Solutions

November 9, 2020

Exercise 1. Which of the following parabolas do intersect the x-axis?

$a.\ y = (x-1)(2x+1)$ $\qquad$ $b.\ y = x^2 - 4x - 21$

$c.\ y = x^2 - (1+\sqrt{2})x + \sqrt{2}$ $\qquad$ $d.\ y = \dfrac{8-x^2}{4} + \dfrac{x(x-1)}{3} - 1$

Solutions

$a.$ It intersects the x-axis, at $x_1 = -\frac{1}{2}, x_2 = 1$ $\qquad$ $b.$ It intersects the x-axis, at $x_1 = -3, x_2 = 7$

$c.$ It intersects the x-axis, at $x_1 = 1, x_2 = \sqrt{2}$ $\qquad$ $d.$ It does not intersect the x-axis

Exercise 2. Solve the following equations, which can be transformed in suitable second-degree equations.

$a.\ 3x^4 - 1 = 0$ $\qquad$ $b.\ x^6 - 7x^3 - 8 = 0$

$c.\ x^6 - 3x^3 + 2 = 0$ $\qquad$ $d.\ 3x^4 - 4x^2 - 4 = 0$

$e.\ x^4 - 6x^2 + 8 = 0$ $\qquad$ $f.\ x^6 - 7x^3 + 12 = 0$

Solutions

$a.\ \mathcal{S} = \{\mp\frac{1}{\sqrt[4]{3}}\}$ $\qquad$ $b.\ \mathcal{S} = \{-1, 2\}$

$c.\ \mathcal{S} = \{1, \sqrt[3]{2}\}$ $\qquad$ $d.\ \mathcal{S} = \{\mp\sqrt{2}\}$

$e.\ \mathcal{S} = \{-2, -\sqrt{2}, \sqrt{2}, 2\}$ $\qquad$ $f.\ \mathcal{S} = \{\sqrt[3]{3}, \sqrt[3]{4}\}$

Exercise 3. Solve the following rational equations.

$a.\ \dfrac{x}{x+1} = \dfrac{1}{x} - 1$ $\qquad$ $b.\ \dfrac{10}{x^2+1} = 6 - x^2$

$c.\ \dfrac{8}{x+6} + \dfrac{12-x}{x-6} = 1$ $\qquad$ $d.\ \dfrac{2x+2}{x-1} + \dfrac{3x-2}{x} = 1$

$e.\ \dfrac{2x-1}{x+2} = \dfrac{x+1}{x^2-4} + \dfrac{1}{5x-10}$ $\qquad$ $f.\ \dfrac{2}{x^2-x+1} - \dfrac{1}{x+1} = \dfrac{2x-1}{x^3+1}$

Solutions

$a.\ \mathcal{S} = \{\mp\frac{1}{\sqrt{2}}\}$ $\qquad$ $b.\ \mathcal{S} = \{-2, -1, 1, 2\}$

$c.\ \mathcal{S} = \{-3, 10\}$ $\qquad$ $d.\ \mathcal{S} = \emptyset$

$e.\ \mathcal{S} = \{\frac{1}{10}, 3\}$ $\qquad$ $f.\ \mathcal{S} = \{2\}$

Exercise 4. Solve the following inequalities, which can be transformed in suitable second-degree inequalities.

$a.\ -9x^2 + 12x - 4 \geq 0$ $\qquad$ $b.\ 2(3x+5)(8-x) < 0$

$c.\ -x^4 + 7x^2 - 12 > 0$ $\qquad$ $d.\ x^4 - 2x^2 - 8 \leq 0$

Solutions

$a.\ \mathcal{S} = \{\frac{2}{3}\}$

$b.\ \mathcal{S} =]-\infty, -\frac{5}{3}[\ \cup\ ]8, +\infty[$

$c.\ \mathcal{S} =]-2, -\sqrt{3}[\ \cup\ ]\sqrt{3}, 2[$

$d.\ \mathcal{S} = [-2, 2]$

Exercise 5. Solve the following inequalities.

$a.\ (x^2 + 2x - 8)(x + 1) > 0$

$b.\ (x^2 - 2)(x + 1)(1 - x) \geq 0$

$c.\ x(x^2 + 2)(2x - 1) < 0$

$d.\ \dfrac{x + 1}{2x - 1} > 1$

$e.\ \dfrac{2x - 8}{1 - x - x^2} > 0$

$f.\ \dfrac{x^2 - x - 2}{x^2 - 3x} \leq 0$

Solutions

$a.\ \mathcal{S} =]-4, -1[\ \cup\ ]2, +\infty[$

$b.\ \mathcal{S} = [-\sqrt{2}, -1] \cup [1, \sqrt{2}]$

$c.\ \mathcal{S} =]0, \frac{1}{2}[$

$d.\ \mathcal{S} =]\frac{1}{2}, 2[$

$e.\ \mathcal{S} =]-\infty, -\frac{1+\sqrt{5}}{2}[\ \cup\ ]\frac{\sqrt{5}-1}{2}, 4[$

$f.\ \mathcal{S} = [-1, 0[\ \cup\ [2, 3[$

Exercise 6. Solve the following inequalities.

$a.\ \sqrt{3x - 7} > 3$

$b.\ x > \sqrt{x + 2}$

$c.\ \sqrt{x^2 - 4x + 2} > x + 3$

$d.\ \sqrt{1 + x} + \sqrt{1 - x} > 1$

$e.\ \sqrt{x - 1} - \sqrt{x - 2} \geq 2$

$f.\ x < \sqrt{|x^2 - 1|}$

Solutions

$a.\ \mathcal{S} =]\frac{16}{3}, +\infty[$

$b.\ \mathcal{S} =]2, +\infty[$

$c.\ \mathcal{S} =]-\infty, -\frac{7}{10}[$

$d.\ \mathcal{S} = [-1, 1]$

$e.\ \mathcal{S} = \emptyset$

$f.\ \mathcal{S} =]-\infty, \frac{1}{\sqrt{2}}[$