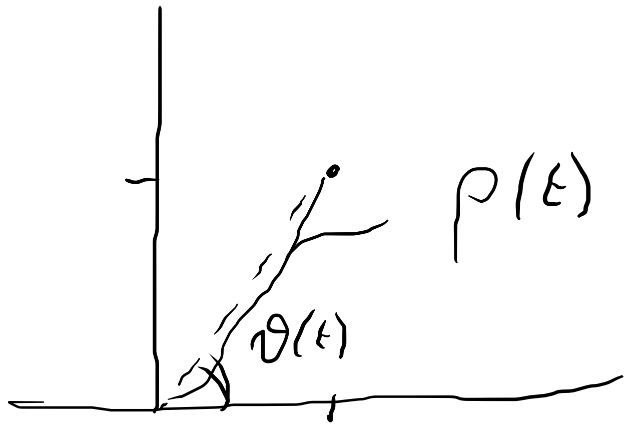
Curve in forma polare

$$\gamma(t) = (x(t), y(t))^T \rightsquigarrow (\rho(t), \vartheta(t))^T$$



$$\begin{cases} x(t) = \rho(t)\cos(\vartheta(t)) \\ y(t) = \rho(t)\,\mathrm{sen}(\vartheta(t)) \end{cases}$$

$$\vartheta(t) = t$$

$$\boxed{\vartheta = t} \rightsquigarrow \begin{matrix} x = \rho(t)\cos t \\ y = \rho(t)\,\mathrm{sen}\, t \end{matrix} \qquad \rho(t)$$

$$x' = \rho'(t)\cos t - \rho(t)\,\mathrm{sen}\, t$$

$$y' = \rho'(t)\,\mathrm{sen}\, t + \rho(t)\cos t$$

$$\|\gamma'(t)\| = \sqrt{x'(t)^2 + y'(t)^2} =$$

$$= \sqrt{(\rho'(t)\cos t - \rho(t)\,\mathrm{sen}\, t)^2 + (\rho'(t)\,\mathrm{sen}\, t + \rho(t)\cos t)^2} = \sqrt{\rho'(t)^2 + \rho(t)^2}$$

$$\rho'(t)^2\cos^2 t \; \cancel{-2\rho'(t)\rho(t)}_{\cancel{\cos t\,\mathrm{sen}\, t}} + \rho(t)^2\,\mathrm{sen}^2(t) + \rho'(t)^2\,\mathrm{sen}^2 t \; \cancel{+ 2\rho'(t)\rho(t)\,\mathrm{sen}\, t} \cos t + \rho(t)^2\cos^2 t$$

$$\|\gamma'(t)\| = \sqrt{\rho'(t)^2 + \rho(t)^2} \qquad \int_\gamma ds = \int_{t\in[a,b]} \sqrt{\rho'(t)^2 + \rho(t)^2}\, dt$$

Esempio: spirale di Archimede

$$\rho(\vartheta) = a\cdot\vartheta \qquad \vartheta \in [0, b] \qquad \rho'(\vartheta) = a$$

$$\ell(\gamma) = \int_0^b \sqrt{a^2 + (a\vartheta)^2}\, d\vartheta = a\int_0^b \sqrt{1+\vartheta^2}\, d\vartheta = a\int_0^{b^*} \sqrt{\cosh^2 u}\cdot \cosh u\, du = *$$

$$\vartheta = \operatorname{senh} u \qquad d\vartheta = \cosh u\, du$$

$$\cosh^2 u - \operatorname{senh}^2 u = 1 \qquad 1+\vartheta^2 = \cosh^2 u$$

$$\vartheta = 0 \rightsquigarrow u = 0$$

$$\vartheta = b \rightsquigarrow u = \underline{\operatorname{settsenh} b} = b^*$$

$$u = \operatorname{settsenh} \vartheta$$

$$* = a\int_0^{b^*} \cosh^2 u\, du = \frac{a}{2}\int_0^{b^*} [1+\cosh(2u)]\, du = \frac{a}{2}\cdot\left(b^* + \frac{1}{2}\operatorname{senh}(2b^*)\right)$$

$$\cosh^2 u = \frac{1}{2}(1+\cosh 2u)$$

Cardioide

$\rho(\vartheta) = 1 + \cos\vartheta \qquad \vartheta \in [0, 2\pi]$ $\qquad \rho'(\vartheta) = -\operatorname{sen}\vartheta$

$$l(\gamma) = \int_0^{2\pi} \sqrt{\rho'(\vartheta)^2 + \rho(\vartheta)^2}\, d\vartheta = \int_0^{2\pi} \sqrt{\left[\operatorname{sen}^2\vartheta + \underbrace{(1+\cos\vartheta)^2}_{1+2\cos\vartheta+\cos^2\vartheta}\right]}\, d\vartheta = \int_0^{2\pi} \sqrt{(2 + 2\cos\vartheta)}\, d\vartheta = \sqrt{2}\int_0^{2\pi} \sqrt{\underbrace{(1+\cos\vartheta)}_{=[\ \ ]^2}}\, d\vartheta = *$$

$$\cos(2x) = \cos^2 x - \underbrace{\operatorname{sen}^2 x}_{=1-\cos^2 x} = 2\cos^2 x - 1$$

$$1 + \cos(2x) = 2\cos^2 x$$

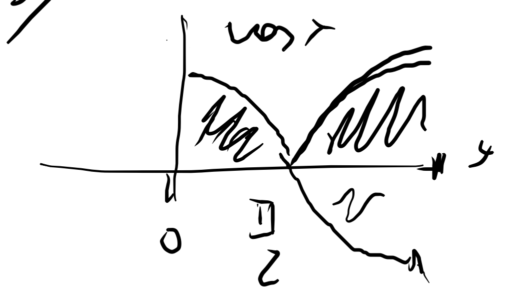
$\vartheta = 2x$

$$* = \sqrt{2}\int_0^{2\pi} \sqrt{2\cos^2(\vartheta/2)}\, d\vartheta = 2\int_0^{2\pi} |\cos(\vartheta/2)|\, d\vartheta = \odot$$

$\vartheta/2 = u \qquad d\vartheta = 2\,du$

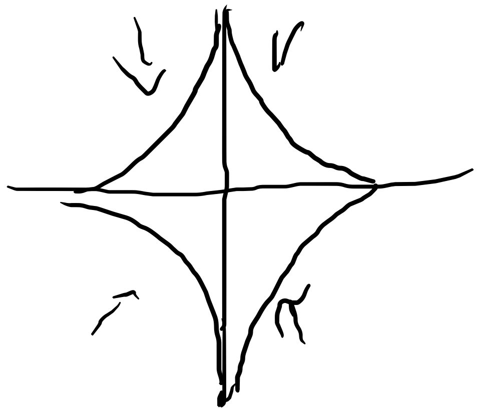
$$\odot = 4\int_0^{\pi} |\cos u|\, du = 8$$

$$2\int_0^{\pi/2} \cos u\, du$$



$$l(\gamma) = 8$$

Es: astroide



$\gamma(t) = (\cos^3 t, \ \text{sen}^3 t)^T$

γ non è regolare ma è regolare a tratti

$$\ell(\gamma) = 4 \cdot \ell(\gamma_1)$$

$\gamma_1 : [0, \frac{\pi}{2}] \to \mathbb{R}^2$ $\qquad \gamma'(t) = (-3 \,\text{sen}\, t \cos^2 t, \ 3 \cos t \,\text{sen}^2 t)^T$

$$\ell(\gamma) = 4 \int_0^{\pi/2} \sqrt{\underbrace{9 \,\text{sen}^2 t \cos^4 t + 9 \cos^2 t \,\text{sen}^4 t}_{\text{sen}^2 t \cos^2 t (\underbrace{\cos^2 t + \text{sen}^2 t}_{=1})}} \, dt = 12 \int_0^{\pi/2} \sqrt{\text{sen}^2 t \cos^2 t} \, dt = 12 \int_0^{\pi/2} \text{sen}\, t \cos t \, dt = *$$

$\text{sen}\, t \geq 0 \quad \cos t \geq 0$

$$* = 6 \cdot \left[\text{sen}^2 t \right]_0^{\pi/2} = 6$$

Superfici in $\mathbb{R}^3$

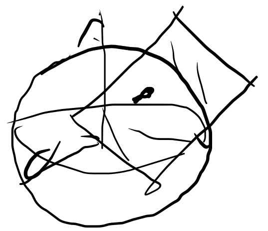
$K \subset \mathbb{R}^2$ dominio regolare a tratti, ∂K è una curva regolare a tratti $[K = A \cup \partial A]$

$\boxed{\sigma : K \to \mathbb{R}^3}$ C^1 ; σ si dice regolare se $\operatorname{rank} J\sigma(s,t) = 2 \quad \forall (s,t)^T \in K$

Si ha in ogni punto il piano tangente e il vettore normale $\boxed{\nu(s,t) = \frac{\partial \sigma}{\partial s}(s,t) \wedge \frac{\partial \sigma}{\partial t}(s,t)}$

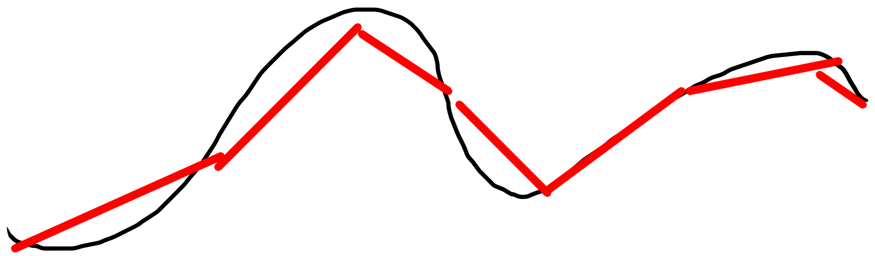
Equazione del piano tangente

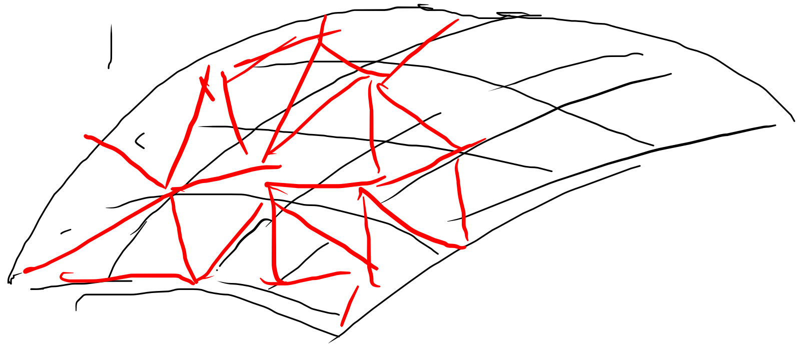
$$\nu(s,t) = (a, b, c)^T$$



$$\boxed{a(x-x_0) + b(y-y_0) + c(z-z_0) = 0}$$ eq. cartesiana

eq. parametrica $$\boxed{\pi(u,v) = \sigma(s_0,t_0) + u\frac{\partial \sigma}{\partial s}(s_0,t_0) + v\frac{\partial \sigma}{\partial t}(s_0,t_0)}$$ $(u,v)^T \in \mathbb{R}^2$





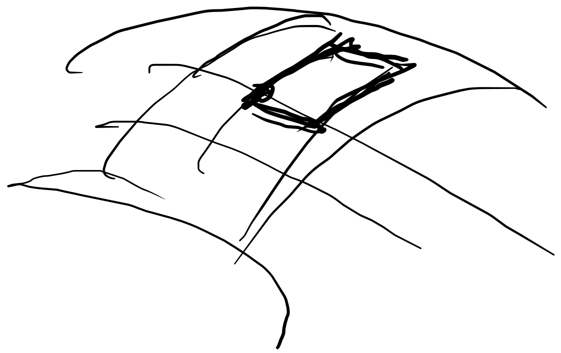
Area di una superficie:

$$\boxed{\text{Area} = \iint_K \| w(s,t) \| \, ds\, dt}$$

dove $\quad w(s,t) = \dfrac{\partial \sigma}{\partial s} \wedge \dfrac{\partial \sigma}{\partial t}(s,t)$

Area del parallelogramma

$= \| v_1 \wedge v_2 \|$



Integrale di superficie di un campo scalare

$f : A \to \mathbb{R} \qquad \Phi : K \subseteq \mathbb{R}^2 \to A \subseteq \mathbb{R}^3$

$$\boxed{\iint_{\Phi} f \, d\sigma = \iint_K f(\Phi(s,t)) \cdot \| w(s,t) \| \, ds\, dt}$$

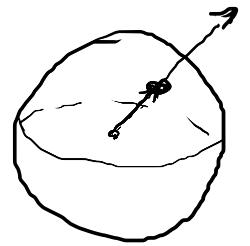
è solo un simbolo $\qquad w = \dfrac{\partial \Phi}{\partial s} \wedge \dfrac{\partial \Phi}{\partial t}$

$$\left(\int_{\gamma} f \, ds = \int_I f(\gamma(t)) \cdot \| \gamma'(t) \| \, dt \right)$$

$\cdot$ Area $= \iint_K \|w(s,t)\|\, ds\, dt$ $\qquad \iint_\Phi f\, d\sigma = \iint_K f(\Phi(s,t)) \cdot \|w(s,t)\|\, ds\, dt$ $\qquad w = \frac{\partial \Phi}{\partial s} \wedge \frac{\partial \Phi}{\partial t}$

Esempi: area della sfera $\quad \Psi(\varphi, \vartheta) = (R \operatorname{sen}\varphi \cos\vartheta,\ R \operatorname{sen}\varphi \operatorname{sen}\vartheta,\ R\cos\varphi)^T \quad \varphi \in [0,\pi] \quad \vartheta \in [0, 2\pi]$

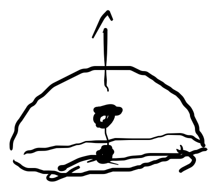
$\|w(\varphi,\vartheta)\| = R^2 \operatorname{sen}\varphi \qquad w(\varphi,\vartheta) = R^2 \operatorname{sen}\varphi\, (\ \ldots\ )^T$



$$\text{Area} = \int_0^\pi \left(\int_0^{2\pi} R^2 \operatorname{sen}\varphi\, d\vartheta \right) d\varphi = 2\pi \cdot R^2 \cdot 2 = 4\pi R^2$$

Massa superficiale, baricentro, momenti di inerzia

Es: baricentro della semisfera



$\hat{x} = \hat{y} = 0 \qquad \hat{z} = \frac{1}{M} \iint_\Psi z\, d\sigma$

$M = 2\pi R^2$

$$\iint_\Psi z\, d\sigma = \int_0^{\pi/2} \left(\int_0^{2\pi} R\cos\varphi \cdot R^2 \operatorname{sen}\varphi\, d\vartheta \right) d\varphi = 2\pi R^3 \cdot \int_0^{\pi/2} \operatorname{sen}\varphi \cos\varphi\, d\varphi =$$

$$= \pi R^3 \left[\frac{\operatorname{sen}^2\varphi}{2} \right]_0^{\pi/2} = \pi R^3 \qquad \hat{z} = \frac{\pi R^3}{2\pi R^2} = \frac{R}{2}$$

Superfici particolari

- grafici $g : K \subset \mathbb{R}^2 \to \mathbb{R}$

$$\phi(s,t) = (s, t, g(s,t))^T$$

$$\phi(x,y) = (x, y, g(x,y))^T$$

$$w = \frac{\partial \phi}{\partial s} \wedge \frac{\partial \phi}{\partial t}$$

$$J\phi = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \frac{\partial g}{\partial s} & \frac{\partial g}{\partial t} \end{pmatrix}$$

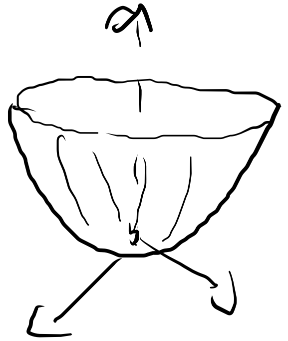
$$w(s,t) = \left(-\frac{\partial g}{\partial t}, -\frac{\partial g}{\partial t}, 1 \right)^T \qquad \|w\| = \sqrt{\overbrace{\frac{\partial g}{\partial t}^2 + \frac{\partial g}{\partial s}^2}^{\|\nabla g\|^2} + 1}$$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & \frac{\partial g}{\partial s} \\ 0 & 1 & \frac{\partial g}{\partial t} \end{vmatrix}$$

$$\boxed{\text{Area} = \iint_K \sqrt{1 + \|\nabla g(x,y)\|^2}\, dx\, dy}$$

Es. $g(x,y) = x^2 + y^2$

$g : B(0, R) \to \mathbb{R}$



Baricentro $\hat{x} = \hat{y} = 0$

$$\hat{z} = \boxed{\frac{1}{M}} \cdot \iint_\phi z \, d\sigma$$

Area $\quad g(x,y) = \underbrace{x^2+y^2} \qquad \nabla g(x,y) = 2\,(x,y)^T \qquad \|\nabla g\|^2 = 4\,(x^2+y^2)$

$$\text{Area} = \iint_{B(0,R)} \sqrt{1+4(x^2+y^2)}\;dx\,dy = \int_0^R \left(\int_0^{2\pi} \sqrt{1+4\rho^2}\cdot \rho\, d\vartheta \right) d\rho =$$

coordinate polari $x = \rho\cos\vartheta$, $y = \rho\sin\vartheta$

$$= 2\pi \cdot \left[(1+4\rho^2)^{3/2} \cdot \frac{1}{12} \right]_0^R = \frac{\pi}{6}\left((1+4R^2)^{3/2} - 1 \right)$$

$\frac{3}{2}(1+4\rho^2)\cdot \cancel{8}^{4}\rho$

$z = g(x,y)$

$$\hat{z} = \frac{1}{\text{Area}} \cdot \iint_{\Phi} z\, d\sigma = \frac{1}{\text{Area}} \iint_{B(0,R)} \underbrace{(x^2+y^2)}_{\rho^2} \cdot \sqrt{1+4(x^2+y^2)}\; \underbrace{dx\,dy}_{\rho\, d\rho\, d\vartheta} =$$

$$= \frac{1}{\text{Area}} \cdot \int_0^R \left(\int_0^{2\pi} \rho^3 \cdot \underbrace{(1+4\rho^2)^{1/2}}\; d\rho \right) d\vartheta = \frac{2\pi}{\text{Area}} \int_0^R \cdots = *$$

$u = (1+4\rho^2)^{1/2} \qquad du = \frac{1}{2}\,\frac{1}{(1+4\rho^2)^{1/2}} \cdot 8\rho\, d\rho = \frac{1}{(1+4\rho^2)^{1/2}}\cdot 4\rho\, d\rho$

$u^2 = 1+4\rho^2 \qquad \rho^2 = \frac{u^2-1}{4}$

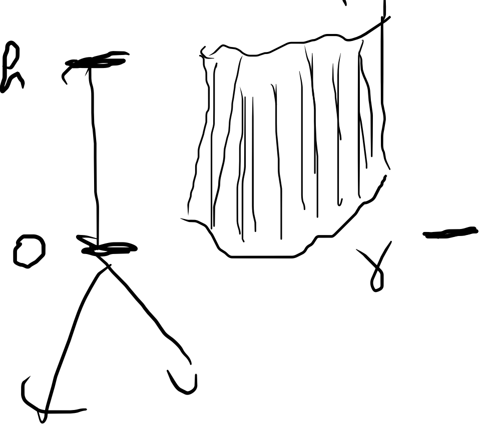
$$* = \frac{2\pi}{\text{Area}} \int_0^R \rho^2 \cdot (1+4\rho^2)^{1/2}\, \frac{(1+4\rho^2)^{1/2}}{4} \boxed{\frac{1}{(1+4\rho^2)^{1/2}} \cdot 4\rho\, d\rho} = \frac{2\pi}{\text{Area}} \int_1^{\sqrt{1+4R^2}} \frac{u^2-1}{4} \cdot \frac{u^2}{4}\, du = \circledast$$

$\rho = 0 \rightsquigarrow u = 1$

$\rho = R \rightsquigarrow u = \sqrt{1+4R^2}$

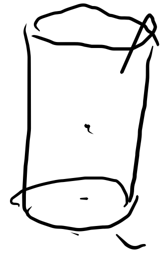
$$\circledast = \frac{\pi}{8\,\text{Area}} \cdot \int_1^{\sqrt{1+4R^2}} (u^4 - u^2)\, du = \frac{\pi}{8\,\text{Area}} \cdot \left[\frac{1}{5}u^5 - \frac{1}{3}u^3 \right]_1^{\sqrt{1+4R^2}}$$

• Superficie cilindrica



$$\gamma = \gamma(t) = (x(t), y(t))^T \quad t \in [a,b]$$

$$0 \le z \le h$$

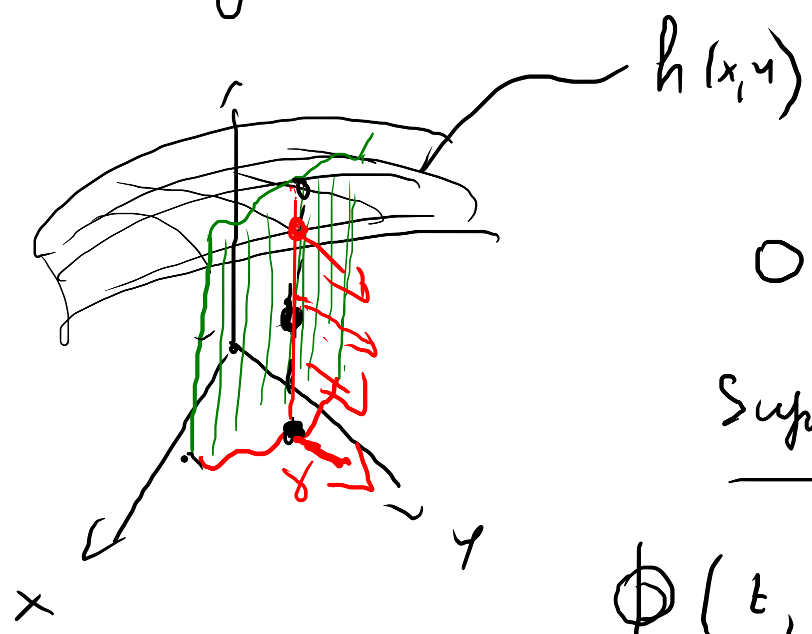


$2\pi r \, h$

Area laterale del cilindro di altezza h e punti con $(x,y)^T$ sulla curva γ

$$= \ell(\gamma) \cdot h$$

$$\begin{matrix} i & j & k \\ x' & y' & 0 \\ 0 & 0 & 1 \end{matrix}$$



$$0 \le z \le h(x,y)$$

Superficie cilindrica $\{ (x(t), y(t), z)^T : t \in [a,b], \; 0 \le z \le h(x(t), y(t)) \}$

$$\Phi(t, z) = (\underbrace{x(t), y(t)}_{\gamma(t)}, z)^T \qquad J\Phi(t,z) = \begin{pmatrix} x'(t) & 0 \\ y'(t) & 0 \\ 0 & 1 \end{pmatrix} \qquad \boxed{W(t,z) = (y'(t), -x'(t), 0)^T}$$

$$\|w(t,z)\| = \sqrt{y'(t)^2 + (-x'(t))^2 + 0} = \sqrt{x'^2 + y'^2} = \underline{\|\gamma'(t)\|}$$

La norma del vettore normale della superficie è uguale alla norma del vettore tangente della curva

$$\underline{\text{Area superficie}} = \iint_K \|w(t,z)\|\, dt\, dz = \int_a^b \left(\int_0^{h(\gamma(t))} \underline{\|\gamma'(t)\|}\, dz \right) dt = *$$

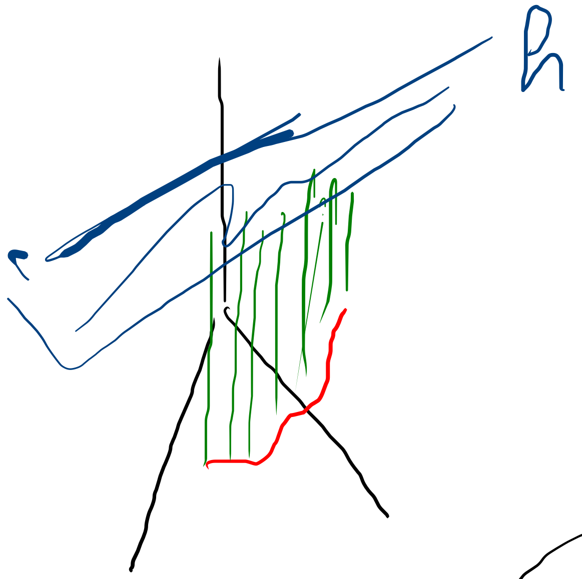
$$K = \{(t,z)^T : t \in [a,b],\ 0 \leq z \leq h(\underbrace{x(t), y(t)}_{\gamma(t)})\}$$

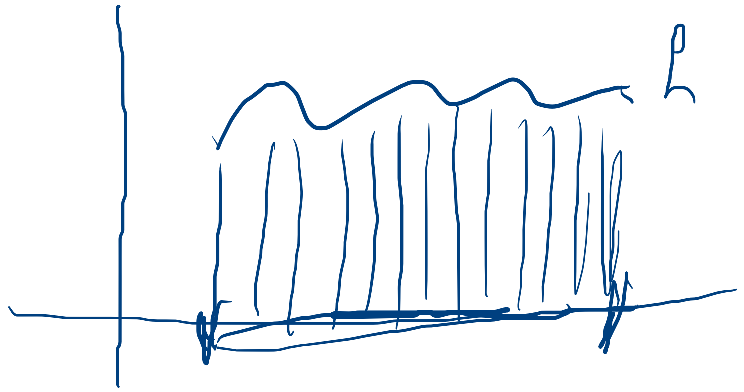
è un dominio normale rispetto all'asse t

$$0 \leq z \leq \underline{(h \circ \gamma)(t)}$$

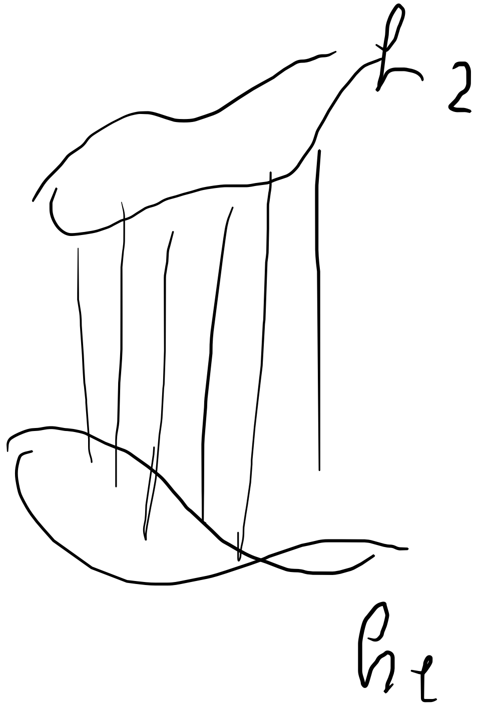
$$* = \int_a^b h(\gamma(t)) \cdot \|\gamma'(t)\|\, dt = \int_\gamma h\, ds$$

$= ?$





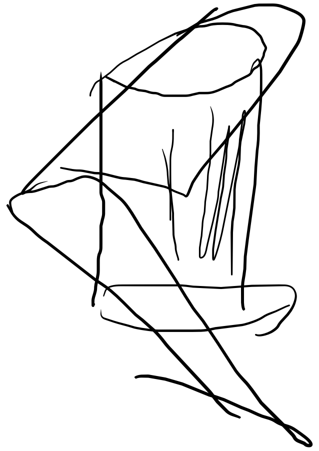
In generale

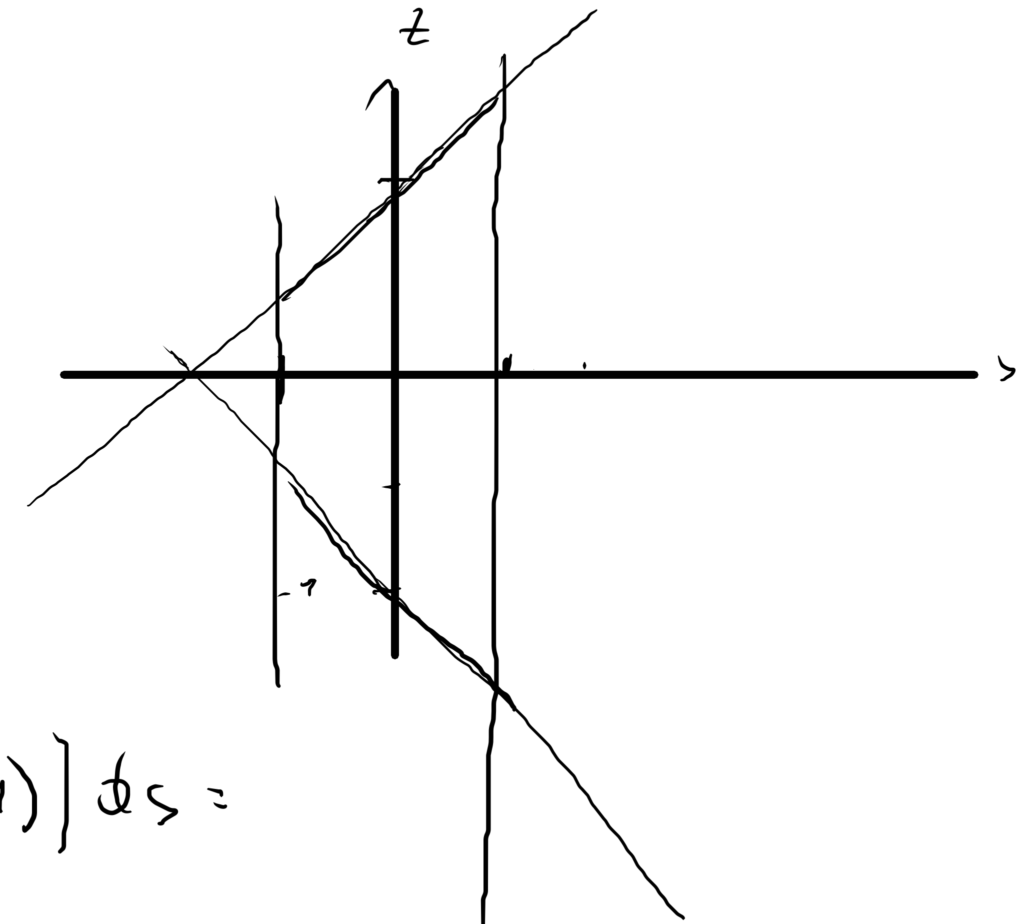


$$\{(x,y,z)^T \in \mathbb{R}^3 : \quad (x,y)^T = \gamma(t), \; t \in [a,b], \; h_1(x,y) \le z \le h_2(x,y)\}$$

$$\boxed{\text{Area} = \int_\gamma (h_2 - h_1)\, ds}$$

$$\begin{cases} x^2+y^2=1 \\ z = 2+x+y \\ z = -2-x-y \end{cases}$$





$y=0$

$x^2=1$

$z=2+x$

$z=-2-x$

Area laterale

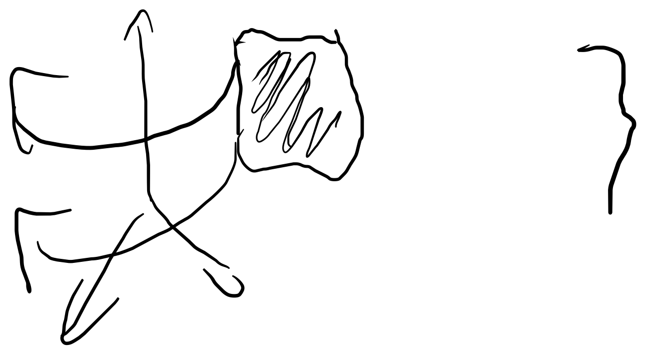
$$\int_\gamma (h_2-h_1)\,ds = \int_\gamma \left[(2+x+y)-(-2-x-y)\right] ds =$$

$$= 2\int_\gamma (2+x+y)\,ds = 2\int_0^{2\pi} (2+\cos t+\operatorname{sen} t)\,dt = 8\pi$$

$\gamma(t) = (\cos t, \operatorname{sen} t)^T$

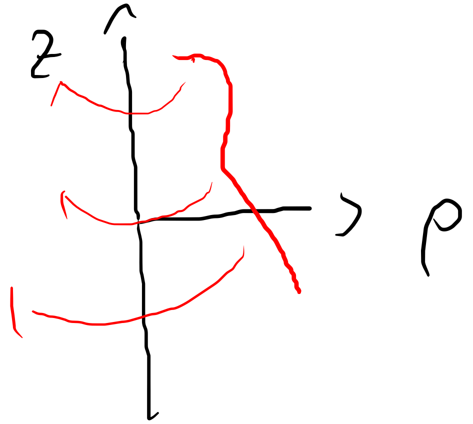
$\|\gamma'(t)\| = 1$

- Superfici di rotazione $\quad \gamma : [a,b] \to \mathbb{R}^2$



Sia $\gamma(t) = (\rho(t), z(t))^T$ una curva nel piano ρz con $\rho(t) \geq 0 \quad t \in [a,b]$

Ruotando la curva γ attorno all'asse z otteniamo la superficie



$$\begin{cases} x = \rho(t) \cos\vartheta \\ y = \rho(t) \operatorname{sen}\vartheta \\ z = z(t) \end{cases}$$

$$\Phi : [a,b] \times [0, 2\pi] \to \mathbb{R}^3$$

$$\Phi(t,\vartheta) = (\rho(t)\cos\vartheta, \rho(t)\operatorname{sen}\vartheta, z(t))^T$$

$$J(\Phi)^T(t,\vartheta) = \begin{array}{c} \begin{array}{ccc} i & j & k \end{array} \\ \begin{pmatrix} \rho'(t)\cos\vartheta & \rho'(t)\operatorname{sen}\vartheta & z'(t) \\ -\rho(t)\operatorname{sen}\vartheta & \rho(t)\cos\vartheta & 0 \end{pmatrix} \end{array}$$

$$N(t,\vartheta) = \begin{pmatrix} -\rho(t)z'(t)\cos\vartheta \\ -\rho(t)z'(t)\operatorname{sen}\vartheta \\ \rho'(t)\rho(t) \end{pmatrix}$$

$$\|w(t,\vartheta)\| = \sqrt{\rho(t)^2 z'(t)^2 \cos^2\vartheta + \rho(t)^2 z'(t)^2 \operatorname{sen}^2\vartheta + \rho'(t)^2 \rho(t)^2} =$$

$$= \rho(t) \underbrace{\sqrt{z'(t)^2 + \rho'(t)^2}}_{= \|\gamma'(t)\|}$$

Area della superficie $= \displaystyle\iint_{[a,b]\times[0,2\pi]} \rho(t)\, \|\gamma'(t)\|\, dt\, d\vartheta = \int_0^{2\pi} \left(\int_a^b \rho(t) \|\gamma'(t)\|\, dt \right) d\vartheta$

$$= 2\pi \cdot \int_a^b \rho(t)\, \|\gamma'(t)\|\, dt = \ell(\gamma) \cdot \hat{x}$$

ascissa del baricentro

$$\frac{1}{\ell(\gamma)} \int_\gamma x\, ds$$

$$\int_a^b x(t)\, \|\gamma'(t)\|\, dt = \boxed{\int_\gamma x\, ds}$$

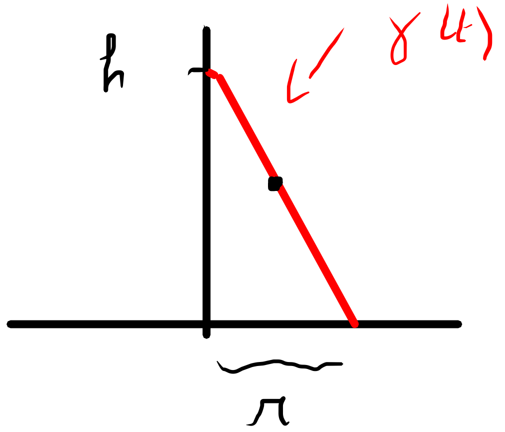
Teorema di Pappo-Guldino per le aree

Sia $\Phi: K \to \mathbb{R}^3$ una superficie di rotazione ottenuta ruotando attorno all'asse z di un angolo $\alpha \in \left]0, 2\pi\right]$ una curva regolare a tratti $\gamma: I \to \mathbb{R}^2$

$\gamma(t) = (x(t), z(t))^T$ con $x(t) \geq 0$. Allora l'area di Φ è data da

$$\text{Area} = \alpha \cdot \ell(\gamma) \cdot \hat{x}$$ dove $\hat{x}$ è l'ascissa del baricentro di γ.

<u>Esempi</u>: Area laterale del cono:



$$\text{Area} = 2\pi \cdot \ell(\gamma) \cdot \hat{x}$$

$$\ell(\gamma) = \sqrt{r^2 + h^2}$$

$$\hat{x} = \frac{1}{2} r$$

$$A = \cancel{2}\pi \cdot \sqrt{r^2 + h^2} \cdot \frac{1}{\cancel{2}} r = \pi r \sqrt{h^2 + r^2}$$

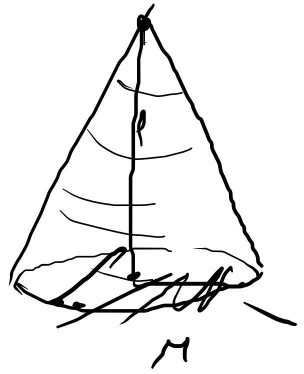
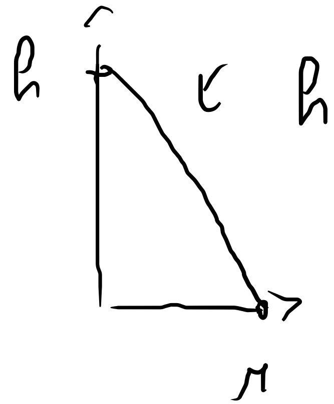


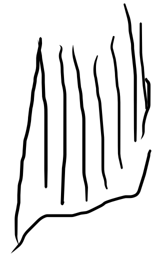
grafico di $f(x,y) = \underbrace{h - \frac{h}{r}\sqrt{x^2+y^2}}$

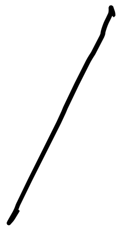
$f : B(0,r) \to \mathbb{R}$



$\leftarrow h - \frac{h}{r} \cdot x$

$$\nabla f = \left(-\frac{h}{r} \frac{x}{\sqrt{x^2+y^2}} , -\frac{h}{r} \frac{y}{\sqrt{x^2+y^2}} \right)^T$$

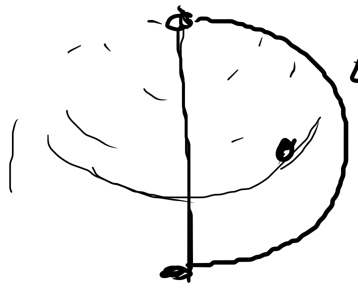




$$\text{Area} = \iint_{B(0,r)} \sqrt{1 + \|\nabla f\|^2}\, dx\,dy = \iint_{B(0,r)} \sqrt{1 + \frac{h^2}{r^2} \cdot \cancel{\frac{x^2+y^2}{x^2+y^2}}}\, dx\,dy$$

$= 1$

$$= \sqrt{1 + \frac{h^2}{r^2}} \cdot \pi r^2 = \pi r \sqrt{r^2+h^2}$$



$\leftarrow \gamma(t) = (R\cos t, R \operatorname{sen} t)^T$

$t \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$

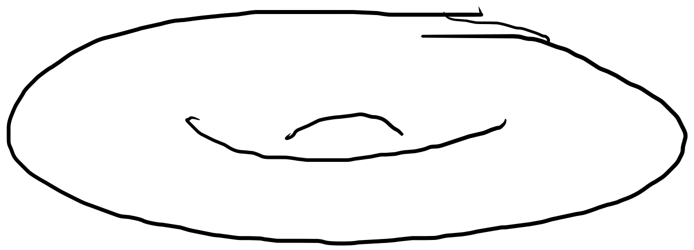
$\frac{1}{r}\sqrt{r^2+h^2}$

$$4\pi R^2 = \text{Area} = 2\pi \cdot \ell(\gamma) \cdot \hat{x} = 2\pi \cdot \pi r \cdot \hat{x} = 2\pi^2 R \hat{x}$$

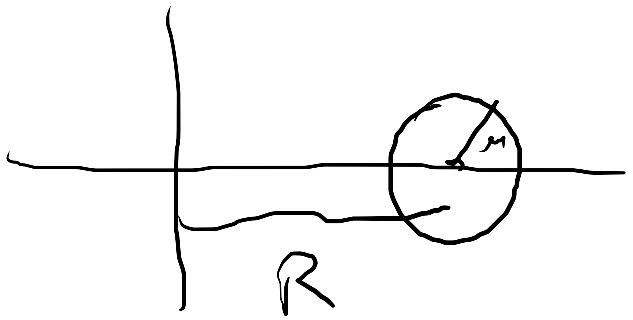
$$\hat{x} = \frac{\cancel{4}^{2}\pi R^2}{\cancel{2}\pi^2 R} = \cancel{2\pi R}$$

$$\frac{2R}{\pi}$$

Toro



$$A = 2\pi \cdot \underset{\substack{\parallel \\ 2\pi r}}{l(\gamma)} \cdot \underset{R}{\hat{x}} = \underbrace{4\pi^2 r R}$$

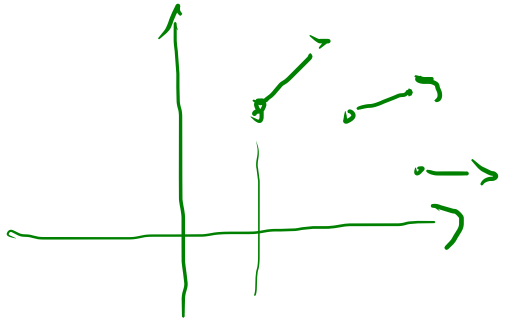


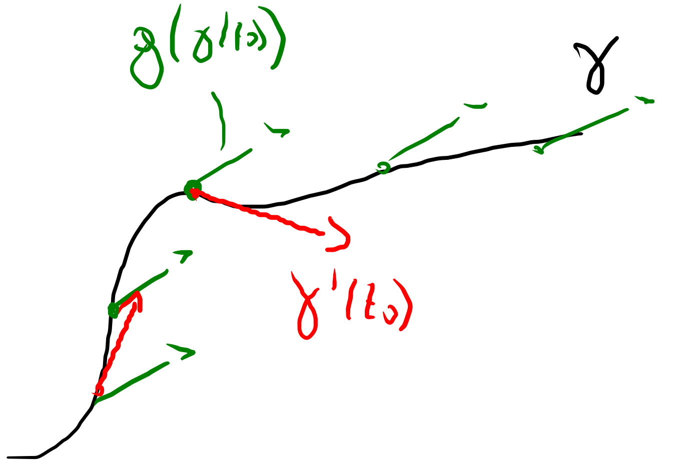
Campi vettoriali

$$g : \mathbb{R}^2 \to \mathbb{R}^2 \qquad g : \mathbb{R}^3 \to \mathbb{R}^3$$

$\gamma : I \to \mathbb{R}^2$

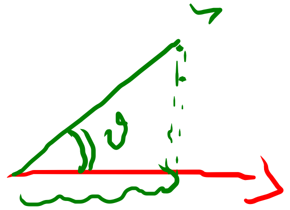
$g : \mathbb{R}^2 \to \mathbb{R}^2$





$(x,y)^T \qquad g(x,y) = (g_1(x,y), g_2(x,y))^T$

la componente del campo g lungo la curva (cioè la proiezione sulla tangente)



$$\int \langle g(\gamma(t)), \gamma'(t) \rangle$$

Sia $g : A \subseteq \mathbb{R}^n \to \mathbb{R}^n$ $\qquad \gamma : I \to A$ regolare (a tratti)

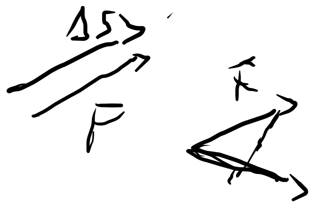
Diciamo integrale della componente tangenziale del campo g lungo γ

l'integrale

$$\boxed{\int_\gamma \langle g, \tau \rangle \, ds := \int_I \langle g(\gamma(t)), \gamma'(t) \rangle \, dt}$$

"elemento di lunghezza d'arco"

versore tangente della curva $\quad \tau(t) = \dfrac{\gamma'(t)}{\|\gamma'(t)\|}$



$L = F \cdot \Delta s$

$\langle F, \Delta s \rangle$

$\cancel{\int \langle F}$

$\int g \cdot ds$

Significato fisico: il lavoro compiuto dal campo g sulla particella che si muove lungo γ

Notazione come forma differenziale

$N=2$ $\quad \boxed{\int_\gamma \langle g, \tau \rangle \, ds} = \int_I \langle g(\gamma(t)), \gamma'(t) \rangle \, dt = *$

$g(x,y) = \left(X(x,y), Y(x,y) \right)^T \qquad \gamma(t) = (x(t), y(t))^T \qquad \gamma'(t) = (x'(t), y'(t))^T$

$$* = \int_I \left\langle \left(X(x(t), y(t)), Y(x(t), y(t)) \right)^T, (x'(t), y'(t))^T \right\rangle dt =$$

$$= \int_I \left[X(x(t), y(t)) \, x'(t) + Y(x(t), y(t)) \, y'(t) \right] dt$$

$x'(t) = \frac{dx}{dt} \qquad y'(t) = \frac{dy}{dt}$

$x'(t) \, dt = dx \qquad y'(t) \, dt = dy$

$$\boxed{= \int_\gamma X \, dx + Y \, dy} \qquad + Z \, dz$$

$X dx + Y dy = \omega$

g γ

$$\boxed{\int g \cdot ds}$$ $$\int g \cdot \tau \, ds$$

$$g \cdot \Delta s \rightsquigarrow \int_\gamma g \cdot ds$$