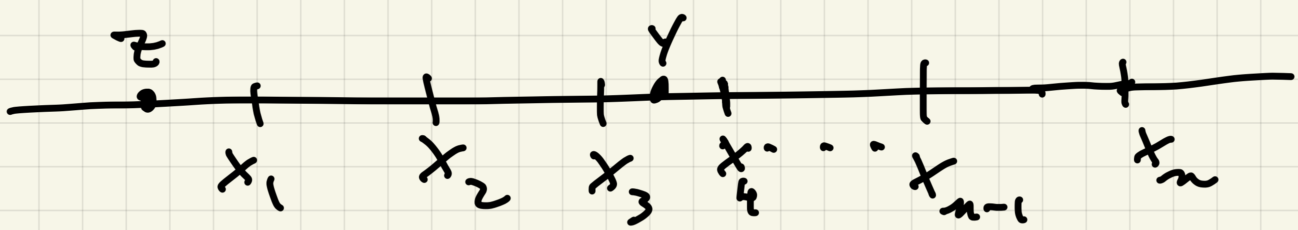
$f \in C^0(\mathbb{R})$ $\qquad f'(x) \geq 0 \quad \forall x \notin \{x_1, \dots, x_n\}$

Devo dimostrare che se $z < y$ allora $f(z) \leq f(y)$.

Se z e y appartengono allo stesso intervallo allora applica Lagrange:

$\exists \; c \in (z, y)$ t.c.

$$0 \leq f'(c) = \frac{f(y) - f(z)}{y - z} \Rightarrow f(y) \geq f(z)$$



Applica Lagrange a

z	x_1	$f(z) \leq f(x_1)$
x_1	x_2	$f(x_1) \leq f(x_2)$
x_2	x_3	$f(x_2) \leq f(x_3)$
x_3	y	$f(x_3) \leq f(y)$

$$\Rightarrow f(z) \leq f(y)$$

f periodica di periodo $T > 0$ non costante. Dimostrare che $\lim_{x \to +\infty} f(x)$ non esiste

Presi $x_1 \neq x_2$ t.c. $f(x_1) \neq f(x_2)$

$f(x_1 + T) = f(x_1)$

$$\boxed{\text{Sia per assurdo } \lim_{x \to +\infty} f(x) = L \in \mathbb{R}}$$

Allora $\forall \varepsilon > 0 \; \exists M_\varepsilon$ t.c. $x > M_\varepsilon$

$$|f(x) - L| < \varepsilon.$$

$$\lim_{x \to +\infty} f(x) = L \Rightarrow \lim_{n \to +\infty} f(x_1 + nT) = L$$

$$\lim_{n \to +\infty} f(x_2 + nT) = L$$

$f(x_1 + nT) = f(x_1)$

$\neq$ per ipotesi

$f(x_2 + nT) = f(x_2)$

$L = f(x_1)$ Conclusione

$L = f(x_2)$ $"$

Assurdo.

Esercizio 7.30

$P_{2n+1}(\mathbb{R})$ è un intervallo

$$\lim_{x\to+\infty} P_{2n+1}(x) = +\infty \quad o \quad -\infty$$

$$\lim_{x\to-\infty} P_{2n+1}(x) = -\infty \quad o \quad +\infty$$

$$\sup P_{2n+1}(\mathbb{R}) = +\infty$$

$$\inf P_{2n+1}(\mathbb{R}) = -\infty$$

$$\Rightarrow \quad P_{2n+1}(\mathbb{R}) = \mathbb{R}$$

8.29 $\quad 1 \Rightarrow 2$

$$f'(x_0) = \lim_{x\to x_0} \frac{f(x)-f(x_0)}{x-x_0} = \overbrace{\lim_{x\to x_0^+} \frac{f(x)-f(x_0)}{x-x_0}}^{f'_d(x_0)}$$

$$= \underbrace{\lim_{x\to x_0^-} \frac{f(x)-f(x_0)}{x-x_0}}_{f'_s(x)}$$

$\exists\, f'(x_0) \Rightarrow \exists\, f'_d(x_0), f'_s(x_0)$ con

$$f'(x_0) = f'_d(x_0) = f'_s(x_0)$$

$2 \Rightarrow 1$

$$f'_d(x_0) = \lim_{x \to x_0^+} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to x_0^-} \frac{f(x) - f(x_0)}{x - x_0} = f'_s(x_0)$$

$$\Downarrow$$

$$\exists \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = f'(x_0)$$

$$\boxed{\begin{array}{l} f, g \qquad g(x) = f(x - x_0) \\ g^{(k)}(x) = f^{(k)}(x - x_0) \end{array}}$$

$$\prod_{j=1}^{n} (a - j + 1)\, x^{a-n} = (x^a)^{(n)}$$

$$\prod_{j=1}^{n} (a - j + 1)\, (x - x_0)^{a-n} = \left((x - x_0)^a \right)^{(n)}$$

$$\frac{\prod_{j=1}^{n} (a - j + 1)\, x^{a-n}}{\prod_{j=1}^{n} (a - j + 1)\, (x - x_0)^{a-n}} = \frac{x^{a-n}}{(x - x_0)^{a-n}}$$

$$\frac{(x-x_0)^{a-n}}{x^{a-n}} = \frac{\left((x-x_0)^a\right)^{(n)}}{(x^a)^{(n)}}$$

$$g(x) = f(x-x_0)$$

$$g'(x) = f'(x-x_0)\,(x-x_0)' = f'(x-x_0)$$

$$g^{(k-1)}(x) = f^{(k-1)}(x-x_0)$$

$$g^{(k)}(x) = \left(g^{(k-1)}(x)\right)' = \left(f^{(k-1)}(x-x_0)\right)' =$$

$$= \left(f^{(k-1)}\right)'(x-x_0) = f^{(k)}(x-x_0)$$