Image Processing for Physicists

Prof. Pierre Thibault

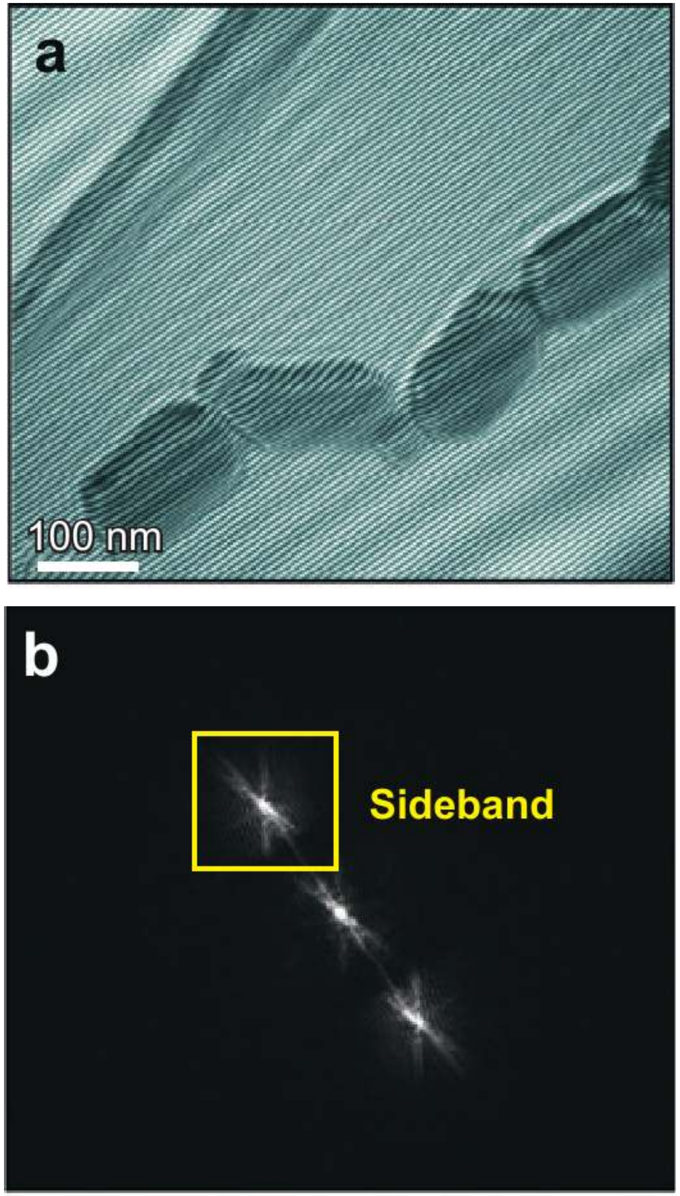
pthibault@units.it

Overview

- Imaging using far-field amplitude measurements
  - Fourier transform holography
  - Coherent diffraction imaging
  - Ptychography

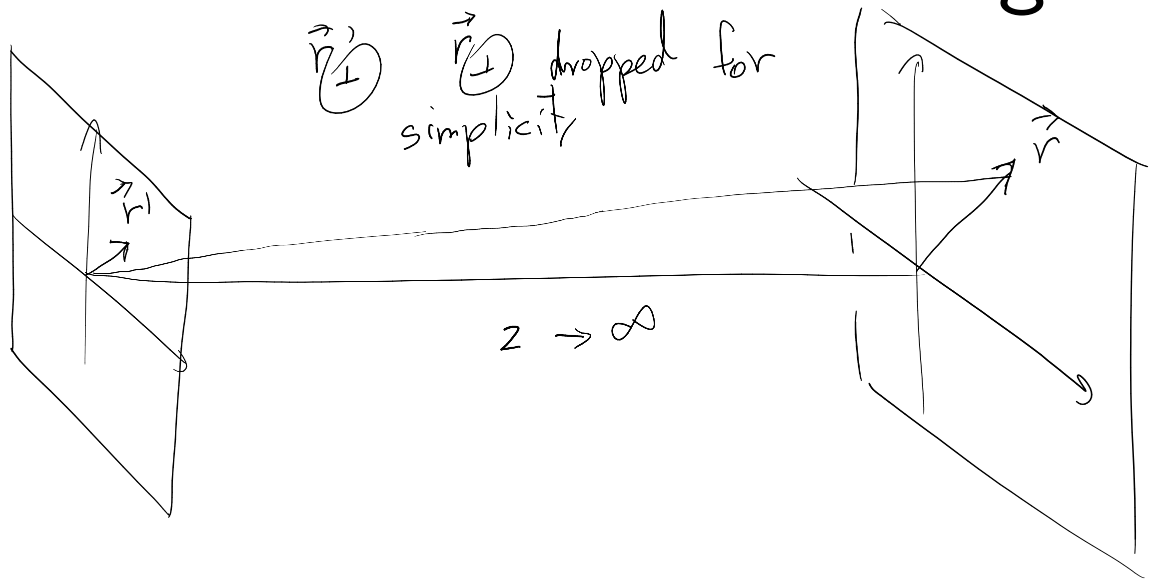
Last week...

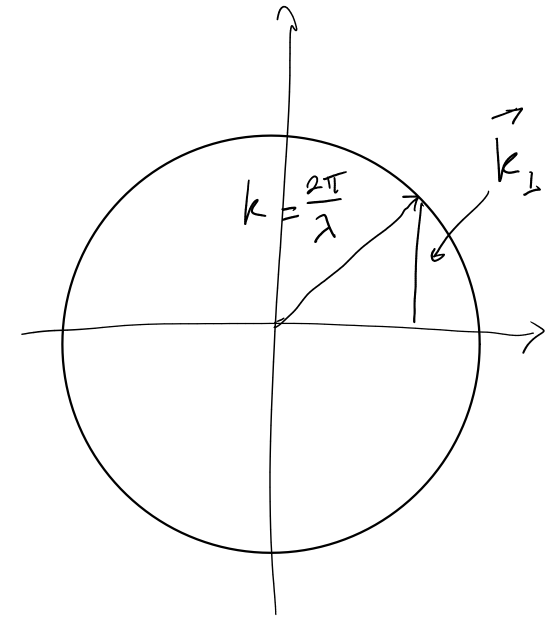
- Off-axis holography:
  - Key idea: encoding a complex-valued wavefield through interference with a (known) tilted planar wave front.
  - Analysis: sidebands in Fourier space



Far-field diffraction

The Fraunhofer regime



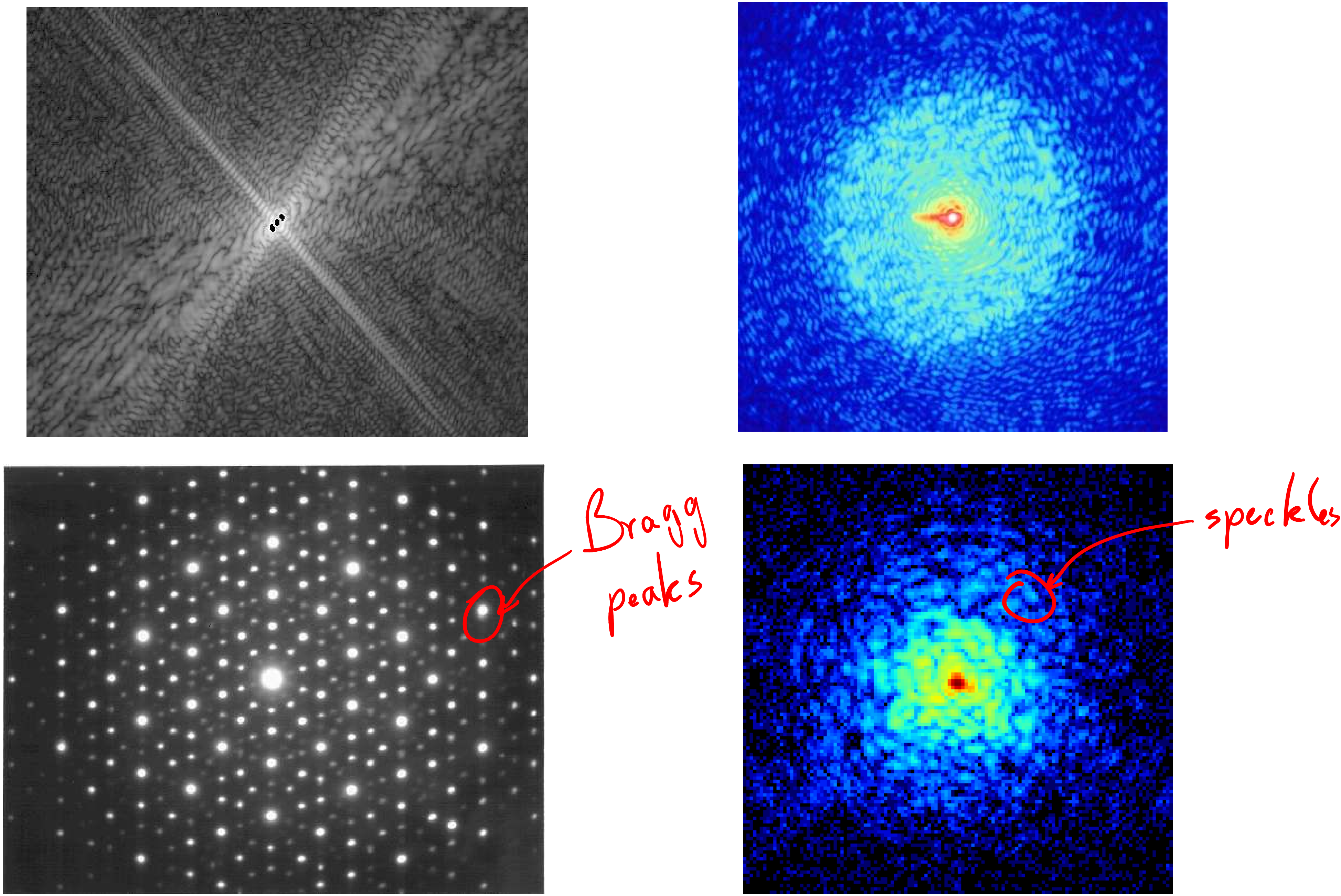


$$\frac{\vec{r}}{z} = \frac{\vec{k}_\perp}{k} = \frac{2\pi\vec{u}}{2\pi/\lambda} = \lambda\vec{u}$$

$$\psi(\vec{r}) = \frac{-i2\pi}{\lambda z}\exp\left(\frac{i\pi r^2}{\lambda z}\right)\int d^2r'\,\psi(\vec{r}')\exp\Big(-2\pi i\,\vec{r}'\cdot\underbrace{\frac{\vec{r}}{\lambda z}}_{\vec{u}}\Big)$$

$$|\psi(r)|^2 \propto |\mathcal{F}\psi|^2 = I(\vec{u})$$

Diffraction patterns

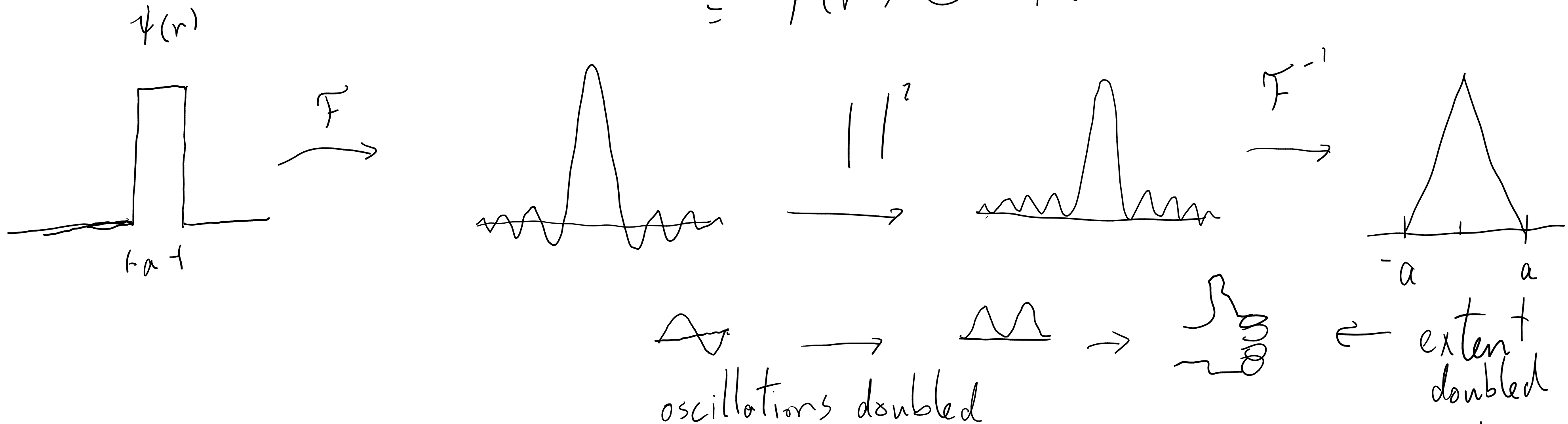


Far-field diffraction

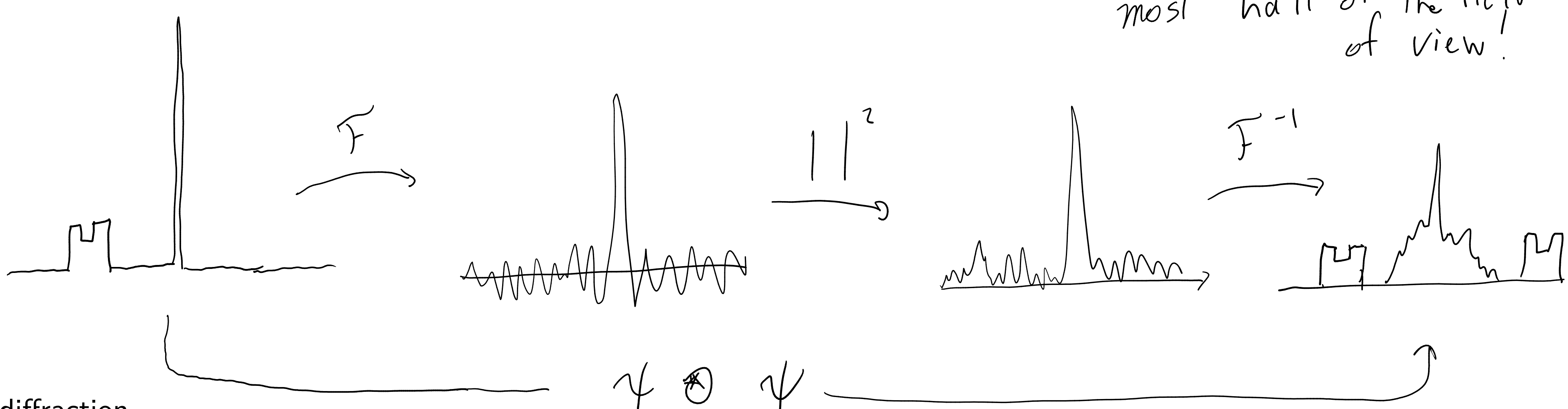
Diffraction and autocorrelation

$$\mathcal{F}^{-1}\{I(\vec{u})\} = \mathcal{F}^{-1}\{\psi(\vec{u})\cdot\psi^*(\vec{u})\}$$

$$= \psi(\vec{r}) \circledast \psi(\vec{r})$$ "autocorrelation"



Nyquist: a must be at most half of the field of view!



Fourier transform holography

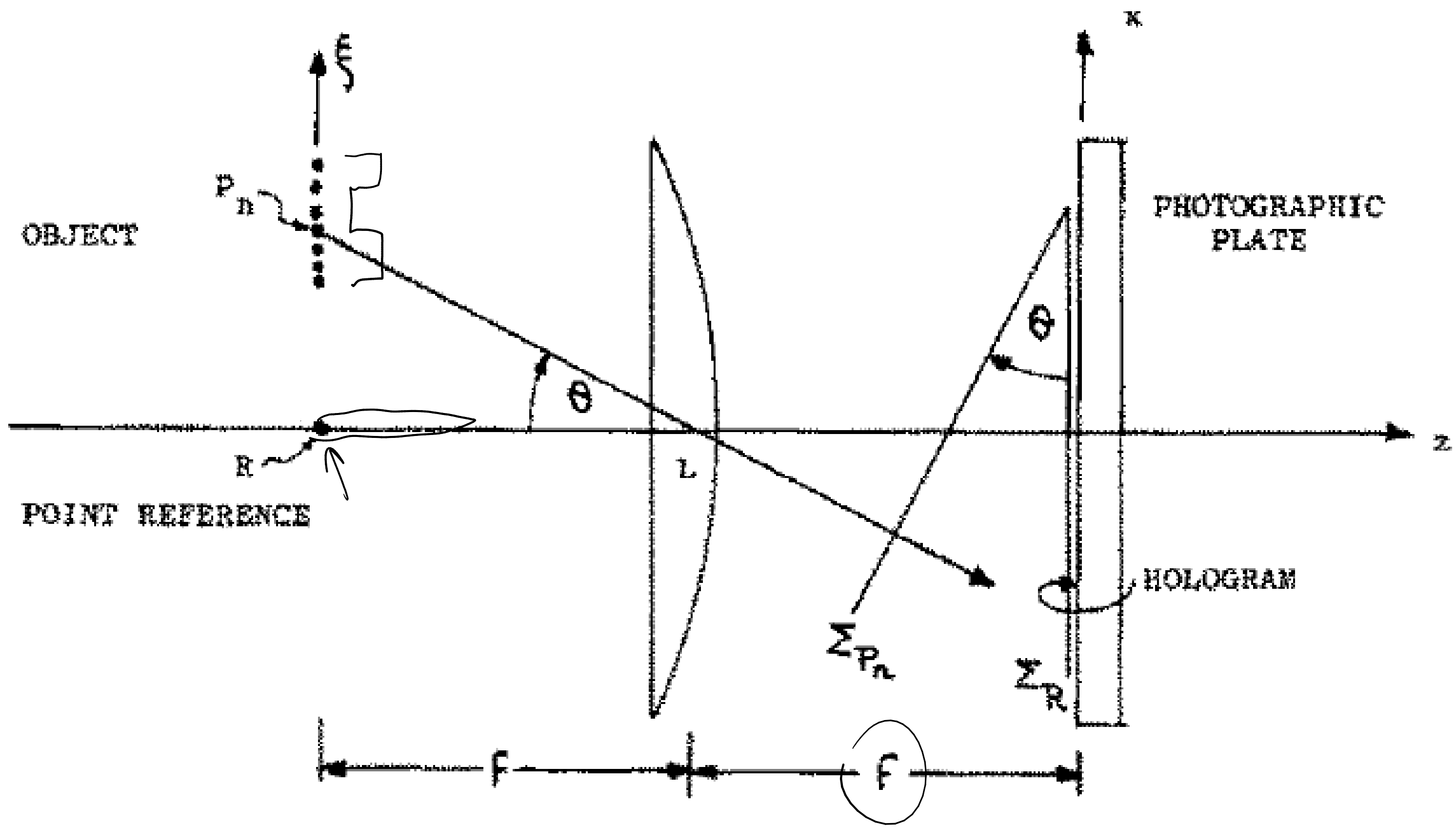
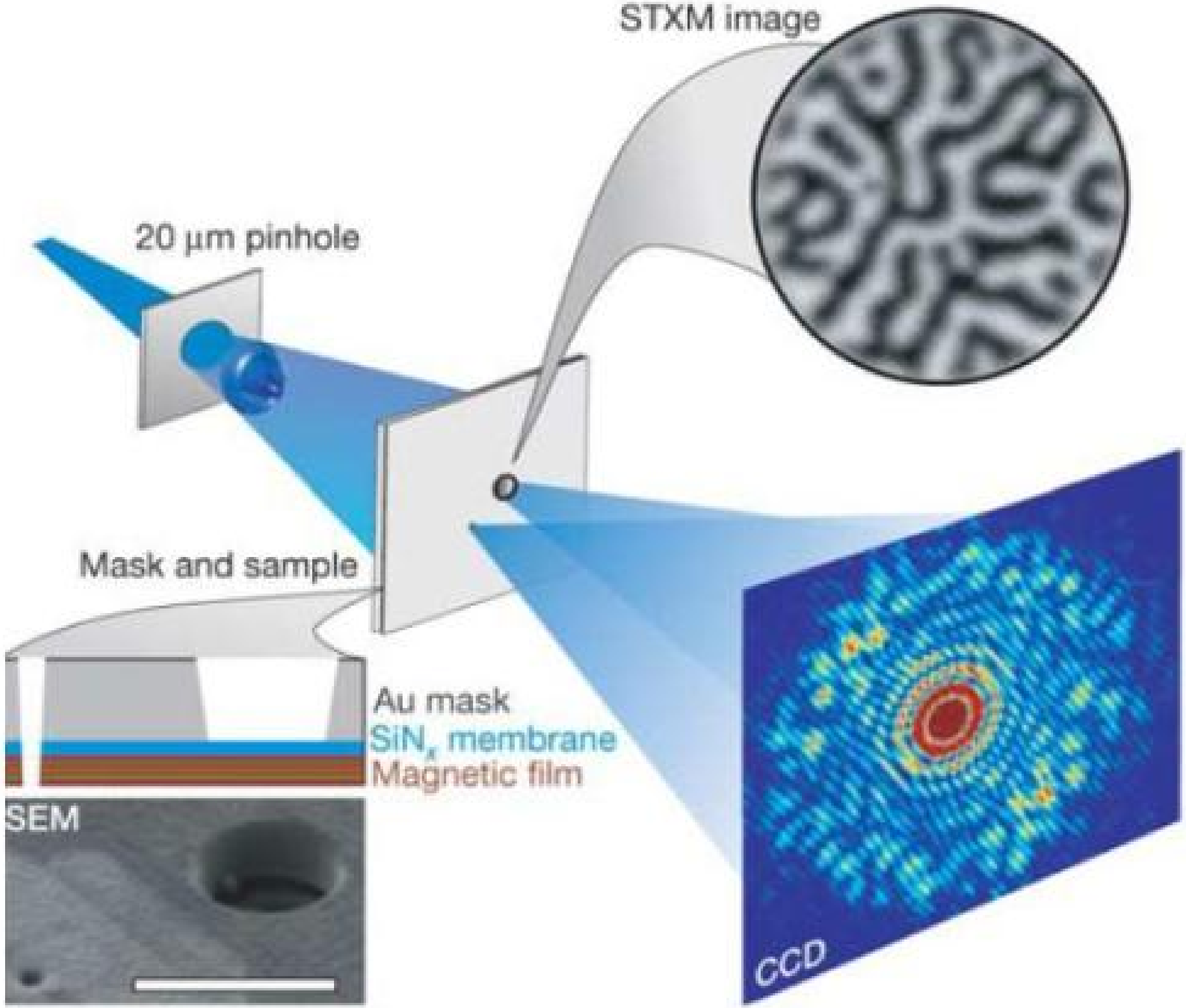


Fig. 1. Recording of a Fourier-transform hologram with a lens L. Σ_R = reference wavefront.

Source: G. Stroke, Appl. Phys. Lett. **6**, 201-203 (1965).

Fourier transform holography



Source: S. Eisebitt et al., Nature **432**, 885-888 (2004).

Fourier transform holography

$$\psi(\vec{r}) = \psi_R(\vec{r}) + \psi_o(\vec{r}) \leftarrow \text{exit wave}$$

$$\downarrow$$

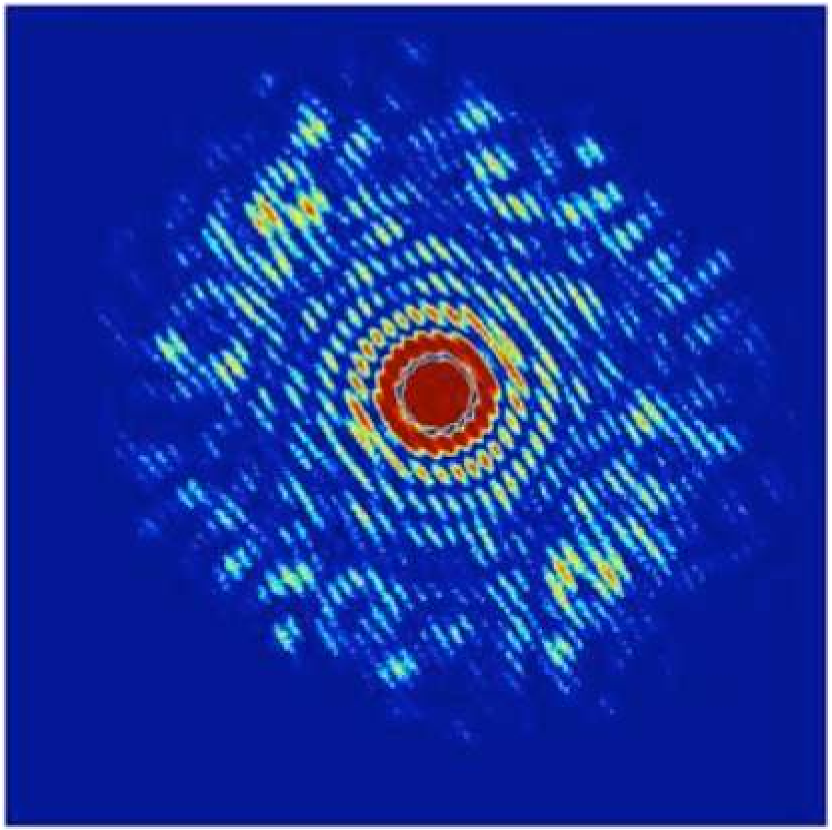
$$\tilde{\psi}(\vec{u}) = \tilde{\psi}_R(\vec{u}) + \tilde{\psi}_o(\vec{u}) \leftarrow \text{far-field}$$

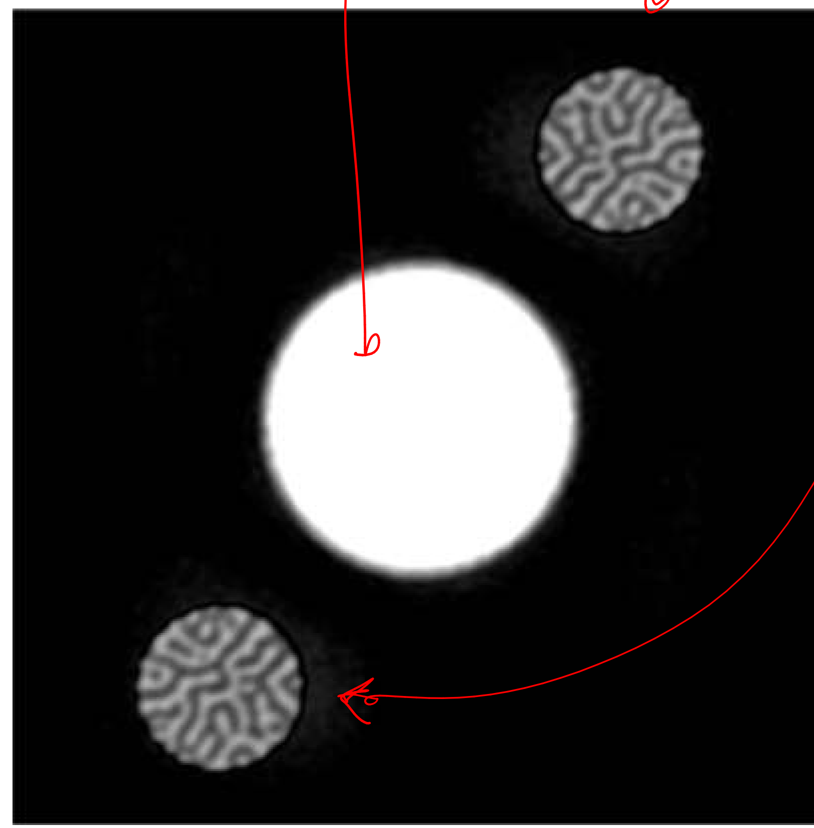
$$I(\vec{u}) = |\tilde{\psi}(\vec{u})|^2 = |\tilde{\psi}_R(\vec{u})|^2 + |\tilde{\psi}_o(\vec{u})|^2 + \underline{\tilde{\psi}_R(\vec{u})\,\tilde{\psi}_o^*(\vec{u}) + c.c.}$$

$\uparrow$ measured intensity

$$\mathcal{F}^{-1}\{I(\vec{u})\} = \underbrace{\psi_R \circledast \psi_R + \psi_o \circledast \psi_o}_{\text{autocorrelations superimposed at origin}} + \underbrace{\psi_R \circledast \psi_o + \psi_o \circledast \psi_R}_{\text{cross-correlations}}$$

Fourier transform holography

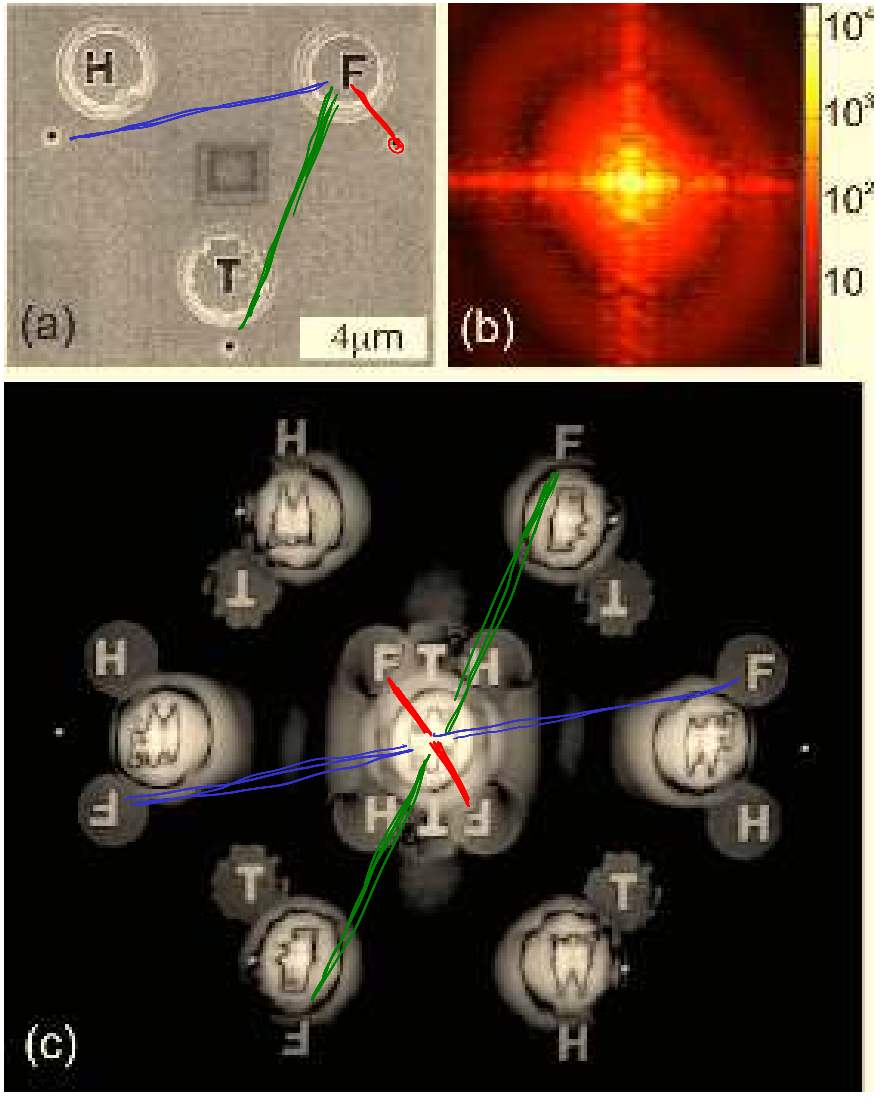




N.B. autocorrelation is always centrosymmetric (equal to itself complex conjugate after 180° rotation) because it is the F.T. of a real object

Fourier transform holography

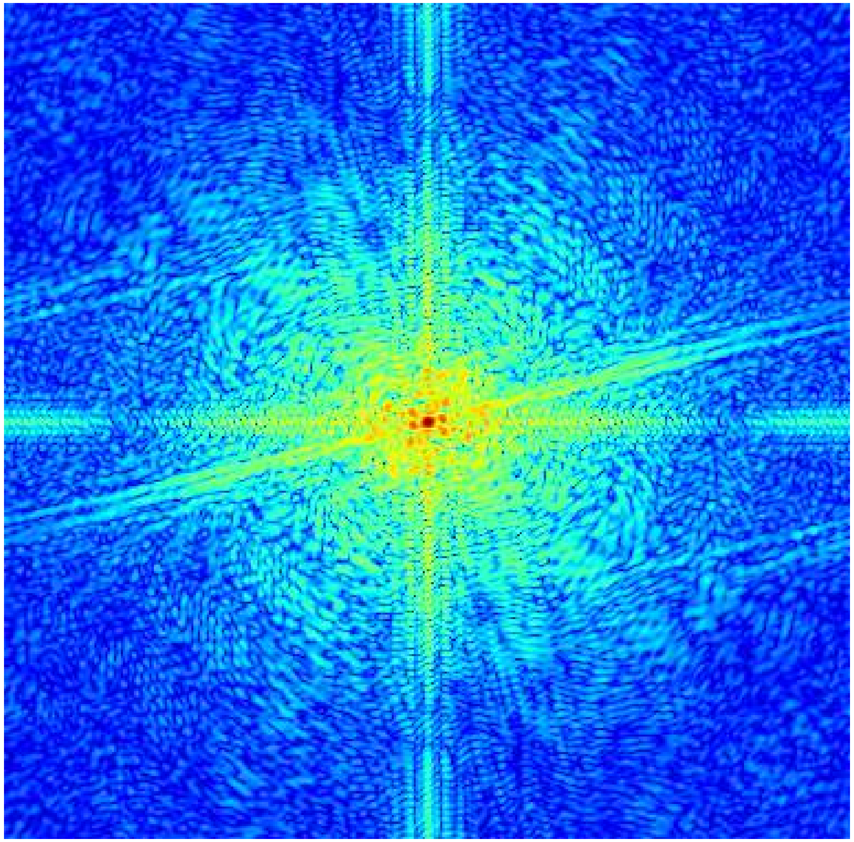
Multiple references



Source: W. Schlotter et al., Opt.. Lett. **21**, 3110-3112 (2006).

Coherent diffractive imaging

Diffraction pattern from isolated sample



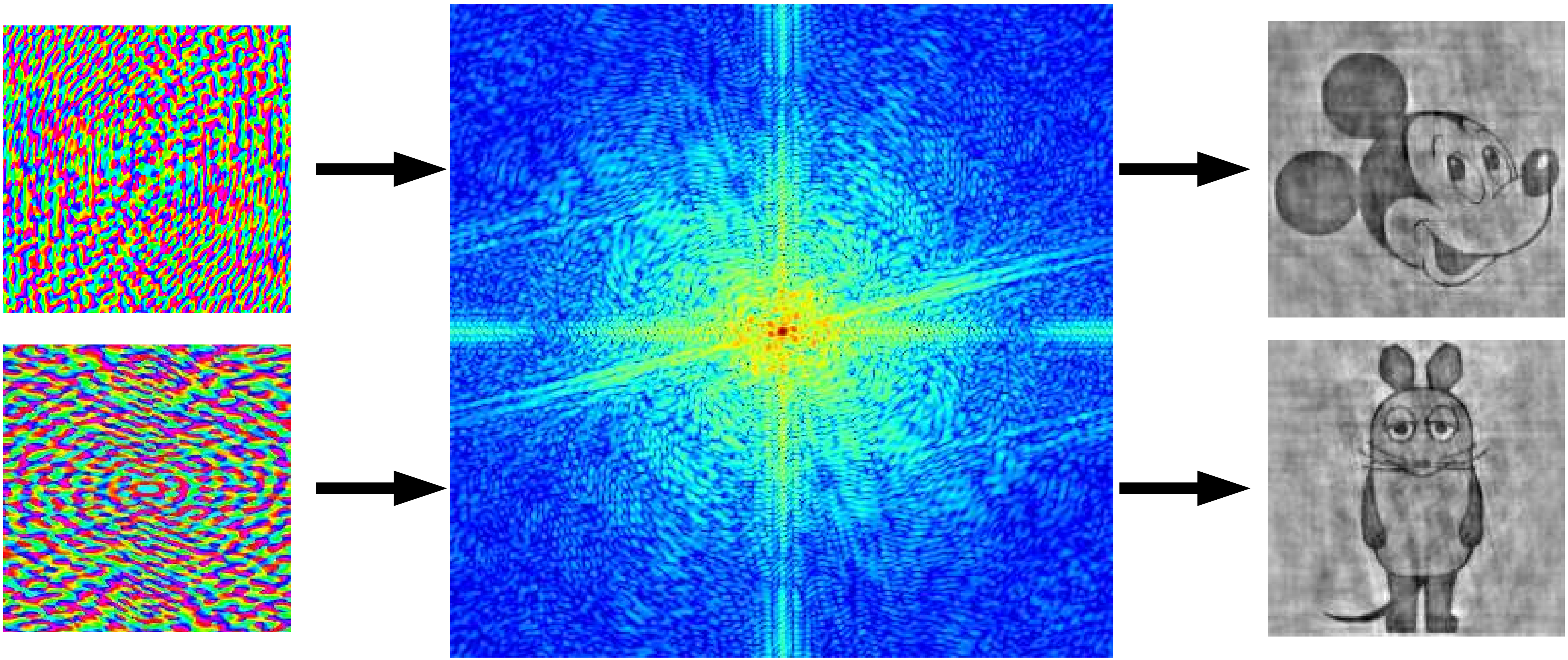
(speckles)

The phase problem

$e^{i\varphi(\vec{u})}$

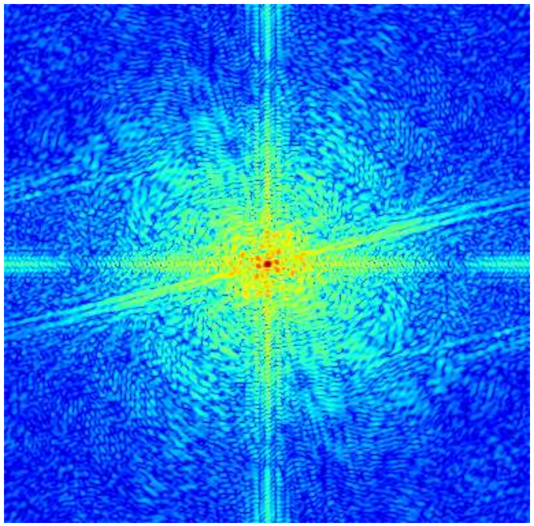
$I(\vec{u}) = |\psi(\vec{u})|^2$

$\psi(\vec{r}) = \mathcal{F}^{-1}\{\sqrt{I}\, e^{i\varphi}\}$

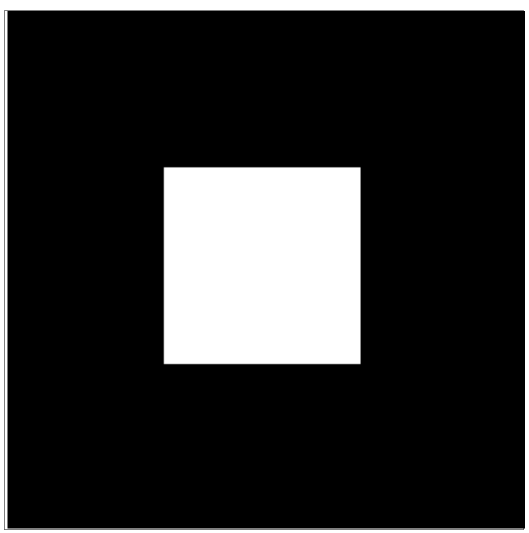


Far-field diffraction

Coherent diffractive imaging



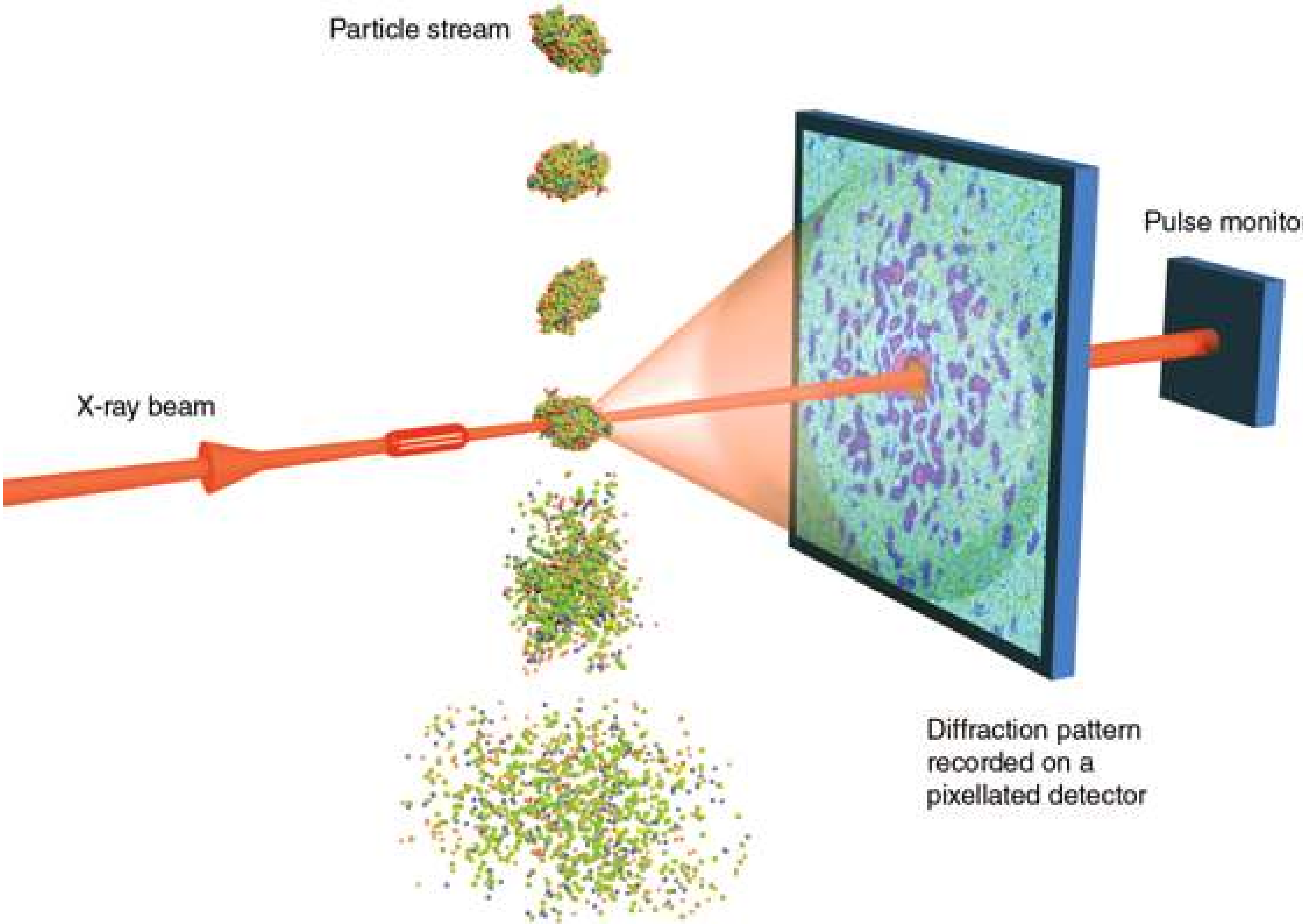
1. Solution has to be consistent with the measured Fourier amplitudes



2. Solution is isolated "support constraint"

Far-field diffraction

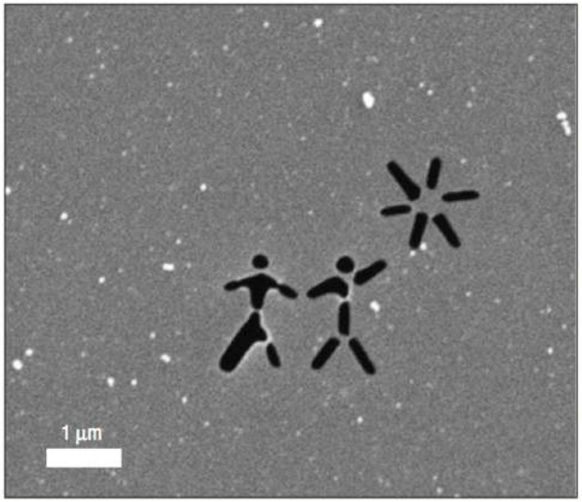
Radiation damage limits on radiation

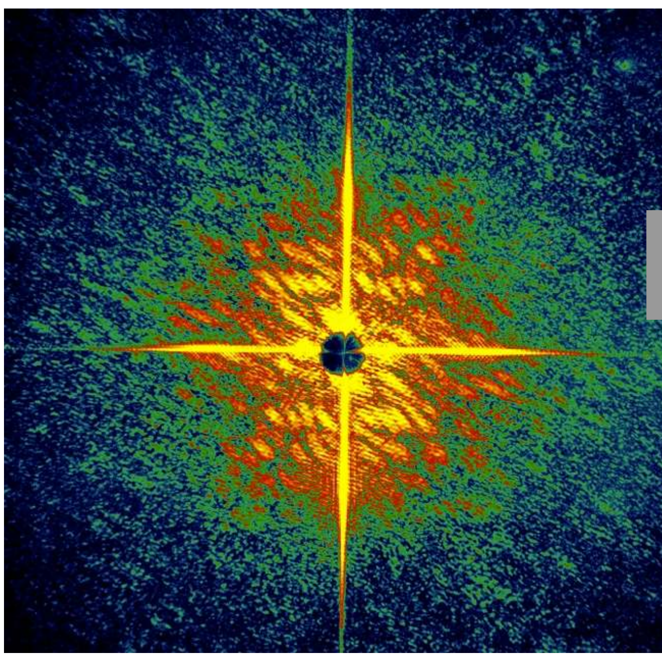


R. Neutze *et al*, Nature **406**, 752 (2000)

K. J. Gaffney *et al*, Science **316**, 1444 (2007)

“Diffraction before destruction”



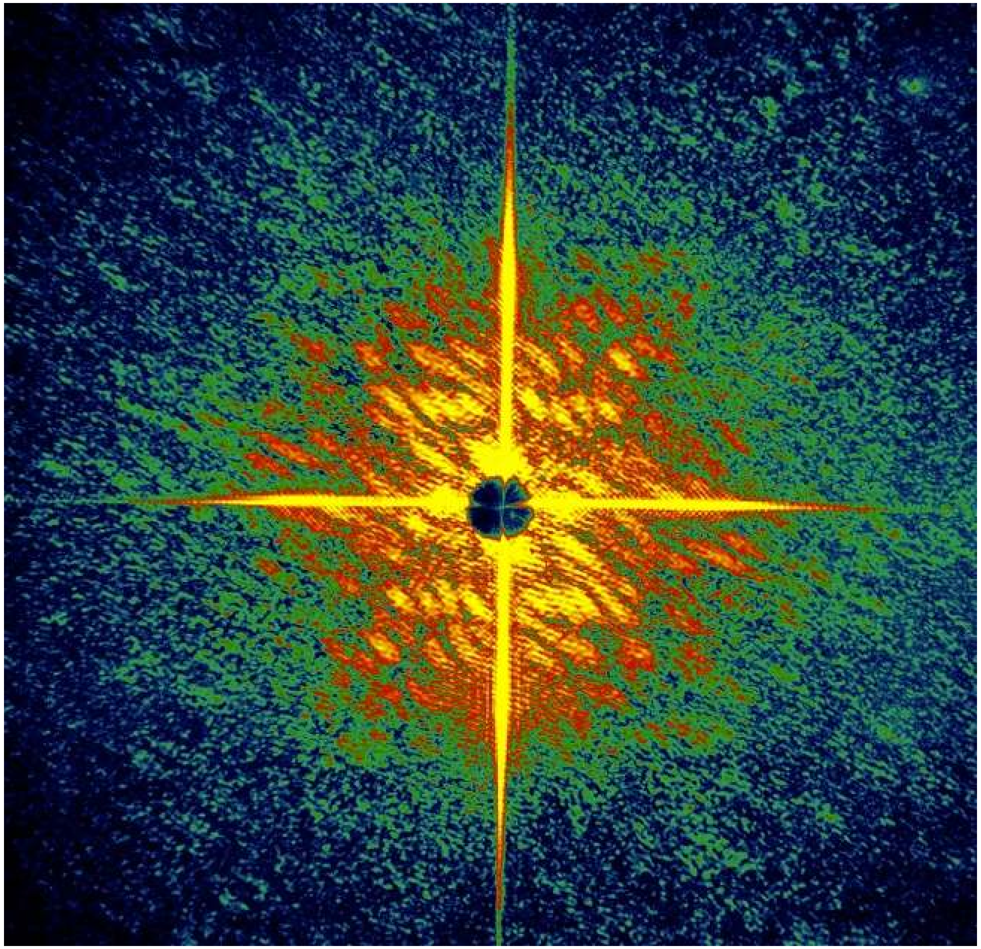


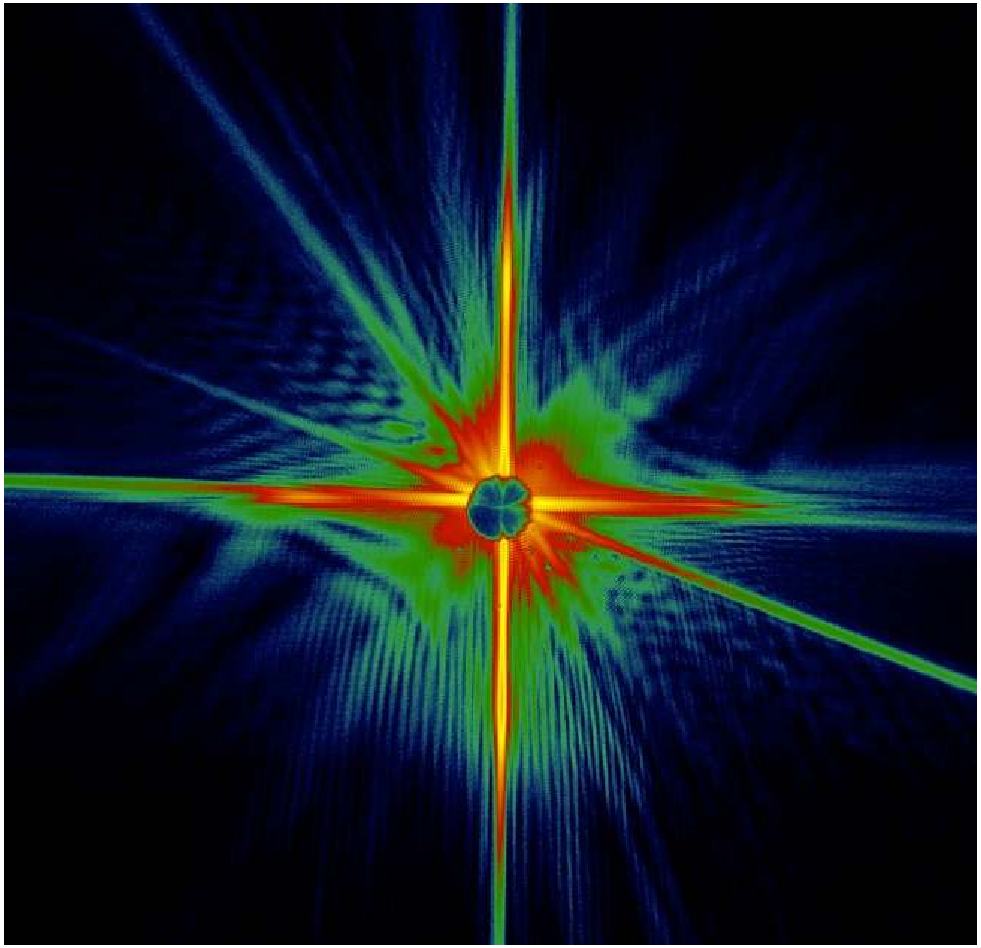
H. N. Chapman *et al*, Nat. Phys. **2**, 839 (2006)

Far-field diffraction

"Diffraction before destruction"

The imaging pulse vaporized the sample





H. N. Chapman *et al*, Nat. Phys. **2**, 839 (2006)

Ptychography

- Scanning an isolated illumination on an extended specimen
- Measure full coherent diffraction pattern at each scan point
- Combine everything to get a reconstruction

Dynamische Theorie der Kristallstrukturanalyse durch Elektronenbeugung im inhomogenen Primärstrahlwellenfeld

Von R. Hegerl und W. Hoppe

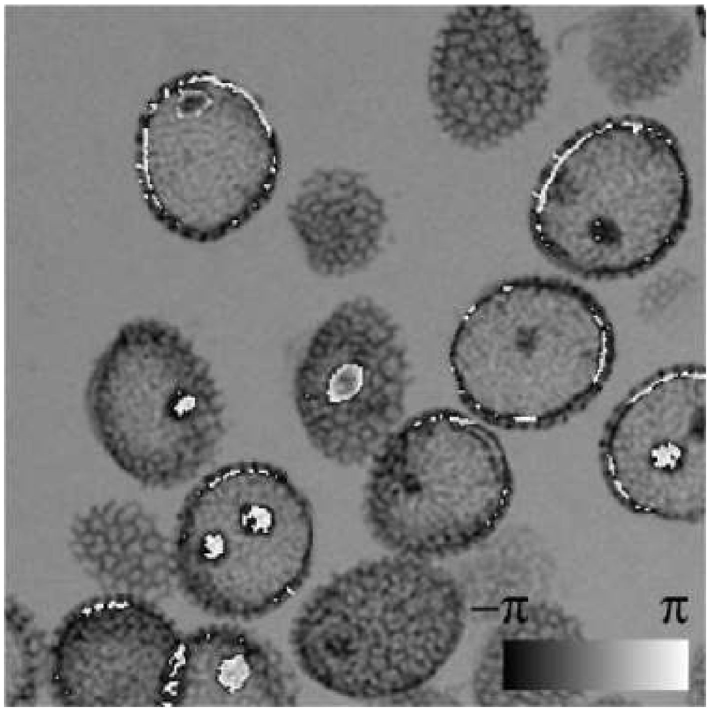
Some time ago a new principle was proposed for the registration of the complete information (amplitudes and phases) in a diffraction diagram, which does not – as does Holography – require the interference of the scattered waves with a single reference wave. The basis of the principle lies in the interference of neighbouring scattered waves which result when the object function $\varrho(x, y)$ is multiplied by a generalized primary wave function $p(x, y)$ in Fourier space (diffraction diagram) this is a convolution of the Fourier transforms of these functions. The above mentioned interferences necessary for the phase determination can be obtained by suitable choice of the shape of $p(x, y)$. To distinguish it from holography this procedure is designated "ptychography" (πτυξ = fold). The procedure is applicable to periodic and aperiodic structures. The relationships are simplest for plane lattices. In this paper the theory is extended to space lattices both with and without consideration of the dynamic theory. The resulting effects are demonstrated using a practical example.

(1970)

Ptychography

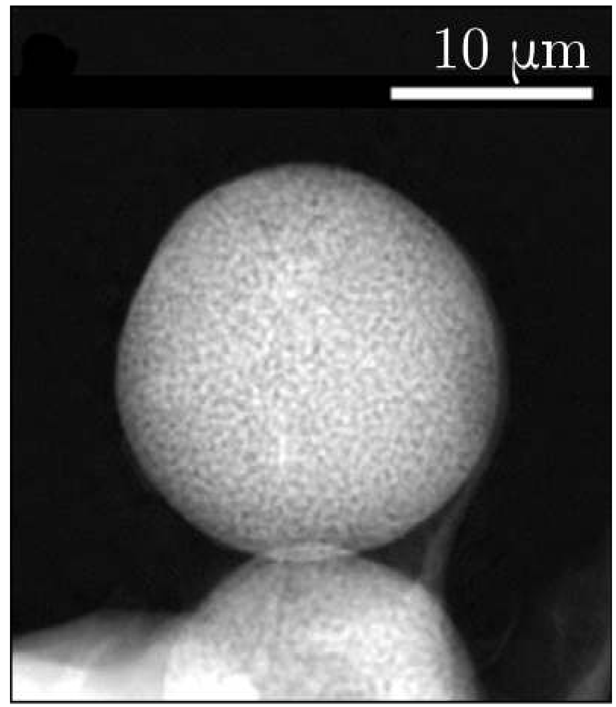
A few examples

Visible light



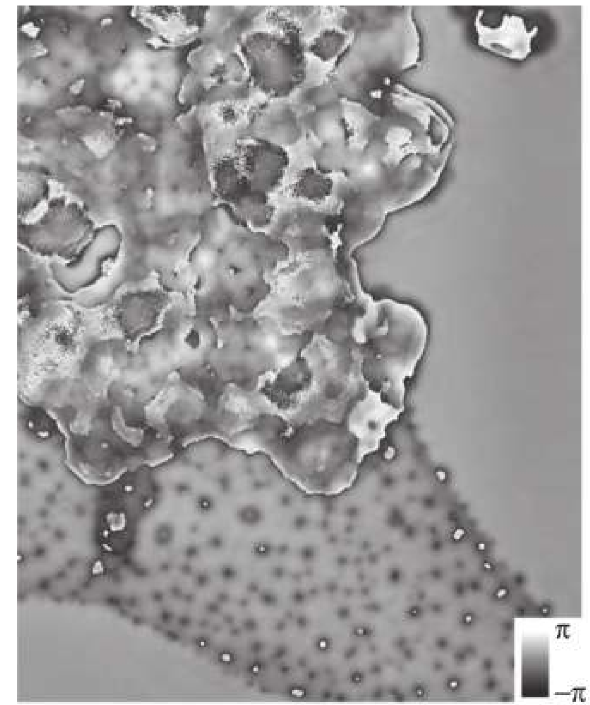
A. Maiden *et al.*, Opt. Lett. **35**, 2585-2587 (2010).

X-rays



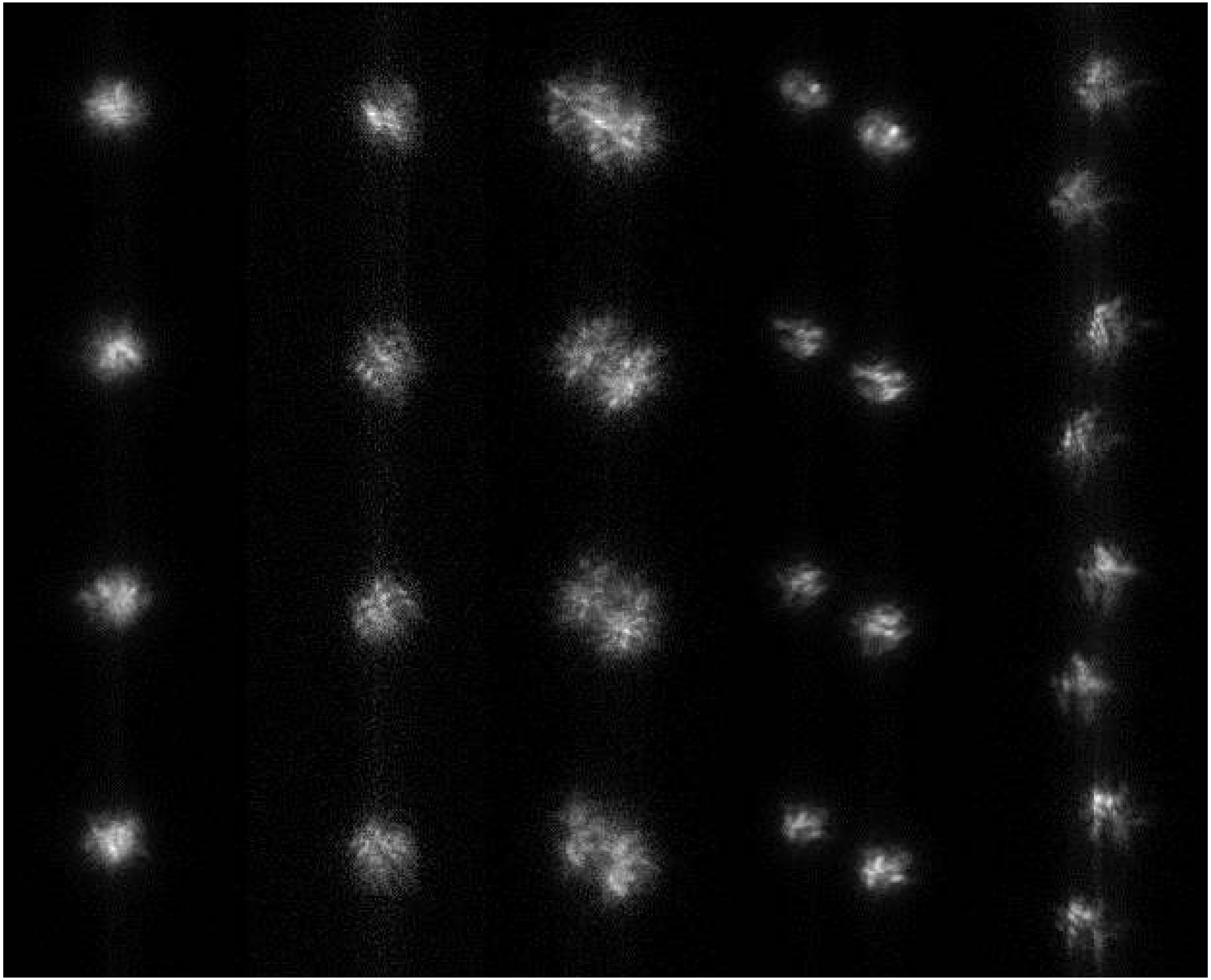
P. Thibault *et al.*, New J. Phys **14**, 063004 (2012).

electrons



M. Humphry *et al.*, Nat. Comm. **3**, 730 (2012).

Speckle imaging in astronomy



Source:http://www.cis.rit.edu/research/thesis/bs/2000/hoffmann/thesis.html

Speckle imaging in astronomy

Model

(incoherent imaging system)

$$I = O * |P|^2$$

$|P|^2$ ← "instantaneous PSF"

$$\tilde{I} = \tilde{O} \cdot P_A$$

P_A ← autocorrelation of PSF

$$|\tilde{I}|^2 = |\tilde{O}|^2 \cdot |P_A|^2$$

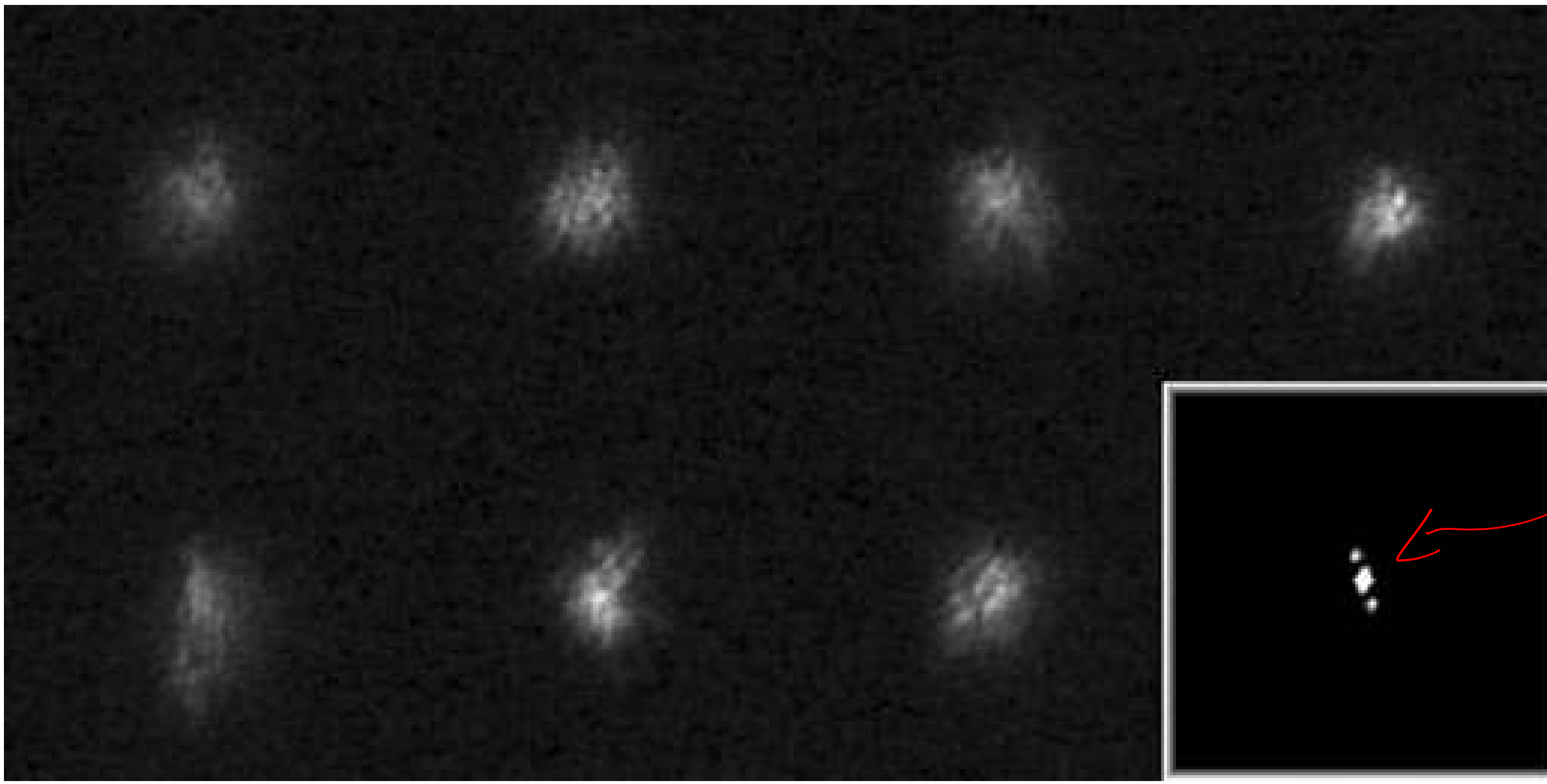
$$\langle |\tilde{I}|^2 \rangle = |\tilde{O}|^2 \langle |P_A|^2 \rangle$$

$\langle |P_A|^2 \rangle$ ← can be characterized (can be considered as a known quantity)

$|\tilde{O}|^2$ ↑ reconstruct O from $|\tilde{O}|^2$

Speckle imaging in astronomy

Retrieval of the autocorrelation



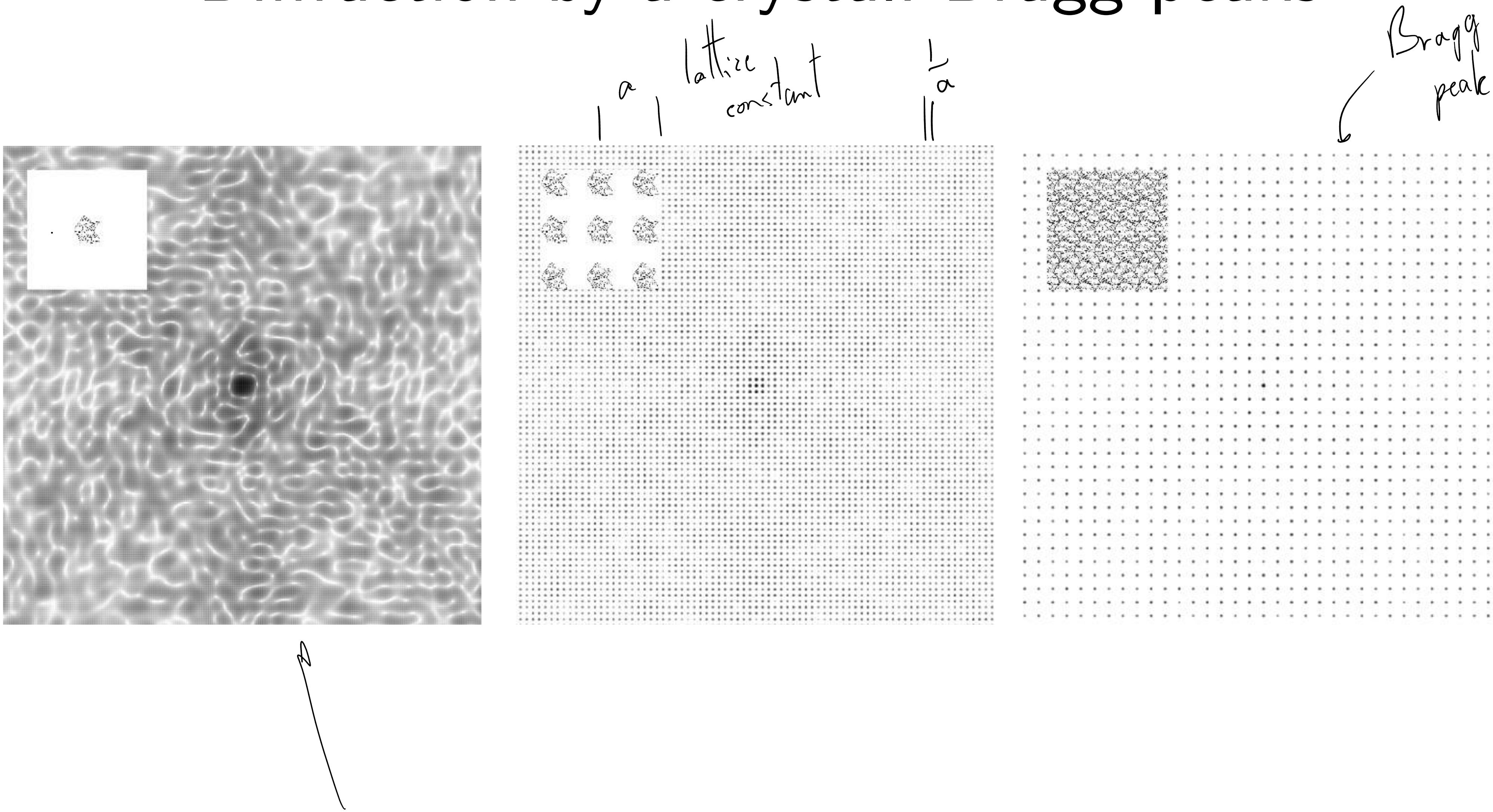
autocorrelation

(binary system)

Source: http://www.astrosurf.com/hfosaf/uk/speckle10.htm

Crystallography

Diffraction by a crystal: Bragg peaks

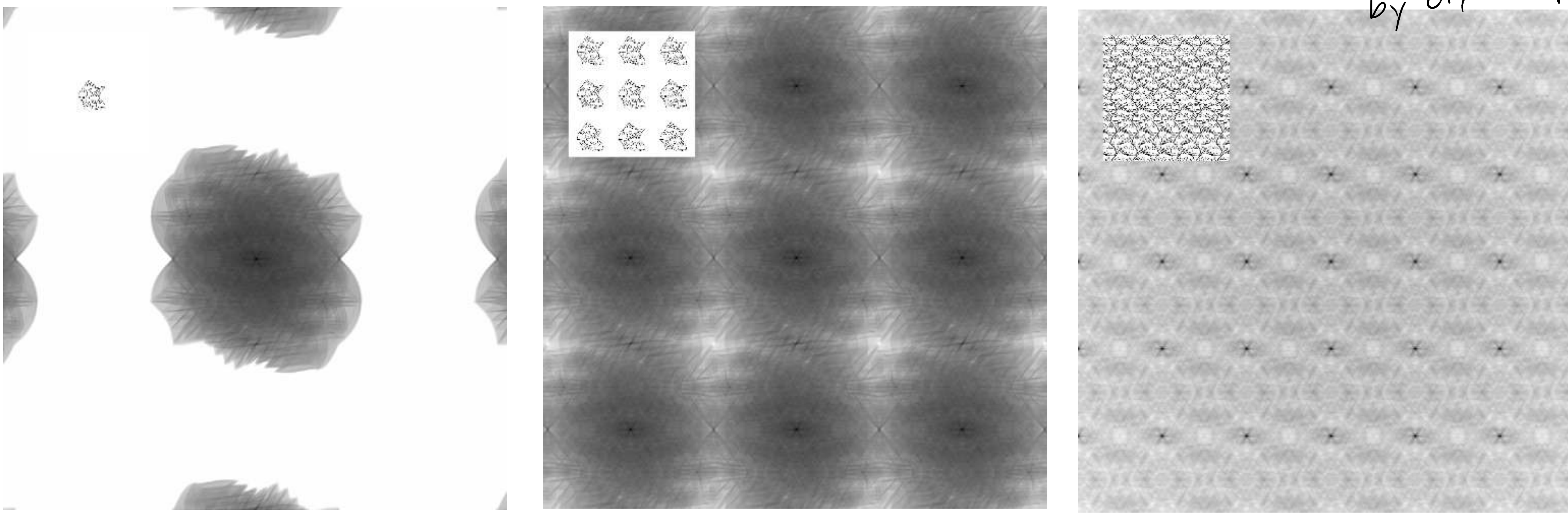


Far-field diffraction

Crystallography

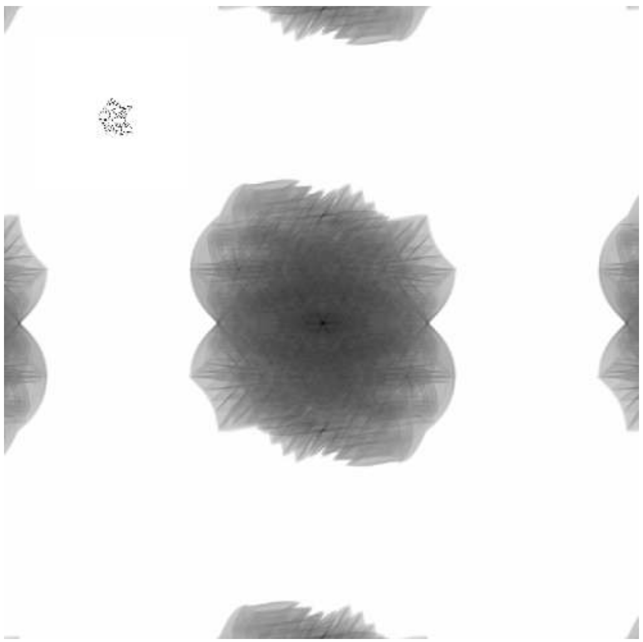
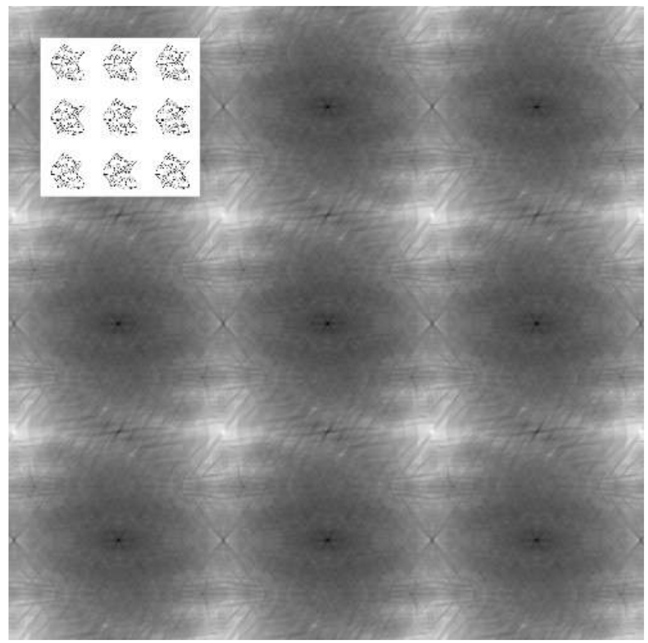
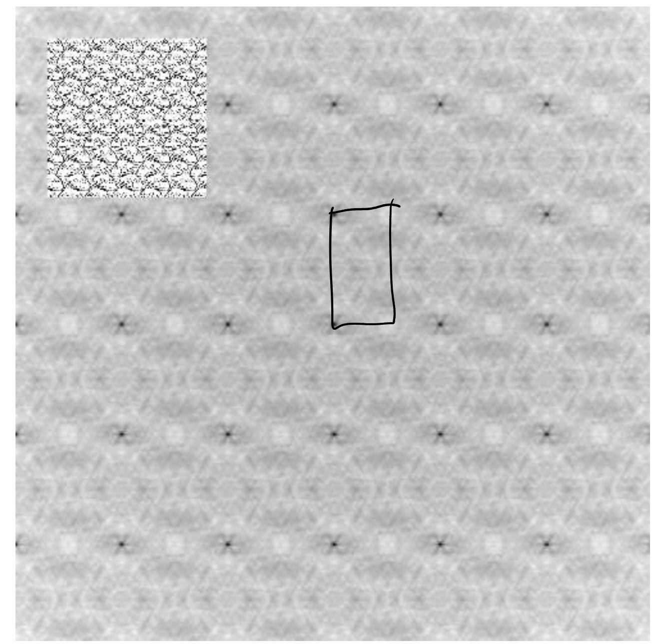
Fourier transform of intensity: autocorrelation

aliased autocorrelation
called Patterson map
by crystallographers



Crystallography is not Nyquist sampled

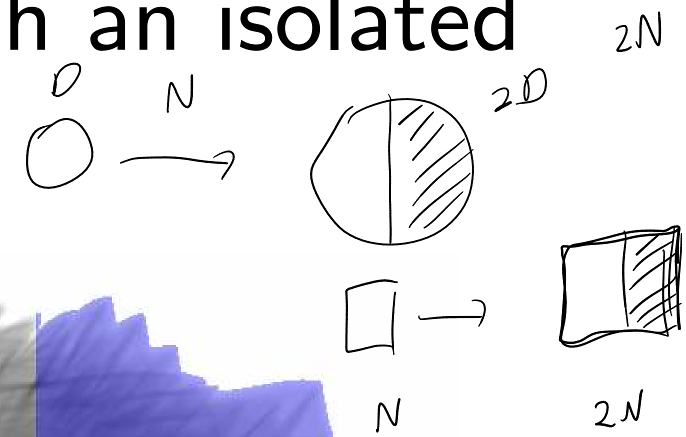
Crystallography

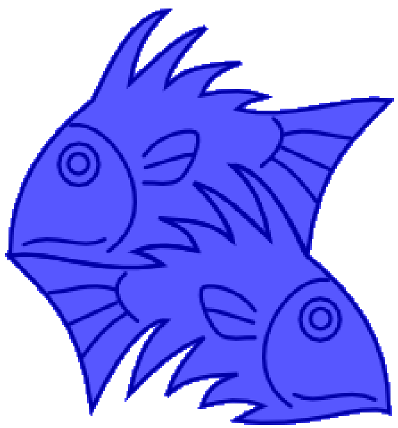




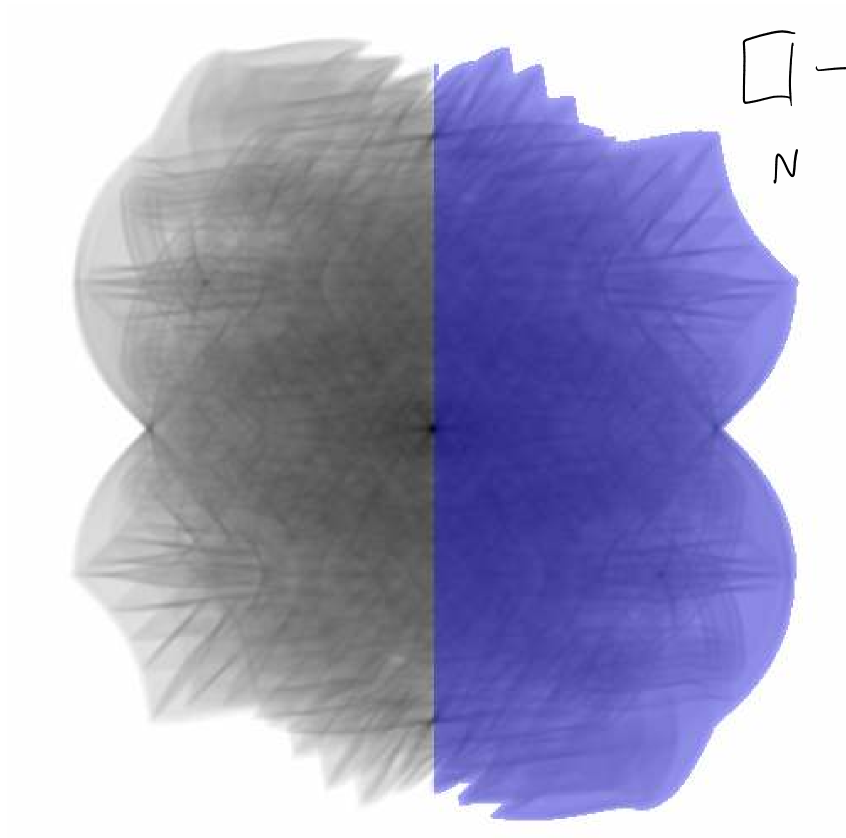
Far-field diffraction

Crystallography

Problem is overconstrained with an isolated sample







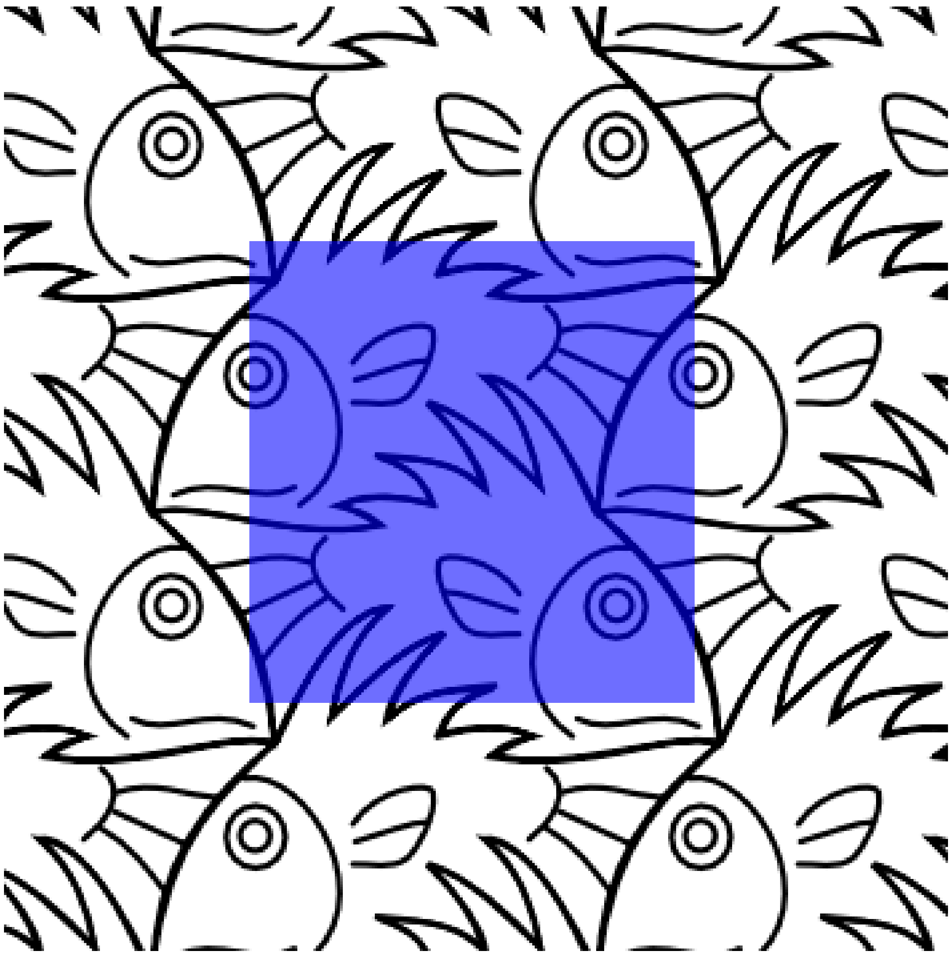
unknowns = N

constraints ≥ 2N

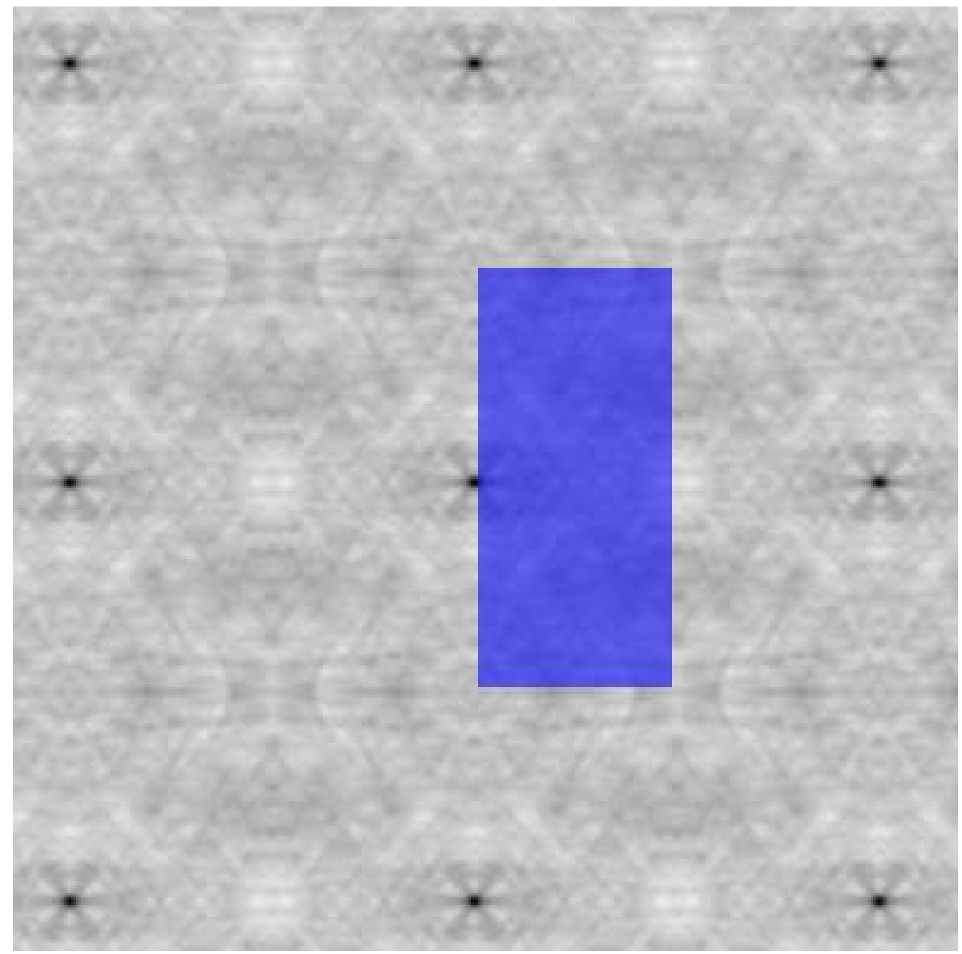
Far-field diffraction

Crystallography

Problem is **under**constrained with a crystal

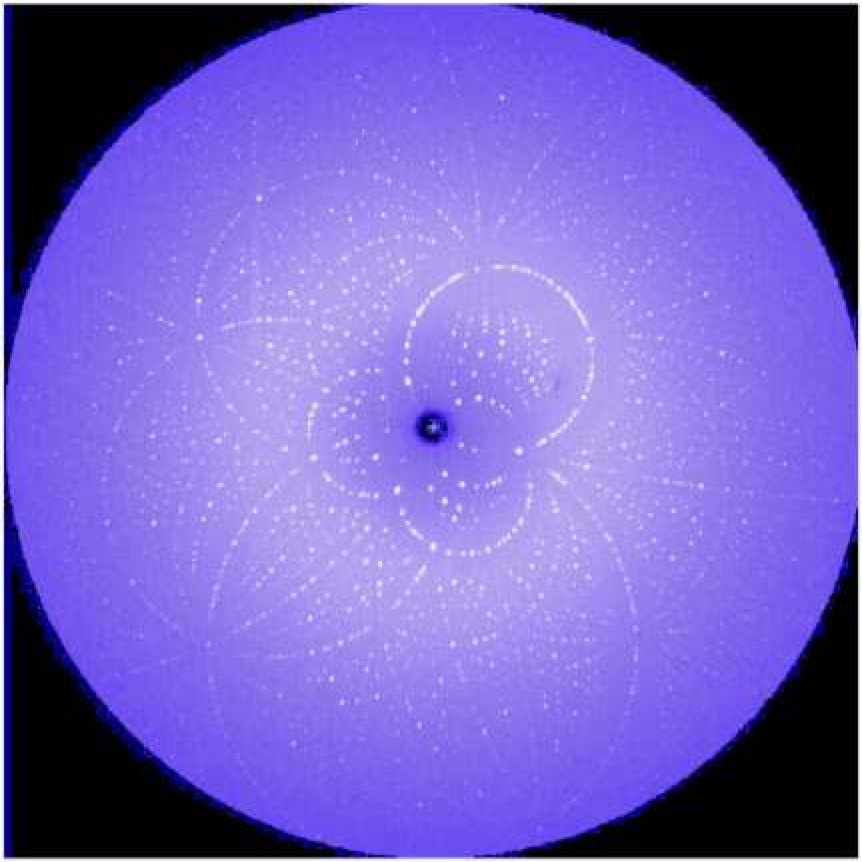


unknowns = N



constraints = N/2

Crystallography
Structure determination



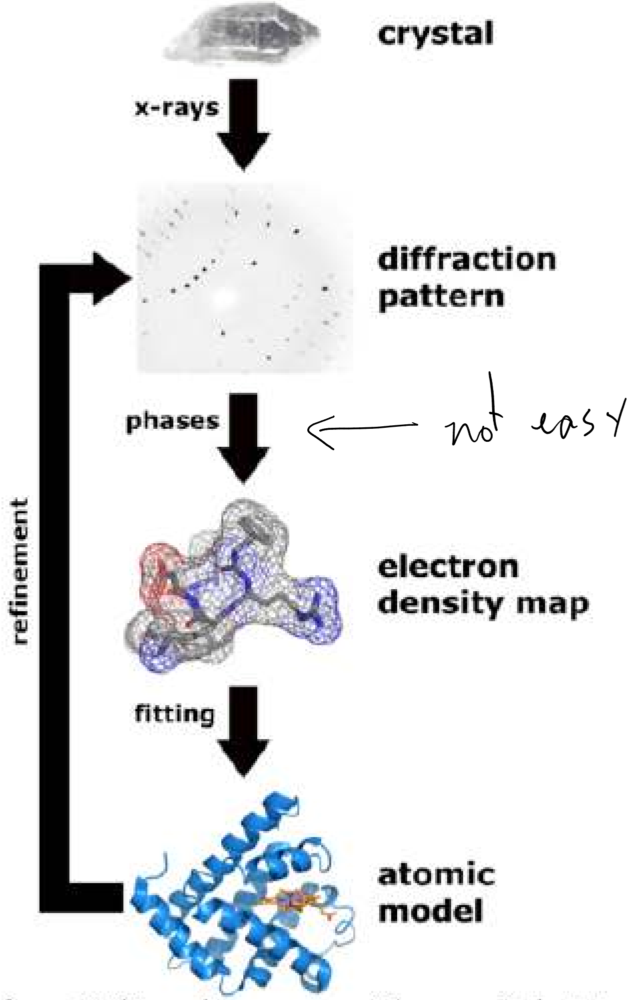


Image from Wikimedia courtesy Thomas Splettstoesser

Crystallography

Structure determination

- Hard problem: few measurements for the number of unknowns
- Luckily: crystals are made of atoms $\rightarrow$ strong constraint
- Also common: combining additional measurements (SAD, MAD, isomorphous replacement, ...)

Summary

Imaging from far-field amplitudes

- Used when image-forming lenses are unavailable (or unreliable) or to obtain more quantitative images.
- In general difficult because of the phase problem
- Solved with the help of additional information:
  - Strong *a priori* knowledge (e.g. CDI: support)
  - Multiple measurements (e.g. ptychography)