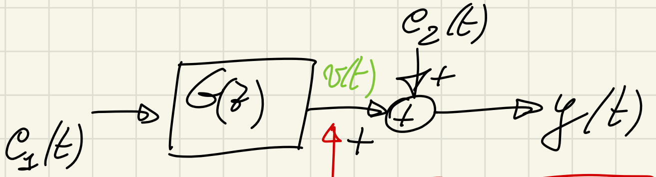
(1)



$e_2(\cdot)$ proc stazionario

a regime

proc stazio AR

$$G(z) = \frac{z}{z - \frac{1}{4}}$$

$e_1(\cdot) \sim WN(0,1)$

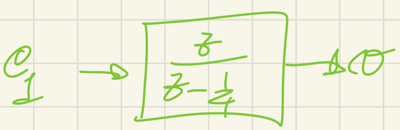
$E[y(t)] = ?$ $\qquad \Gamma_{yy}(\omega) = ?$

(a) $e_2(t) \equiv 0 \quad \forall t$

(b) $e_2(\cdot) \sim WN(0,1)$ $\qquad e_2, e_1$ INDIPENDENTI

(c) $e_2(t) \equiv e_1(t) \quad \forall t$

$$y(t) = \frac{1}{4} y(t-1) + e_1(t) + e_2(t)$$



$$v(t) = \frac{z}{z - \frac{1}{4}} e_1(t)$$

$$v(t) = \frac{1}{1 - \frac{1}{4} z^{-1}} e_1(t)$$

$$\left(1 - \frac{1}{4} z^{-1}\right) v(t) = e_1(t)$$

$v_1 \quad \text{+} \quad E(v_1) = \mu_1 \quad \sigma^2_{v_1} = \lambda^2_1$

$v_2 = v_1 + c$ $\quad c = \text{cost}$

$\text{var}(c) = 0$

$E(v_2) = E(v_1) + c \quad \sigma^2_{v_2} = E\left[\left(v_2 - \bar{v}_2\right)^2\right]$

$= \bar{v}_2 \qquad = \sigma^2_{v_1}$

$\left(1 - \frac{1}{4} z^{-1}\right) v(t) = e_1(t)$

$v(t) - \frac{1}{4} v(t-1) = e_1(t)$

$v(t) = \frac{1}{4} v(t-1) + e_1(t) \qquad AR(1)$

$y(t) = v(t) + e_2(t)$

$\underset{\sim}{y}(t) = \underset{\sim}{\frac{1}{4} y}(t-1) + e_1(t) + e_2(t)$

(a) $e_2(t) \equiv 0$ $\qquad y(t) = \frac{1}{4} y(t-1) + e_1(t) + 0$

$\Longrightarrow AR(1)$ a regime $\Longleftarrow$ $\frac{1}{1-\frac{1}{4}z^{-1}} = \frac{z}{z-\frac{1}{4}}$

$E[y(t)] = 0$

$E[y(t)] = \bar{y}$

$E\{ \quad \}$ $\qquad \bar{y} = \frac{1}{4}\bar{y} + \underbrace{E(e_1)}_{\neq 0} \Rightarrow \bar{y} = 0$

$$\Gamma_y(\omega) = |G(e^{j\omega})|^2 \cdot \lambda_{e_1}^2 =$$

$\lambda_{e_1}^2 = 1$

$$= \frac{|e^{j\omega}|^2}{|e^{j\omega} - \frac{1}{4}|^2} \cdot 1 = \frac{1}{|\cos\omega - \frac{1}{4} + j\operatorname{sen}\omega|^2}$$

$$= \frac{16}{17 - 8\cos\omega} \qquad \omega \in [-\pi \quad \pi]$$

$$\phi_y(z) = W(z)\,W(z^{-1})\,\phi_e(z)$$

$$= G(z)\,G(z^{-1})\,\sigma^2_{e_1}$$

$$\Gamma_y(\omega) = \phi_y(z)\Big|_{z=e^{j\omega}}$$

b $\quad e_2(t) \sim WN(0,1) \qquad e_1, e_2$ INDIP.

$$y_b(t) = y_a(t) + e_2(t)$$

$$E[y_b(t)] = \underbrace{E[y_a(t)]}_{=0} + \underbrace{E[e_2(t)]}_{=0} = 0$$

$y_a(t) + e_2(t) \longrightarrow e_1, e_2$ IND.

dipende da $e_1(t)$ e da $e_1(t-1), e_1(t-2)$ . . . . .

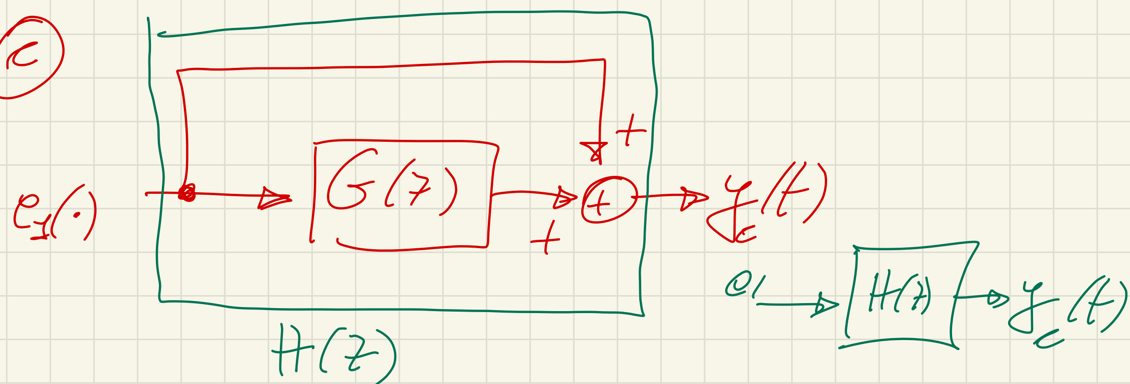
$$\mathrm{var}(y_b) = \mathrm{var}(y_a) + \mathrm{var}(e_2)$$

$y_a, \; e_2$ IND.

$$\Gamma_{y_b}(\omega) = \Gamma_{y_a}(\omega) + \Gamma_{e_2}(\omega) = \frac{16}{17 - 8\cos\omega} + 1 = \frac{33 - 8\cos\omega}{17 - 8\cos\omega}$$

$\omega \in [-\pi ; \pi]$

c



$$H(z) = 1 + \frac{z}{z - \frac{1}{4}} = \frac{2 - \frac{1}{4} z^{-1}}{1 - \frac{1}{4} z^{-1}}$$ ARMA(1,1)

$$y_c(t) = \frac{1}{4} y_c(t-1) + 2 e_1(t) - \frac{1}{4} e_1(t-1)$$

$$H(z) = \frac{2 - \frac{1}{4} z^{-1}}{1 - \frac{1}{4} z^{-1}} \quad \begin{matrix} \leftarrow & z_1 = +\frac{1}{8} \\ \leftarrow & p_1 = \frac{1}{4} \end{matrix}$$

$$W(z) = \frac{1 - \frac{1}{8} z^{-1}}{1 - \frac{1}{4} z^{-1}} \qquad e_1 \to \frac{e_1}{\alpha}$$

$$\left[\frac{1}{2} H(z)\right] \qquad \left[\alpha H(z)\right] \phi_{\bar{e}} = \quad \cancel{\phi_{e_1}} \frac{1}{\alpha^2}$$