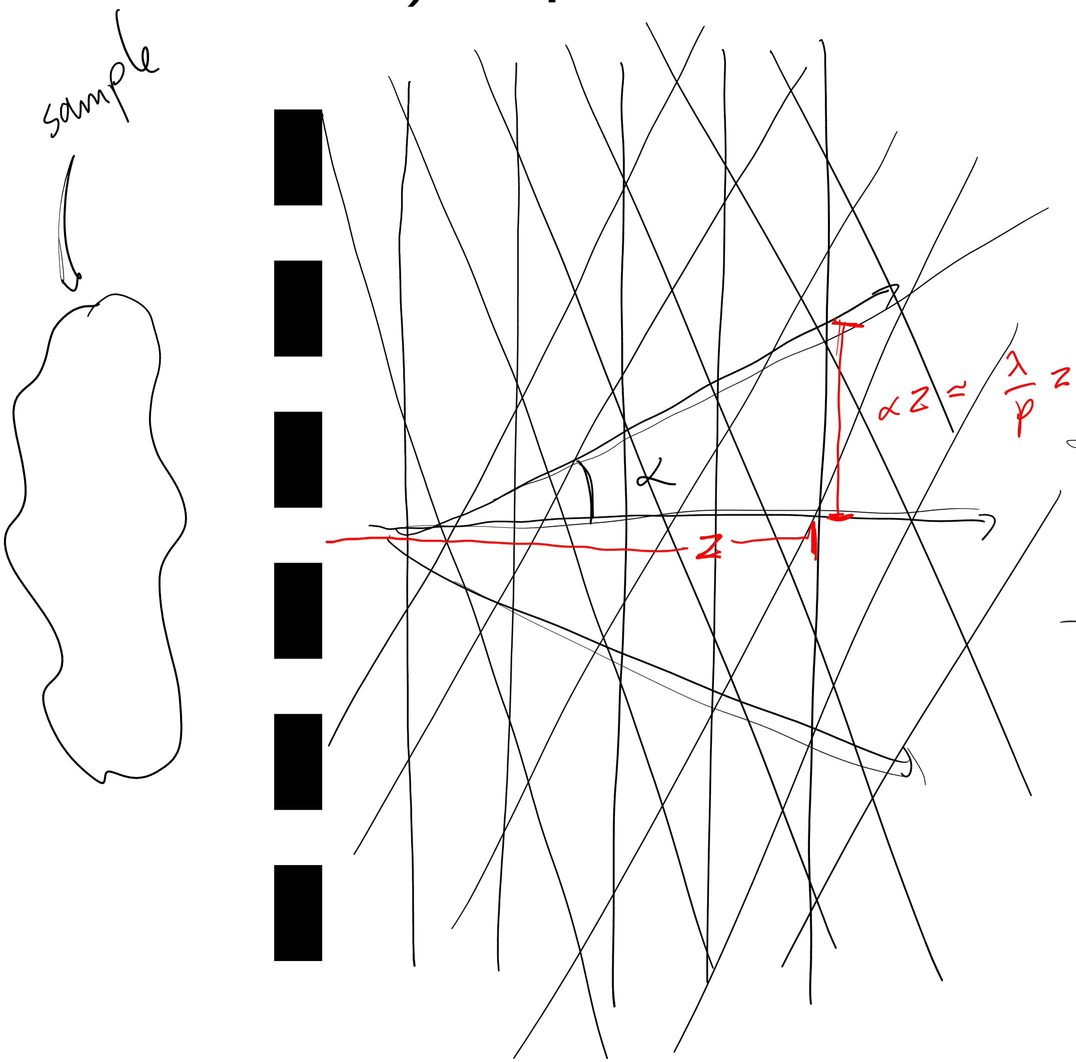
$$G(u) = \sum_{n=-\infty}^{\infty} g_n \, \delta(u - n/p)$$

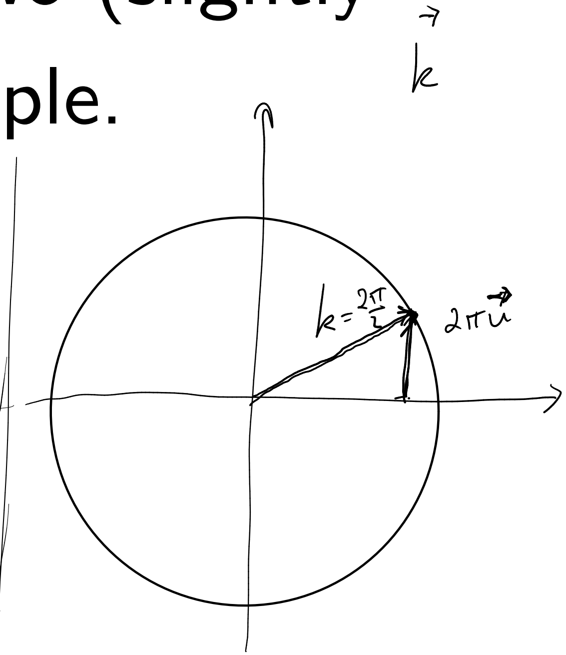
"Dirac comb" (almost)



Grating interferometry

Observing the interference between two (slightly offset) copies of the same sample.





$\left(e^{i \vec{u}_0 \cdot \vec{r}} \right.$ ← leads to translation in Fourier space)

$$\alpha = \frac{2\pi/p}{2\pi/\lambda} = \lambda/p$$

Grating interferometry

Observing the interference between two (slightly offset) copies of the same sample.

e.g. if only orders +1 and −1 are relevant:

$$\psi(r;z) = \psi_0\left(\vec{r} + \frac{\lambda}{p} z \hat{x}\right) e^{i 2\pi x/p} + \psi_0\left(r - \frac{z\lambda}{p}\hat{x}\right) e^{-i2\pi x/p}$$

$$\psi_0(\vec{r}) = a(\vec{r})\, e^{i\varphi(\vec{r})}$$

$$I(r;z) = |\psi(r;z)|^2 = \left|\psi_0\left(\vec{r} + \frac{z\lambda}{p}\hat{x}\right)\right|^2 + \left|\psi_0\left(\vec{r} - \frac{z\lambda}{p}\hat{x}\right)\right|^2$$

$$+ \psi_0\left(\vec{r} + \frac{z\lambda}{p}\hat{x}\right)\psi_0^*\left(r - \frac{z\lambda}{p}\hat{x}\right) e^{4\pi i\, x/p} + \text{C.C.}$$

"differential phase contrast"

(if $\frac{z\lambda}{p}$ is small)

$$= 2a^2(\vec{r}) + 2\underbrace{a\left(\vec{r} + \frac{z\lambda}{p}\hat{x}\right) a\left(\vec{r} - \frac{\lambda z}{p}\hat{x}\right)}_{\simeq a^2(\vec{r})} \cos\left[\overbrace{\varphi\left(r + \frac{z\lambda}{p}\right) - \varphi\left(r - \frac{z\lambda}{p}\right)}^{\simeq 2\frac{z\lambda}{p}\frac{\partial}{\partial x}\varphi} + 4\pi x/p\right]$$